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Optimal Control of Deterministic Chaos

Doctoral thesis

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RESUMÉ

Hlavním cílem disertační práce je ukázat, že výkonný nástroj, jakými jsou zcela určitě evoluční algoritmy, je možno v praxi použít k optimalizaci řízení deterministického chaosu. Tato práce je především zaměřena na vysvětlení jak správně použít evoluční algoritmy, jak nadefinovat účelovou funkci a dále je zaměřena na výběr vhodné řídící metody a samozřejmě na vysvětlení všech možných problémů, které mohou nastat v tak obtížné úloze, jakou je řízení chaosu.

Nejdříve jsou zde popsány nejběžnější a zároveň nejpoužívanější metody řízení chaosu – Linearizace Poincarého mapy (OGY metoda), metoda zpožděné zpětné vazby (Pyragasova metoda) a zmíněny jsou mnohé další, jež jsou často využívány v mnoha vědeckých a výzkumných pracích. Druhá zmíněná, Pyragasova metoda byla zvolena jako vhodná pro otestování optimalizace řízení chaosu a byla použita při experimentech v rámci této práce.

Další část této práce je zaměřena na popis nejznámějších příkladů chaotických systémů, jednak diskrétních (Logistická rovnice, Henonova mapa), které jsou použity jako modely chaotického systému při optimalizaci řízení chaosu v této práci, a jednak stručně jsou popsány spojité systémy, kde mezi nejznámější patří Lorenzův a Rösslerův systém.

Následující a největší část je zaměřena na popis dosažených výsledku optimalizace řízení chaosu. Skládá se ze sedmi samostatných případových studií. Každá z nich je zaměřena na testování návrhu účelové funkce, která byla použita ve veškerých optimalizacích v rámci dané případové studie. Získané výsledky jsou vždy průběžně popsány a diskutovány v závěru každé případové studie.

Tato práce se zabývá zkoumáním optimalizace řízení chaosu za pomoci evolučních algoritmů a návrhu účelové funkce, jež by měla zajistit nalezení optimálních výsledků, které by vedly ke zlepšení chování systému a rychlému dosažení žádaného stavu. Řízení probíhá za pomocí dvou Pyragasových metod – TDAS a rozšířené ETDAS verze. Jako model chaotického systému byla zvolena jedno-dimenzionální logistická rovnice a dvou-dimenzionální Henonova mapa. Jako evoluční algoritmy byly použity tyto: algoritmus SOMA (Samo-Organizační-Migrační-Algoritmus) ve čtyřech verzích a Diferenciální Evoluce (DE) v šesti verzích. Pro každou verzi byly simulace opakovány několikrát, aby se ukázala a prověřila účinnost a robustnost použité metody.

Na konci praktické části jsou všechny dosažené výsledky porovnány mezi sebou a taktéž již v jednotlivých případových studiích jsou uvedeny dílčí shrnutí výsledků, přičemž srovnání s klasickou řídící technikou – OGY je v této práci též uvedeno, a to na konci případové studie č.1.

Na základě získaných výsledků je možno tvrdit, že všechny simulace podaly velmi uspokojivé výsledky, a také že evoluční algoritmy jsou schopné řešit i tak složitý problém, a především, že kvalita výsledků nezávisí jen na problému, který je řešen, ale je extrémně závislá na správné definici účelové funkce, výběru řídící techniky, či nastavení samotného evolučního algoritmu.

ABSTRACT

The main aim of this dissertation is to show that powerful optimizing tools like evolutionary algorithms can be in reality used for the optimization of deterministic chaos control. This work is aimed on explanation of how to use evolutionary algorithms (EAs) and how to properly define the cost function (CF). It is also focused on selection of control method and, the explanation of all possible problems with optimization which comes together in such a difficult task, which is chaos control.

Firstly, the most common and used chaos control methods are described – Linearization of Poincaré Map (OGY method), Time – Delayed Feedback (Pyragas method) and many others which are often used in many variations in research works are mentioned. The second one (Pyragas Method) was chosen as a suitable method for successful optimization and is used in this study.

The next part is focused on the description of the most known examples of chaotic systems, discrete – time systems (Logistic equation, Henon map), which are also used in this thesis as a model of chaotic systems used in optimization of chaos control; and also briefly is focused on time – continuous systems (Lorenz system, Rossler system).

The following and the biggest part describes the results of optimization of chaos control. It consists of seven case studies and each one is aimed on testing the proposal of cost function used for optimizations within this case study. The obtained results are continuously described in partial conclusions at the end of each case study.

This work deals with an investigation on the optimization of the control of chaos by means of EA and constructing of the cost function securing the improvement of system behavior and faster stabilization to desired periodic orbits. The control law is based on two Pyragas methods: Delay feedback control – TDAS and Extended delay feedback control – ETDAS. As models of deterministic chaotic systems, one dimensional Logistic equation and two dimensional Henon map were used. The evolutionary algorithm SOMA (Self-Organizing Migrating Algorithm) was used in four versions and Differential Evolution (DE) in six versions. For each version, simulations were repeated several times to show and check robustness of used method. At the end of this work, the results of optimized chaos control for each case study are compared

and also the comparison with classical control technique – OGY is presented at the end of case study 1.

From the obtained results, it is possible to say that all simulations gave satisfactory results and thus evolutionary algorithms are capable of solving this class of difficult problems and the quality of results does not depend only on the problem being solved, but they are extremely sensitive on the proper definition of the CF, selection of control method or parameter settings of evolutionary algorithms.

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NOMENCLATURE

List of Abbrevations

UPO	Unstable Periodic Orbit
OGY	Ott-Grebogi-York method for controlling Chaos
TDAS	Time Delay Auto Synchronization
ETDAS	Extended Time Delay Auto Synchronization
EA	Evolutionary Algorithm
CF	Cost Function
CFE	Cost Function Evaluation
NA	Version of Cost function, also mentioned as Non-Auto
DE	Differential Evolution
SOMA	Self Organizing Migrating Algorithm
LQ	Logistic Equation – one dimensional chaotic system
ATO	All To One strategy of SOMA, all individuals search in the direction to Leader
ATR	All To one Rand strategy of SOMA, all individuals search in the direction to one randomly selected
ATA	All To All strategy of SOMA, all individual search in the direction to all individuals
ATAA	All To All Adaptive strategy of SOMA, all individuals search in the direction to all by means of adaptive way
DERand1Bin	Version of DE
DERand2Bin	Version of DE
DEBest2Bin	Version of DE
DELocalToBest	Version of DE
DERand1DIter	Version of DE
DEBest1JIter	Version of DE

List of symbols

t	time
x	space variable
y	space variable
R	weight constant in ETDAS method
K	weight constant in TDAS/ETDAS method
F_{max}	Limitation of perturbation in TDAS/ETDAS method
n	step in iteration
τ	time – delay in TDAS/ETDAS method
τ_i	simulation interval
τ_s	short time interval
r, a, b	constants
PopSize	Number of individuals in population
CR	Control parameter of DE, crossover constant
F	Control parameter of DE, mutable constant
Generations	Stopping parameter of DE, number of loops in all evolution
Migrations	Stopping parameter of SOMA, number of migration loops
MinDiv	Stopping parameter of SOMA, minimal accepted error between the best and worst individual in population
NP	Number of individuals in population
PathLength	Control parameter of SOMA; it determines the stopping position of the movement of an individual
PRTVector	Vector of zeros and ones, it interacts with the movement of an individual
Step	Control parameter of SOMA, length of step of an individual during search
i, j	indexes
IStab	Number of iterations required for stabilization of system

AbgIStab Average number of iterations required for stabilization of system

CFVal Cost Function Value

AvgCFVal Average Cost Function Value for EA version runs.

1 INTRODUCTION AND STATE OF ART

The chaos theory together with quintals theory and universal relativity theory are most considerable products of physics in the 20th century. Latest of these theories is chaos theory.

In mathematics and physics, chaos theory deals with the certain nonlinear dynamical systems that under certain conditions, the phenomenon known as chaos, exhibits in their behavior. This is most famously characterized by sensitivity to initial conditions. Examples of such systems can be seen in almost every science disciplines include the atmosphere, the solar system, turbulent fluids, economies, and population growth.

Systems that exhibit mathematical chaos are deterministic, and thus orderly in some sense, which suggests that the use of the word chaos is at odds with common meaning, which suggests complete disorder. When we say that chaos theory studies deterministic systems, it means that these systems are exactly given by the system of mathematic relations and in spite of it, exhibits chaotic behavior.

Since the early 1990's, many methods for control of chaos [1] have been developed and based on the original OGY control method [2]. But there is generally one big disadvantage of OGY and it is the long initial chaotic transient before trajectories are stabilized. Consequently many targeting algorithms were introduced [3 - 11] to shorten the time of stabilization. Unlike the OGY the Pyragas's delayed feedback control technique [12], [13] can be simply considered as targeting and stabilizing algorithm together in one package. From the point of view of soft computing and optimizations another big advantage of Pyragas method is the amount of accessible control parameters, which are set up by using a priori knowledge or mathematical analysis. This is very advantageous for successful use of optimization of parameters set up by means of EA, leading to improvement of system behavior and better and faster stabilization to the desired periodic orbits.

These days the evolutionary algorithms (EA) are known as powerful tool for almost any difficult and complex optimization problem. But the quality of optimization process results mostly depends on proper design of used cost function, especially when the EAs are used for optimization of chaos control. It is well known that deterministic chaos in general and also any technique to control of chaos are sensitive to parameter set up, initial conditions and in the case of optimization they are also extremely sensitive to the construction of used cost function.

The main aim of the dissertation will be focused on the examples of EA implementation to methods for chaos control for the purpose of obtaining better results, which means faster reaching of desired stable state and superior stabilization, which could be robust and effective to optimize difficult problems in the world. Some research in this field has been recently done using the evolutionary algorithms for optimization of local control of chaos based on a Lyapunov approach [14], [15]. But the approach described here is unique and novel and up to date were not used or mentioned anywhere. We use EA to search for optimal setting of adjustable parameters of arbitrary control method to reach desired state or behavior of chaotic system.

The work is divided in the eight main numbered chapters.

The first chapter gives the overview of the research area of chaos control, whereas *the second chapter* formulates the main goals of this dissertation.

The following section, *number three* is focused on the description and theoretical knowledge about the most common and used chaos control methods.

The next part *number four* is focused on description of the most known examples of chaotic systems, discrete – time systems, which are also used in this thesis; and also briefly is focused on time – continuous systems.

The *fifth chapter* deals with the description of evolutionary algorithms used in the work.

The sixth chapter is the biggest one from the main part of the work (except Appendix) and shows the results of optimization of chaos control. This chapter consists of seven case studies and at the end all results are compared and discussed

The seventh part gives the brief discussion of the main obtained optimization results from all case studies and conclusion of the achieved goals in this work together with an outlook for the future research.

The last part, Appendix contains complete overview of all simulation results.

2 DISSERTATION GOALS

In this dissertation I would like to show how evolutionary algorithms can be used in the challenging task of optimization of deterministic chaos control. The main aim of the dissertation will be focused on the investigation of EA implementation to methods for chaos control in order to reach better results from the point of view of speed and quality of stabilization (i.e. control process).

As can be seen from the results and conclusions of all presented case studies, evolution algorithms were able to find optimal solution for selected control technique Thus to avoid complicated mathematical analysis of chaotic system due to the finding of settings for control method.

In my further work, I would like to continue with research in this field of optimization of chaos control by means of evolutionary algorithms. The dissertation goals could be summarized into following points.

- To prove that EAs are able to find optimal solution in case of chaos control.
- To test several examples of chaotic systems (one and higher dimensional).
- To test a stabilization for various states (stable state – a fixed point) or higher dimensional periodic orbits.
- To compare the results between different EAs
- To try a various designs of cost functions and compare their performance (faster stabilization – targeting to close neighborhood of desired UPO, solving the small problems described in the individual case study).

THEORETICAL FRAMEWORK

3 METHODS FOR CONTROLLING OF CHAOS

In general, methods for control of chaos deal with a process wherein a tiny perturbation is applied to a chaotic system in order to realize a desirable chaotic, periodic or stationary behavior. The problems of control of chaos has attracted the attention of researchers and engineers since the early 1990's.

The idea of chaos control was enunciated at the beginning of the last decade at the University of Maryland. The main principle consisted in waiting for a natural passage of the chaotic orbit close to the desired periodic behavior and then applying a small reasonably chosen perturbation, in order to stabilize such periodic dynamics.

Here is given a list of the most important and often used methods.

3.1 Linearization of Poincaré Map (OGY method)

This control method is in general based on waiting for the entering of attractor into close neighborhood of desired fixed point, or arbitrary Unstable Periodic Orbit – UPO, and thereafter applying a tiny perturbation – p to the system. The size of this perturbation is strictly limited and can be applied only within the small neighborhood due to very limited validity of the linearization [13], [16 - 17].

The fixed point has one stable direction $\vec{e}_s$ with eigenvalue λ_s and one unstable direction $\vec{e}_u$ with eigenvalue λ_u. The OGY scheme for control is easily understood pictorially. Figure 3.1 depicts the unstable fixed point $\vec{x}_f$ with its stable and unstable eigenvectors, and also the path g of the fixed point for small changes in p given by (3.1) [18].

$$\vec{g} = \frac{\partial \vec{x}_f}{\partial p} \tag{3.1}$$

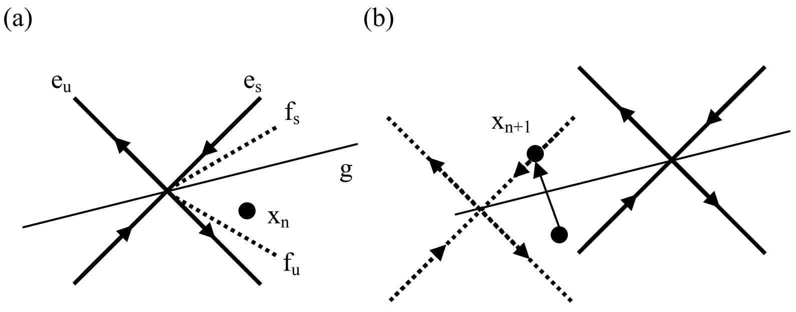


Figure 3.1: The OGY approach to control.

Figure 3.1 (a) shows the unstable fixed point, stable and unstable eigenvectors, $\vec{e}_s$ and $\vec{e}_u$, and their adjoint vectors, $\vec{f}_s$ and $\vec{f}_u$. The dotted line is the path g of the fixed point as the control parameter is varied. Figure 3.1 (b) shows the control process. Shifting the fixed point to the new (solid) position allows x_{n+1} to be directed to the stable manifold of the unshifted (dashed) fixed point.

If an iteration point $\vec{x}_n$ comes close to $\vec{x}_f$ i.e. $\vec{x}_n = \vec{x}_f + \delta\vec{x}_n$ with $\delta\vec{x}_n$ being small (close neighborhood), the parameter p is changed $p = p_0 + \delta p_n$ After the next iteration $\vec{x}_{n+1}$ lies along the stable direction of $\vec{x}_f$. The *n+1* iteration can be calculated by linearization about the moved position of the fixed point (3.2).

$$\delta\vec{x}_{n+1} - \delta p_n \vec{g} = \left(\lambda_u \vec{e}_u \vec{f}_u + \lambda_s \vec{e}_s \vec{f}_s\right) \cdot \left(\delta\vec{x}_n - \delta p_n \vec{g}\right) \tag{3.2}$$

And $\vec{f}_u$ and $\vec{f}_s$ as the unit adjoint eigenvectors are given by (3.3):

$$\vec{f}_u \cdot \vec{e}_s = \vec{f}_s \cdot \vec{e}_u = 0 \tag{3.3}$$

The condition for $\vec{x}_{n+1}$ to lie along the stable direction of $\vec{x}_f$ is then $\partial\vec{x}_{n+1} \cdot \vec{f}_u = 0$ which together with (3.2) and (3.3) gives the final solution for the perturbation *p*.

$$\delta p_n = \frac{\lambda_u}{\lambda_u - 1} \frac{\delta\vec{x}_n \cdot \vec{f}_u}{\vec{g} \cdot \vec{f}_u} \tag{3.4}$$

This is very simple to implement if the analytic knowledge of the map function *F* is known. The control is done in these three stages:

1. Find the desired UPO
2. Calculate the stable and unstable directions at each component of UPO
3. When the trajectory reaches the close neighborhood of the desired UPO, calculate the perturbations in each iteration and take the possible limitation into account.

There also exist more variations of this control technique from the simplest version used for controlling to the fixed point in one-dimensional maps, to higher dimensional control. And of course there exist special versions which uses an element from general linear control theory – the pole placement [19], [20] or the version, which was developed for the purpose of shortening the initial waiting passage and should secure faster targeting to close neighborhood of desired UPO - the targeting OGY, also called SOGY version [10], [11].

3.2 Delayed Feedback Control (Pyragas Method)

This is a method developed to stabilize UPO by means of applying small time- continuous control (perturbation) to a system parameter, while it evolves in continuous time. This is the main difference from OGY method which is suitable for a discrete control at the points of Poincaré map (attractor's crossing of a surface). It is also known as the Time Delayed Auto Synchronization (TDAS method) and it was proven that it is very easy to implement and is effective for the less order UPOs, i.e. orbits with smaller periods. This is one of the most important limitations for this technique. There also exists the discrete version suitable for control the chaos within chaotic maps [21].

It is assumed the system *P* is described by variables *x* with *F* as an external controllable parameter, which has numerical value *F* = 0 in the absence of control (external perturbation) (3.5).

$$\frac{dx}{dt} = P(x) + F(t) \tag{3.5}$$

Desired UPO of period τ which fulfills the following logical condition $x(t+\tau) = x(t)$ can be stabilized by means of delayed feedback control by calculating and applying control parameter *F* to the system based on following control law (3.6).

$$F(t) = K\left[x(t-\tau) - x(t)\right] \tag{3.6}$$

where: the parameter *K* represents the strength of the perturbation. By proper choice of the value of K, the desired UPO may be stabilized. The big advantage of this method lies in the fact, that there is no need of additional information about UPO except its period τ or only its order in case of discrete-time control.

Once the control is achieved, the size of the perturbation is very small, although during the previous chaotic transient passage it may be very large and of have to be limited. But this kind of absence of perturbation can lead to either low quality stabilization or none, especially in case of higher periodic orbits.

Due to this problem, the extended version of delayed feedback control method was developed to solve it. (Also called ETDAS – Extended Time Delayed Auto Synchronization) (3.7) [22]

$$\begin{aligned} \frac{dx}{dt} &= P(x) + F(t) \\ F(t) &= K\left[(1-R)S(t-\tau) - x(t)\right] \\ S(t) &= x(t) + RS(t-\tau) \end{aligned} \tag{3.7}$$

where: R is adjustable constant and S is given by a delay equation utilizing previous states of the system.

This modification particularly solved the problems with stabilization of higher order UPOs in discrete or continuous time systems.

This method is very simple and can be applicable to a wide variety of systems, of course it is possible to use it for discrete-time systems. There are only small changes in the form of equations 3.6 and 3.7. The discrete-time version of TDAS method has the following form (3.8):

$$\begin{aligned} x_{n+1} &= P(x_n) + F_n \\ F_n &= K\left[x_{n-m} - x_n\right] \end{aligned} \tag{3.8}$$

The discrete-time version of ETDAS method has form (3.9):

$$\begin{aligned} x_{n+1} &= P(x_n) + F_n \\ F_n &= K\left[(1-R)S_{n-m} - x_n\right] \\ S_n &= x_n + RS_{n-m} \end{aligned} \tag{3.9}$$

Where the symbol m represents the order of desired UPO.

3.3 Others Methods

During recent years, a lot of other control techniques have been developed. Some of them are based on classical linear control law - Open loop and Open plus loop [17], [23], or they represent new approaches in linear/nonlinear deterministic chaos control such as adaptive nonlinear control [24 - 27], impulsive control [28 - 30], sliding modes technique [31], using of neural network controller [32] or back-stepping design [33 - 34].

4 CHAOTIC SYSTEMS

Here is the description of the most used and investigated examples of chaotic systems divided into two groups – discrete and continuous-time systems.

4.1 Discrete-Time Systems

4.1.1 Logistic Equation

The logistic equation (logistic map) is a one-dimensional discrete-time example of how complex chaotic behavior can arise from very simple non-linear dynamical equation. This chaotic system was introduced and popularized by the biologist Robert May [35]. It was originally introduced as a demographic model by Pierre François Verhulst as a typical predator – prey relationship. Mathematical notation is given by (4.10) [36]:

$$x_{n+1} = rx_n(1 - x_n) \tag{4.10}$$

where (in case of biological meaning) x_n is the population at year n, and r is a positive number, which represents a special parameter - combination of rate for reproduction and starvation.

The chaotic behavior can be observed by varying the parameter r. At $r = 3.57$ is the beginning of chaos, at the end of the period-doubling behavior. At $r > 3.57$ the system exhibit chaotic behavior

All of this behavior can be clearly seen from bifurcation diagram (Figure 4.1). The horizontal axis shows the values of the parameter r while the vertical axis shows the possible long-term values of x.

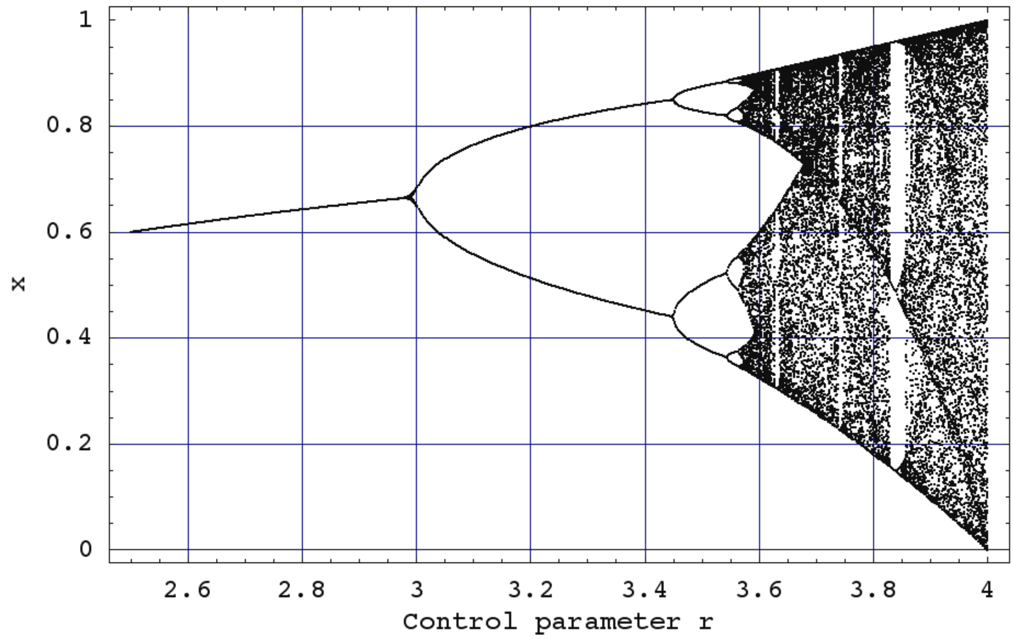


Figure 4.1 Bifurcation diagram of Logistic equation

4.1.2 Henon Map

This is a model invented with a mathematical motivation to investigate chaos. The Henon map is a discrete-time dynamical system, which was introduced as a simplified model of the Poincaré map for the Lorenz system. It is one of the most studied examples of dynamical systems that exhibit chaotic behavior and in fact it is also a two-dimensional extension of the one-dimensional quadratic map.

Mathematical notation is given by (4.11) [36]:

$$x_{n+1} = 1 + y_n - a x_n^2$$
$$y_{n+1} = b x_n \tag{4.11}$$

The map depends on two parameters, a and b, which for the canonical Henon map have values of $a = 1.4$ and $b = 0.3$. For the canonical values the

Henon map is chaotic. For other values of *a* and *b* the map may be chaotic, intermittent, or converge to a periodic orbit.

As a dynamical system, the canonical Henon map is interesting because, unlike the logistic map, its orbits defy a simple description (they are also called strange attractors).

Figure 4.2 shows the bifurcation diagram for the Henon map created by plotting of variable *x* as a function of the one control parameter for the fixed second parameter.

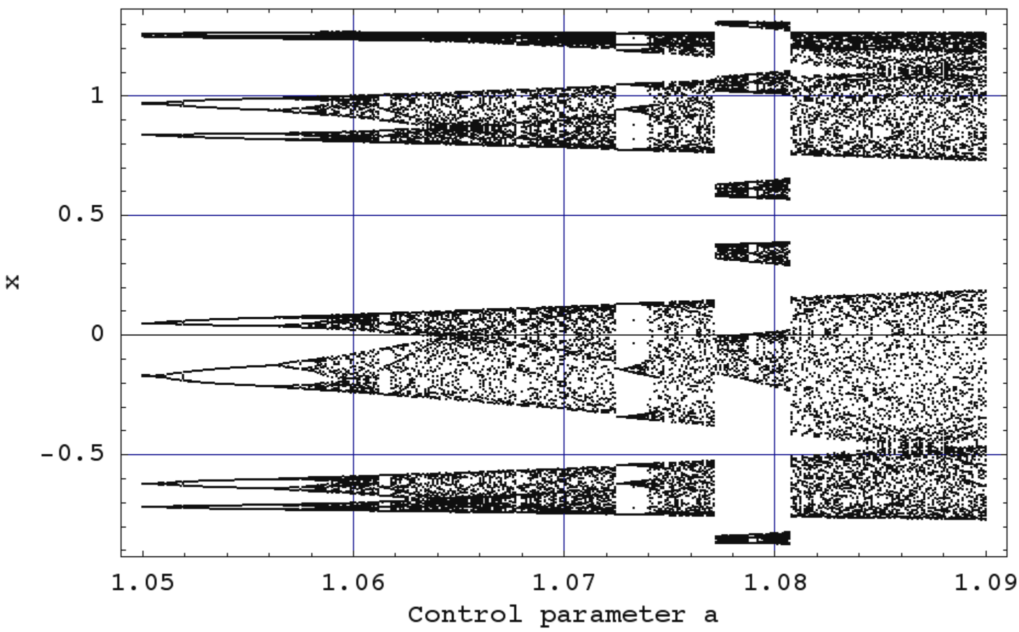


Figure 4.2 Bifurcation diagram of Henon map

4.2 Continuous-Time Systems

4.2.1 Lorenz system

The Lorenz system is a 3-dimensional dynamical flow which exhibits chaotic behavior.

It was introduced by Edward Lorenz in 1963, who derived it from the simplified equations of convection rolls arising in the equations of the atmosphere. It is a very simple model of the dynamics of a fluid heated from below in the gravitation field, thus it was first used to study a problem of weather predictability. The system also arises in lasers, dynamos, and specific waterwheels.

The equations which describes the Lorenz system are given in (4.12) [36]:

$$\begin{aligned} \frac{dx}{dt} &= \sigma(y-x) \\ \frac{dy}{dt} &= x(\rho-z)-y \\ \frac{dz}{dt} &= xy-\beta z \end{aligned} \tag{4.12}$$

where σ is called the Prandtl number and ρ is called the Rayleigh number. The Lorenz attractor is depicted on Figure 4.3.

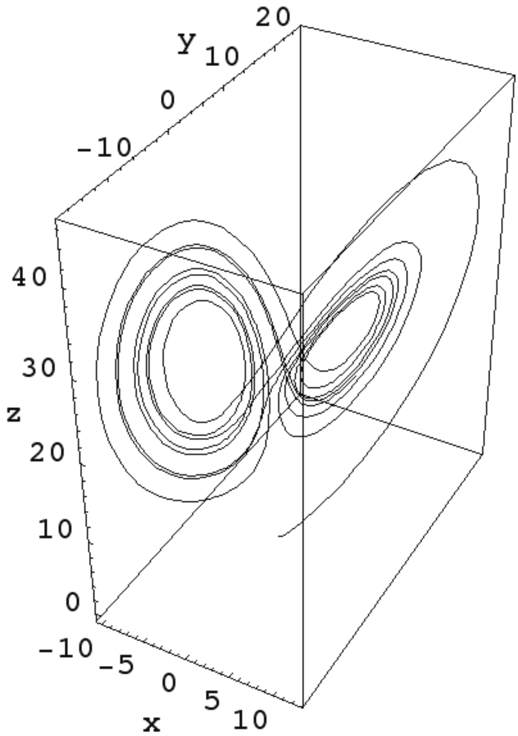


Figure 4.3 Lorenz attractor

4.2.2 Rössler system

The Rössler system is a system of three non-linear ordinary differential equations. These differential equations define a continuous-time dynamical system that exhibits chaotic dynamics associated with the fractal properties of the attractor. It was originally introduced as an example of very simple chaotic flow containing chaos, in fact it was intended to behave similarly to the Lorenz attractor, but also be easier to analyze qualitatively. This attractor has some similarities to the Lorenz attractor, but is simpler and has only one manifold. The attractor was designed in 1976, but the originally theoretical equations were later found to be useful in modeling equilibrium in chemical reactions. The Rössler system is given by following set of equations (4.13) [36]:

$$\frac{dx}{dt} = -y - z$$
$$\frac{dy}{dt} = x + ay \qquad (4.13)$$
$$\frac{dz}{dt} = b + z(x - c)$$

Rössler studied the chaotic attractor with a = 0.2, b = 0.2, and c = 5.7, which shows a simple chaotic attractor with the trajectory rotating around a fixed point. Together with increasing of value a chaotic behavior and period doubling in the attractor is achieved. Figure 4.4 shows the Rössler attractor.

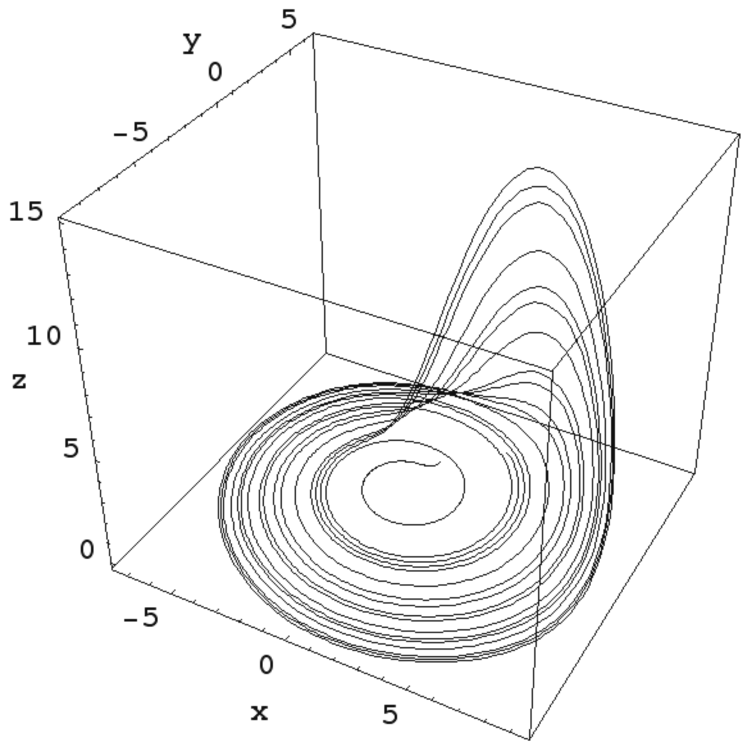


Figure 4.4 Rössler attractor

5 EVOLUTIONARY ALGORITHMS

Here, an overview is given of the algorithms, which were used in all simulations in this dissertation.

5.1 Self Organizing Migrating Algorithm (SOMA)

SOMA works with groups of individuals (population) whose behavior can be described as a competitive – cooperative strategy. The construction of a new population of individuals is not based on evolutionary principles (two parents produce offspring) but on the behavior of social group, e.g. a herd of animals looking for food [37]. This algorithm can be classified as an algorithm of a social environment. To the same group of algorithms particle swarm algorithm can also be put in, sometimes called swarm intelligence. In the case of SOMA, no velocity vector works as in particle swarm algorithm, only the position of individuals in the search space is changed during one generation, here called 'Migration loop'.

The rules are as follows: In every migration loop, the best individual is chosen, i.e. individual with the minimum cost value, which is called "Leader". An active individual from the population moves in the direction towards Leader in the search space. At the end of the movement, the position of the individual with minimum cost value is chosen. If the cost value of the new position is better than the cost value of an individual from the old population, the new individual appears in new population. Otherwise the old one remains for the next migration loop. The graphical explanation of movement can be seen on Figure 5.1.

There exist four versions of SOMA – AllToOne (ATO), AllToOneRand (ATR), AllToAll (ATA), AllToAllAdaptive (ATAA). [38]

All of them are used in this work despite the fact that the versions AllToAll and AllToAllAdaptive are much better in searching. They can search a wider area of solutions and the possibility of finding the global optimum is then more probable. On the other hand, these two variations of SOMA need more time for its successful end of evolution. Thus the combination of properties of versions ATO / ATR (fast searching) and ATA / ATAA (thorough searching in CF surface) was used with success in this work, where the CF surfaces are very complex and erratic (See Appendix, section 8.1 – 8.6).

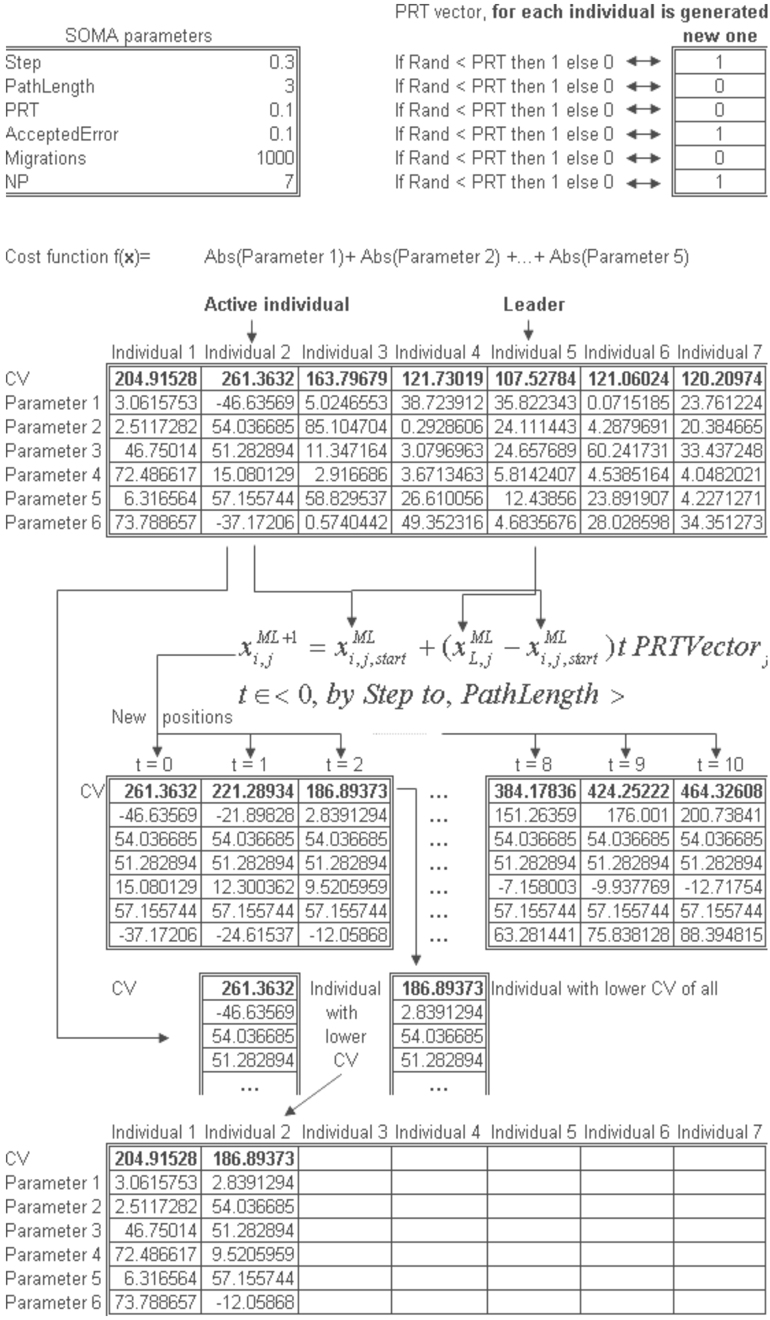


Figure 5.1 Principle of SOMA

5.2 Differential Evolution (DE)

DE works also with a population of individuals but there is one exception compared to other evolution algorithms. Four parents are used to produce offspring, not only two parents as the norm. Three individuals are randomly chosen with the active one as the current solution from the population. Then one of randomly chosen parents is subtracted from the second parent and so called differential vector is produced. This one multiplies by a mutable constant and the result of this operation is a weighted differential vector. The third parent plus the weighted differential vector gives a noise vector. After that, trial vector is created where there appears some arguments from the active individual and some from the noise vector. The probability of selection depends on a crossover constant *CR*. All these actions are repeated in each generation to find the best solution. [39]. The graphical explanation of this algorithm can be seen on Figure 5.2.

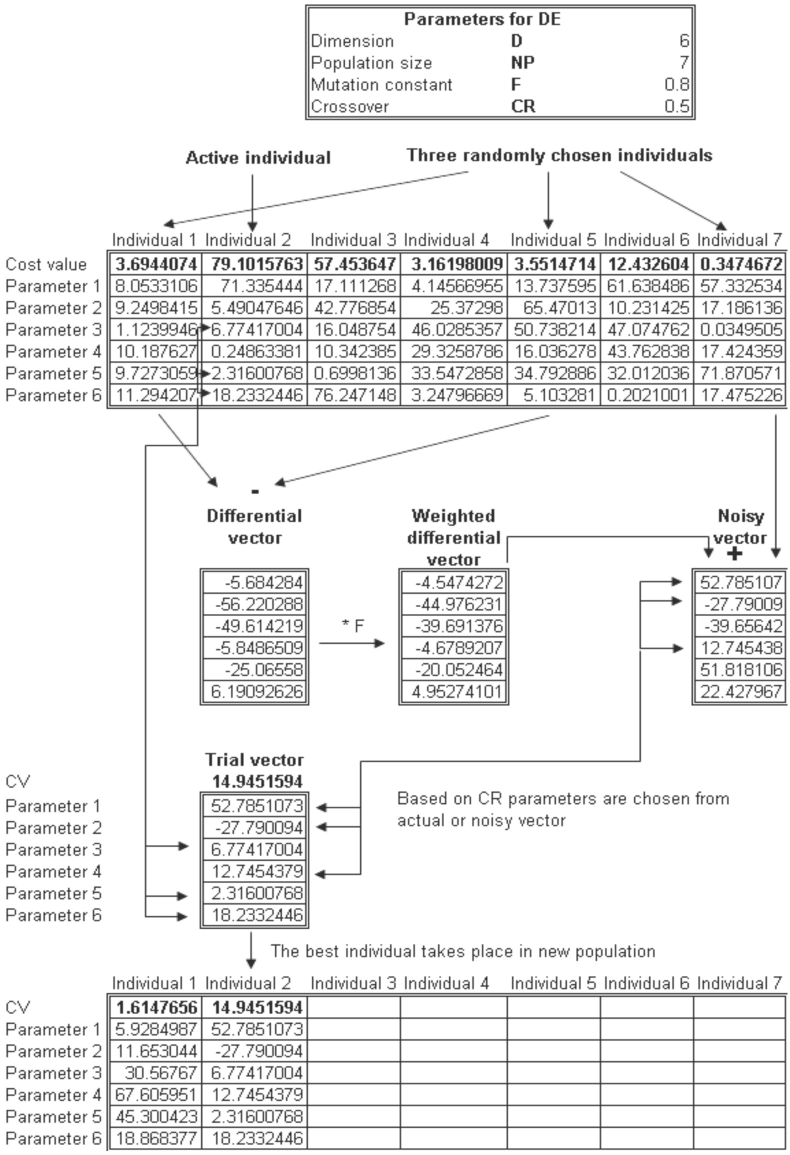


Figure 5.2 Principle of DE

EXPERIMENTAL PART

6 OPTIMIZATION OF CHAOS CONTROL

6.1 Problem design

This chapter primarily consists of seven case studies. All of them are focused on estimation of accessible control parameters for TDAS or EDTAS method for seven proposed Cost Functions used in optimizations to stabilize selected UPOs, which are the following: p-1 (a fixed point), p-2 and p-4 (examples of higher periodic orbits). The chosen examples of chaotic systems were one dimensional Logistic equation in the form (4.10) [36] and two dimensional Henon map in the form (4.11) [36]. Here is the list of desired UPOs:.

Logistic Equation with $r = 3.8$:

p-1 (fixed point): $x_F = 0.73842$

p-2 orbit: $x_1 = 0.3737$, $x_2 = 0.8894$

p-4 orbit: $x_1 = 0.3038$, $x_2 = 0.8037$, $x_3 = -0.5995$, $x_4 = 0.9124$

Henon Map with $a = 1.2$ and $b = 0.3$:

p-1 (fixed point): $x_F = 0.8$

p-2 orbit: $x_1 = -0.562414$, $x_2 = 1.26241$

p-4 orbit: $x_1 = 0.139$, $x_2 = 1.4495$, $x_3 = -0.8595$, $x_4 = 0.8962$

All simulations were mostly repeated 50 times for each EA version (with only one exception – as written in section 6.3), in order to find the actual optimum and to show and check robustness of used method. The control method – original TDAS has the form (3.5, 3.6) [21] and ETDAS has the form (3.7) [22].

In the case of Logistic Equation (LQ), optimization proceeded with these parameters: K and $F_{\max}$ for TDAS control method, which is obtained in the form (6.14) after modification into discrete form suitable for logistic equation.

$$x_{n+1} = rx_n(1 - x_n) + F_n$$
$$F_n = K[x_{n-m} - x_n] \tag{6.14}$$

Due to problems with stabilization of higher periodic orbits described in section 6.3, it was necessary to try the optimization by EA for another control method – ETDAS in the form (6.15) suitable for the logistic equation. Thereafter optimization proceeded with these parameters: *K*, $F_{\max}$ and *R*.

$$x_{n+1} = rx_n(1 - x_n) + F_n$$
$$F_n = K[(1 - R)S_{n-m} - x_n] \tag{6.15}$$
$$S_n = x_n + RS_{n-m}$$

In case of the Henon map, the ETDAS control method was used for all simulations in the form (6.16) after modification into discrete form suitable for the used system.

$$x_{n+1} = a - x_n^2 + by_n + F_n$$
$$F_n = K[(1 - R)S_{n-m} - x_n] \tag{6.16}$$
$$S_n = x_n + RS_{n-m}$$

All results are shown only for variable *x* of two dimensional Henon map because of its form (9), where the variable *y* has the same values as variable *x* but it is only phase shifted.

The perturbation F_n in equations (6.14 – 6.16) may have arbitrarily large value, which can cause diverging of the system outside the interval {0, 1} for logistic equation or {-1.5, 1.5} in the case of Henon map. Therefore, F_n should have a value between $-F_{\max}$, $F_{\max}$ and EA should find an appropriate value of this limitation to avoid diverging of the system.

Four versions of SOMA and six versions of DE were used for all simulations. See Table 6.1 and Table 6.2 for correlation between each version of evolutionary algorithm and index mark in all following Figures and Tables. See also Table 6.3 and Table 6.4 for parameter setting. These parameters for optimizing algorithms were set up in this “common” way in order to reach the same value of maximal CF evaluations. This fact is very important due to further possibility of creating the complete statistical overview of EAs performance. This statistical summary is significant not only for comparison of both used evolutionary algorithms and its versions but for example in the task of the decision, as to which algorithm gives better results for all 50 runs when final CF value of the best individual solution is the same as the CF Value of other best individual solution given by different versions or algorithms. The ranges of all estimated parameters were in general these:

$-2 \le K \le 2$, $0 \le F_{max} \le 0.5$ and $0 \le R \le 0.5$

If other ranges were used, these exceptions are described in concrete cases.

The optimization interval for p-1 orbit was $\tau_i = 100$ iterations, for higher periodic orbits it was mostly $\tau_i = 150$ iterations, exceptions being described in each concrete cases.

Table 6.1 Used versions of SOMA

Index	**Algorithm / Version**
1	SOMA AllToOne
2	SOMA AllToRandom
3	SOMA AllToAll
4	SOMA AllToAllAdaptive

Table 6.2 Used versions of DE

Index	Algorithm / Version
5	DERand1Bin
6	DERand2Bin
7	DEBest2Bin
8	DELocalToBest
9	DERand1DIter
10	DEBest1JIter

Table 6.3 Parameter settings for SOMA

Parameter / Version	ATO/ATR	ATA/ATAA
PathLength	3	3
Step	0.33	0.33
PRT	0.1	0.1
PopSize	25	10
Migrations	25	7
Max. CF Evaluations (CFE)	5400	5670

Table 6.4 Parameter settings for DE

Parameter	
F	0.9
Cr	0.2
PopSize	25
Generations	215
Max. CF Evaluations (CFE)	5375

All the results within the following seven case studies (chapters 6.3 - 6.9) are depicted in this way:

Each case study is divided into two main sections, which are focused to optimizations and simulations with one dimensional system - LQ and two dimensional system - HENON. These two sections are mostly divided into three subsections for three desired UPOs. In two case studies (2 and 3) there is exception, because the designed CF is not suitable for all UPOs. Thus each case study includes six subsections, when the above mentioned exceptions are not taken into consideration. These subsections always contain a brief description, two tables and two figures for further illustration.

The first table shows the best individual solutions for four SOMA versions and the second one shows the best results for six DE versions.

The results shows the following data: Estimated parameters K, $F_{\max}$, R, CF value, Average CF Value, number of iterations required to stabilization (IStab value) and the average IStab value for all repeated simulations of EA version.

The best solution with lowest CF value for SOMA or DE is highlighted by bold number corresponding to EA version in the first column of the table. In the case when the final CF value is the same for more EA versions, then the rule for decision is better distribution of final CF values in histograms depicted in the Appendix.

The two figures shows the simulation of the best individual solutions given by SOMA and DE under identical initial conditions used in optimization (left part of the figures) and for the uniformly distributed initial conditions in the range $0 < x_{initial} < 1$, 100 samples were used in this kind of simulation (right part of the figures).

At the end of each case study there is brief conclusion of all related results. Furthermore the conclusion of the first case study contains the comparison of presented results with classical control method - OGY. The subsequent conclusions include the comparison with previous case studies and the discussion about the most important changes in results of optimizations and possible problems and their elimination.

The results from all seven case studies are finally concluded at the end of this chapter 6.

6.2 Case studies – Cost Function Design

In this work several types of cost function (CF) were developed and tested for stabilization of p-1 orbit (fixed point) and higher periodic orbits (p-2 and p-4). The CF has been calculated in general from the distance between desired state and actual system output. The minimal value of this cost function revealing the best solution is zero. The aim of all the simulations was to find the best solution that returns the cost function value as close as possible to zero.

Each described case study of CF design contains the illustrative example of CF surface depicted as a dependence of CF value on parameters K and $F_{\max}$ (3D graphics) and a dependence of CF value only on parameter K (2D graphics) for two dimensional Henon map. Complete overview of CF surfaces for both systems and all case studies can be found in Appendix (sections 8.1 - 8.7).

6.2.1 Basic CF

This proposal of the basic cost function is in general based on the most simple CF, which could be used only for the stabilization of p-1 orbit. The idea was to minimize the area created by the difference between the required state and the real system output on the whole simulation interval – τ_i.

But another cost function (CF) had to be used for stabilizing of higher periodic orbit. It was synthesized from the simple CF and other terms were added. In this case, it is not possible to use the simple rule of minimizing the area created by the difference between the required and actual state on the whole simulation interval – τ_i, due to the many serious reasons, for example: degrading of the possible best solution by phase shift of periodic orbit.

This CF, is in general based on searching for desired stabilized periodic orbit and thereafter calculation of the difference between desired and found actual periodic orbit on the short time interval - τ_s (approx. 20 - 50 iterations) from the point, where the first min. value of difference between desired and actual system output is found. Such a design of CF should secure the successful stabilization of higher periodic orbit anywise phase shifted.

Furthermore, because of CF values being very close to zero, this CF also allows using of decision rule avoiding very time demanding simulations. This rule stops EA immediately, when the first individual with good parameter structure is reached, thus the value of CF is lower then the acceptable (CF_{acc}) one. Typically $CF_{acc} = 0.001$ at time interval $\tau_s = 20$ iterations, thus difference between desired and actual output has value 0.0005 per iteration – i.e. successful stabilization for used control technique. This CF was also used for p-1 orbit. The CF_{Basic} has the form (6.17).

$$CF_{Basic} = penalization_1 + \sum_{t=\tau 1}^{\tau 2} \left| TS_t - AS_t \right| \tag{6.17}$$

where: TS - target state, AS - actual state

τ_1 - the first min. value of difference between TS and AS

τ_2 – the end of optimalizing interval ($\tau_1 + \tau_s$)

$penalization_1 = 0$ if $\tau_i - \tau_2 \geq \tau_s$;

$penalization_1 = 10*(\tau_i - \tau_2)$ if $\tau_i - \tau_2 < \tau_s$ (i.e. late stabilization)

See Figure 6.1 and Figure 6.2 for the example of CF surfaces, corresponding to this CF design.

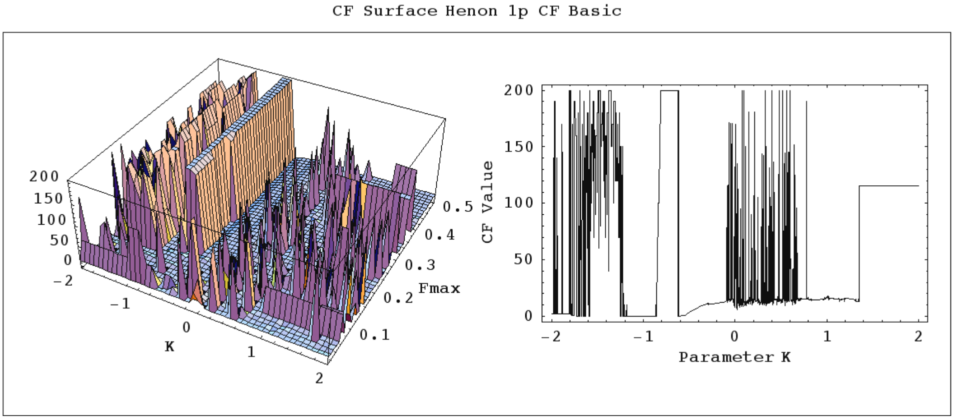


Figure 6.1 Dependence of CF value on parameters K and F_{max} ; R = 0.24 (left); and parameter K; F_{max} = 0.39, R = 0.24 (right); CF Basic, p-1 orbit, $x_{initial}$ = 0.7

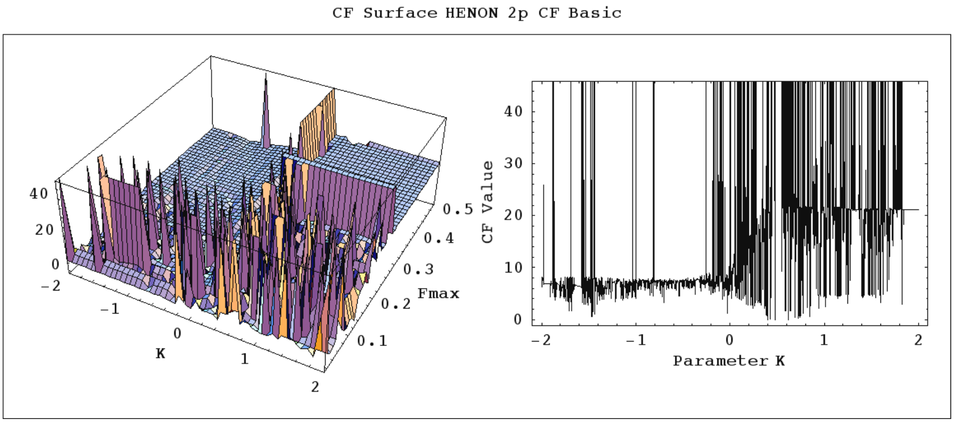


Figure 6.2 Dependence of CF value on parameters K and F_{max} ; R = 0.46 (left); and parameter K, F_{max} = 0.16, R = 0.46 (right); CF Basic, p-2 orbit, $x_{initial}$ = 0.7

6.2.2 Targeting CF

In this section two types of CF were developed and tested.

CF Simple

In this section the most simple CF proposal outlined above was used. It is based on minimizing the area created by the difference between the required state (stabilized fixed point) and the real system output on the whole simulation interval – τ, thus this proposal of CF should secure fast targeting into the close neighborhood of p-1 orbit and its stabilization. The CF_{Simple} is given by (6.18).

$$CF_{Simple} = \sum_{t=0}^{\tau_i} |TS_t - AS_t| \tag{6.18}$$

where: TS - target state, AS - actual state

This CF was used in case study 2 only for optimization in the task of improvement of stabilization of p-1 orbit. Following proposal of CF NA was used for both p-1 and p-2 orbit.

See Figure 6.3 for example of CF simple surface.

CF Surface HENON 1p CF Simple

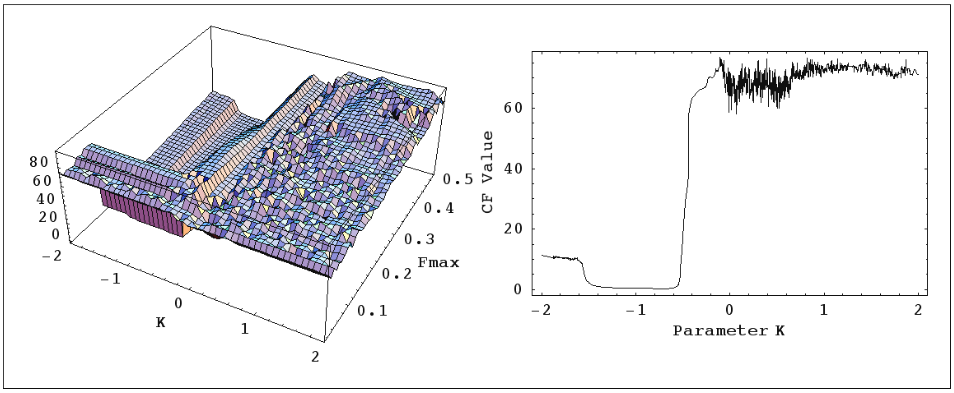


Figure 6.3 Dependence of CF value on parameters *K* and F_{max} ; *R* = 0.17 (left); and parameter *K*; F_{max} = 0.30, *R* = 0.17 (right); CF Simple, p-1 orbit, $x_{initial}$ = 0.7

Targeting CF NA

Due to the reasons outlined in later details (section 6.3), it was necessary to modify the definition of CF in order to decrease the average number of iteration required for successful stabilization and avoidance of any associated problem. The CF_1 is not suitable for adding any term of penalization for slowly stabilizing solutions, thus the CF_2 was modified to use for all required UPOs. The CF value is multiplied by the number of iterations (*NI*) of the first found minimal value of difference between desired and actual system output (i.e. the beginning of fully stabilized UPO). To avoid problems associated with CF returning value 0 and to put the penalization to similar level as the non-penalized CF value, the small constant (*SC*) is added to CF value before penalization (multiplying by *NI*). The modified CF_{NA} has the form (6.19).

$$CF_{NA} = NI\left(SC + penalization1 + \sum_{t=\tau 1}^{\tau 2} \left| TS_t - AS_t \right| \right) \tag{6.19}$$

where: *TS* - target state, *AS* - actual state

$\tau 1$ - the first min. value of difference between *TS* and *AS*

τ_2 – the end of optimalizing interval ($\tau_1 + \tau_s$)

penalization1= 0 if $\tau_i - \tau 2 \geq \tau s$

penalization1= 10*($\tau_i - \tau 2$) if $\tau_i - \tau 2 < \tau s$ (late stabilization)

$SC = 10^{-16}$ for p-1 orbit

$SC = 10^{-8}$ for p-2 orbit

See Figure 6.4 and Figure 6.5 for examples of CF surfaces corresponding to this CF design.

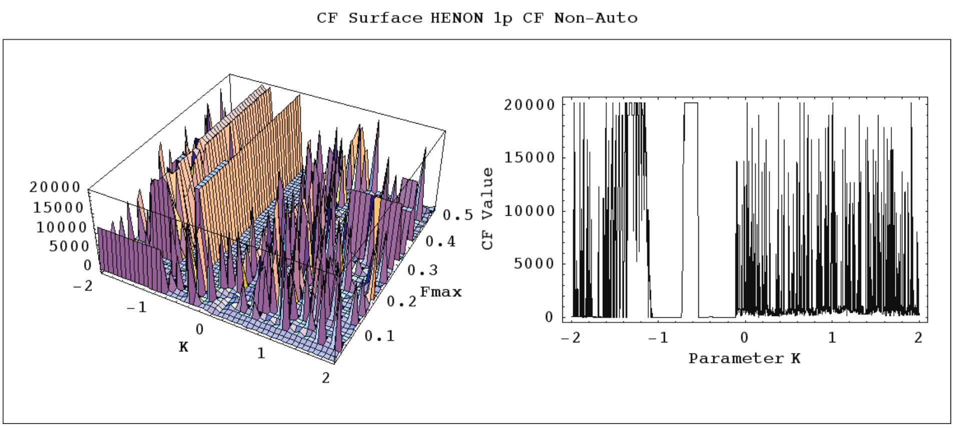


Figure 6.4 Dependence of CF value on parameters *K* and F_{max} ; *R* = 0.22 (left); and parameter *K*; F_{max} = 0.17, *R* = 0.22 (right); CF NA, p-1 orbit, $x_{initial}$ = 0.7

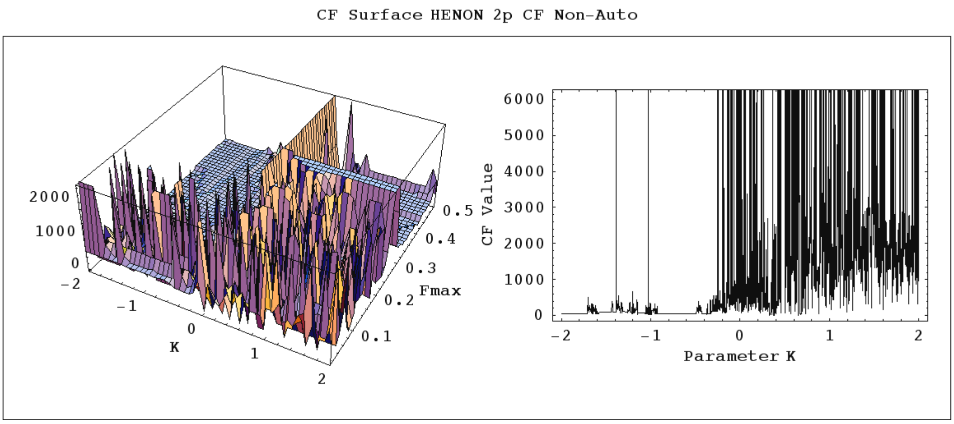


Figure 6.5 Dependence of CF value on parameters K and F_{max} ; R = 0.42 (left); and parameter K; F_{max} = 0.20, R = 0.42 (right); CF Basic, p-1 orbit, $x_{initial}$ = 0.7

6.2.3 Targeting CF – CF Targ1

The next proposal of CF design is based on the previous one with small change, which should avoid any problems with defining the value of small constant *SC* in advance (especially for stabilization of higher periodic orbit). The *SC* value (6.22) here is computed with the aid of power of non-penalized basic part of CF.

In general, there exists two possible ways for applying the multiplication by number of iterations required for stabilization (*NI*). The first version of final design of targeting CF (CF_{TARG1}) has the form (6.20). Here the sum of basic part of CF and automatically computed *SC* is multiplied by *NI*. Consequently, the EA should find the solutions securing the fast targeting into desired behavior of system.

$$CF_{TARG1} = NI\left(SC + penalization1 + \sum_{t=\tau 1}^{\tau 2} \left| TS_t - AS_t \right| \right) \tag{6.20}$$

where:

$$EXPCF = \log_{10}\left(\sum_{t=\tau 1}^{\tau 2} \left| TS_t - AS_t \right| + 10^{-15} \right) \tag{6.21}$$

$$SC = 10^{\mathrm{EXPCF}} \tag{6.22}$$

TS - target state, *AS* - actual state

$\tau 1$ - the first min. value of difference between *TS* and *AS*

τ_2 – the end of optimalizing interval ($\tau_1 + \tau_s$)

penalization1= 0 if τ_i - $\tau 2 \geq \tau s$

penalization1= 10*(τ_i - $\tau 2$) if τ_i - $\tau 2 < \tau s$ (late stabilization)

See Figure 6.6 and Figure 6.7 for illustrative examples of CF Targ1 surfaces.

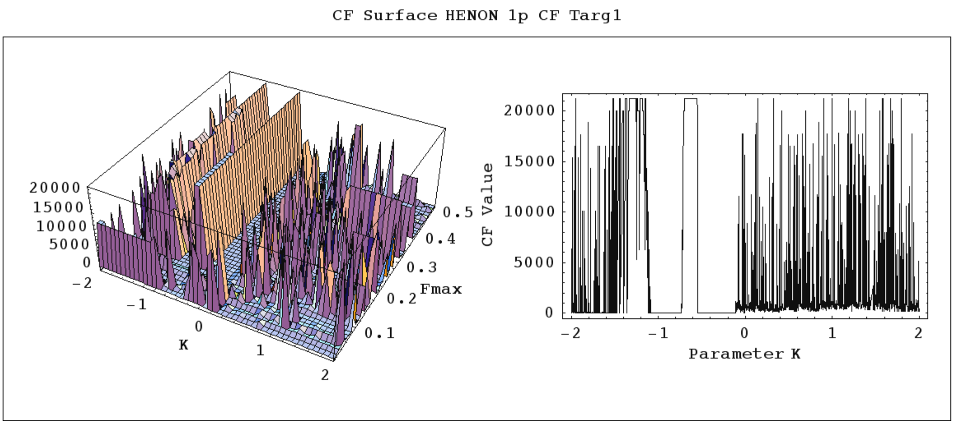


Figure 6.6 Dependence of CF value on parameters *K* and F_{max} ; *R* = 0.23 (left); and parameter *K*; F_{max} = 0.29, *R* = 0.23 (right); CF Targ1, p-1 orbit, $x_{initial}$ = 0.7

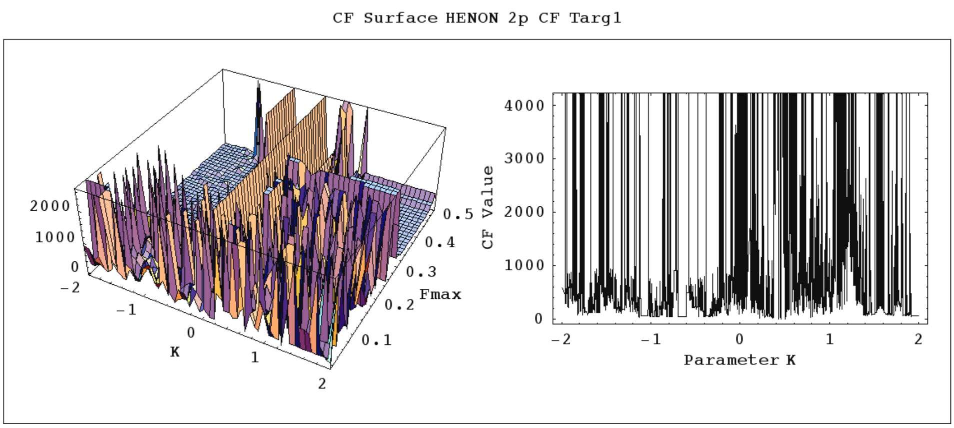


Figure 6.7 Dependence of CF value on parameters *K* and F_{max} ; *R* = 0.50 (left); and parameter *K*; F_{max} = 0.18, *R* = 0.50 (right); CF Targ1, p-2 orbit, $x_{initial}$ = 0.7

6.2.4 Targeting CF – CF Targ2

In the second version of targeting CF (CF_{TARG2}), there is only slight change in comparison with the previous proposal. Here the number of steps for stabilization (*NI*) multiplies only the small constant (*SC*) which is counted in the same way as in the previous case (6.20) This version of targeting CF (CF_{TARG2}) has the form (6.23)

$$CF_{TARG2} = (NI \cdot SC) + penalization1 + \sum_{t=\tau 1}^{\tau 2} |TS_t - AS_t| \qquad (6.23)$$

where:

$$EXPCF = \log_{10}\left(\sum_{t=\tau 1}^{\tau 2} |TS_t - AS_t| + 10^{-15}\right) \qquad (6.24)$$

$$SC = 10^{\mathrm{EXPCF}} \qquad (6.25)$$

TS - target state, *AS* - actual state

$\tau 1$ - the first min. value of difference between *TS* and *AS*

τ_2 – the end of optimalizing interval ($\tau_1 + \tau_s$)

penalization1= 0 if $\tau_i - \tau 2 \geq \tau s$

penalization1= 10*($\tau_i - \tau 2$) if $\tau_i - \tau 2 < \tau s$ (late stabilization)

The differences in performance of both targeting CFs from the point of view of design are depicted in figures and described in details in the chapter 6.10. The examples of CF surfaces are depicted in Figure 6.8 and Figure 6.9.

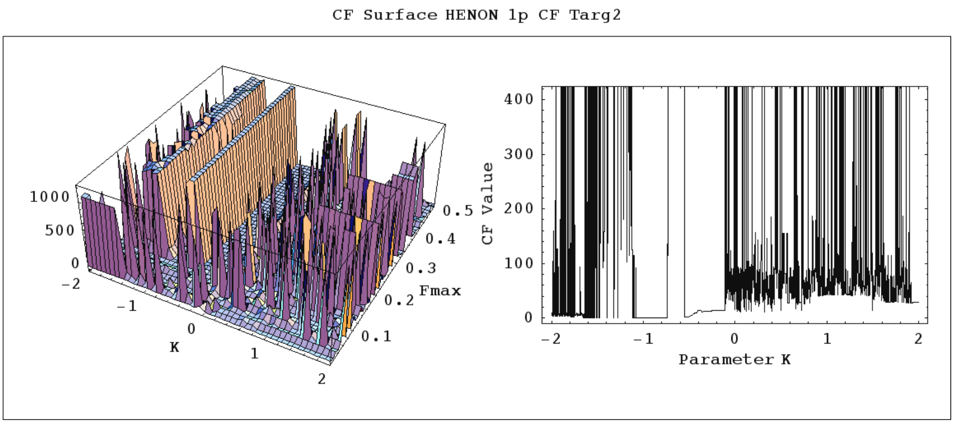


Figure 6.8 Dependence of CF value on parameters K and F_{max} ; R = 0.21 (left); and parameter K; F_{max} = 0.20, R = 0.21 (right); CF Targ2, p-1 orbit, $x_{initial}$ = 0.7

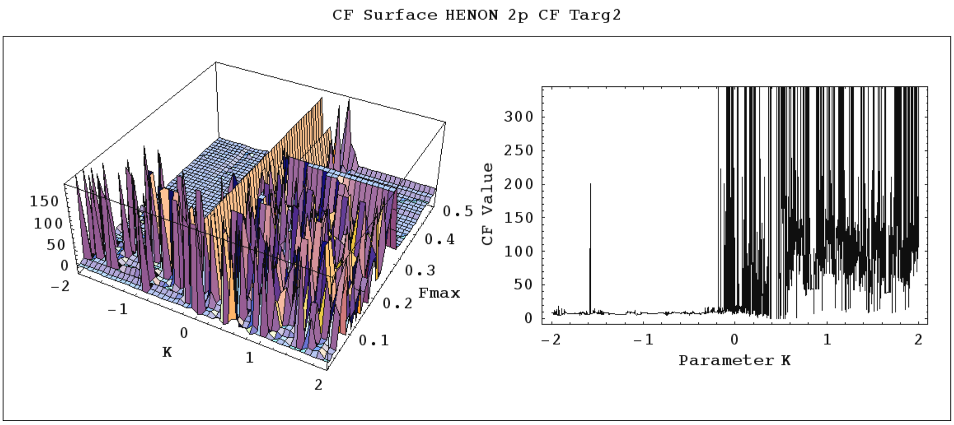


Figure 6.9 Dependence of CF value on parameters K and F_{max} ; R = 0.49 (left); and parameter K; F_{max} = 0.16, R = 0.49 (right); CF Targ2, p-2 orbit, $x_{initial}$ = 0.7

6.2.5 Targeting CF Targ1 & CF Targ2 – Advanced Design

Finally, to avoid the problems with fast stabilization only for limited range of initial conditions two previously proposed targeting Cost functions (CF Targ1 and CF Targ2) were modified. More about this problem is written at the end of each case study and it can be clearly seen from the complex simulations of best individual solutions for uniformly distributed initial conditions in Appendix (sections 8.7 - 8.20) In this case, the final CF value is computed from n repeated simulations of CF Targ1 or CF Targ2 with different initial conditions. The constant SC is computed as in case of CF Targ1 (6.22).

The advanced CF Targ1 has the form (6.26) and the CF surfaces are displayed in Figure 6.10 and Figure 6.11.

$$CF_{T1-Adv} = \sum_{1}^{n} NI\left(SC + penalization1 + \sum_{t=\tau 1}^{\tau 2} \left| TS_t - AS_t \right| \right) \tag{6.26}$$

where: $x_{initial}$ is from the range 0.05 – 0.95 and uses step 0.1.

TS - target state, AS - actual state

$\tau 1$ - the first min. value of difference between TS and AS

τ_2 – the end of optimalizing interval ($\tau_1 + \tau_s$)

penalization1= 0 if τ_i - $\tau 2 \geq \tau s$

penalization1= 10*(τ_i - $\tau 2$) if τ_i - $\tau 2 < \tau s$ (late stabilization)

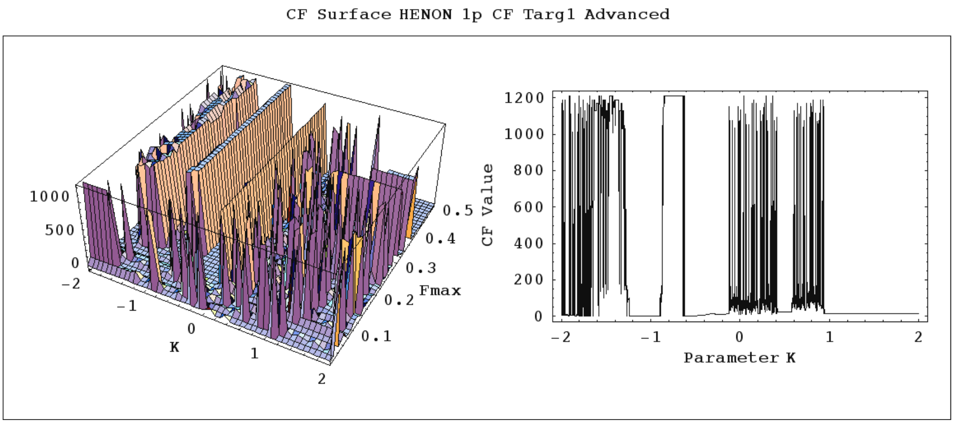


Figure 6.10 Dependence of CF value on parameters K and F_{max} ; R = 0.31 (left); and parameter K; F_{max} = 0.41, R = 0.31 (right); CF Targ1 Advanced, p-1 orbit, $x_{initial}$ = 0.7

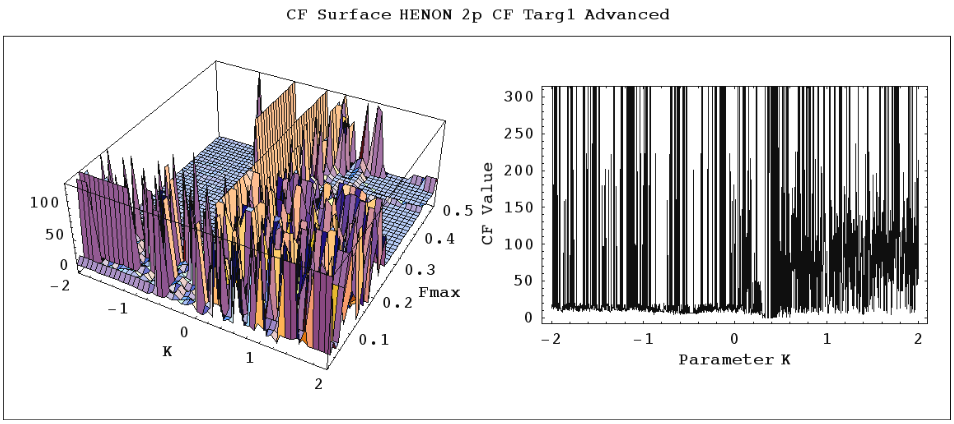


Figure 6.11 Dependence of CF value on parameters K and $F_{\max}$; $R = 0.10$ (left); and parameter K; $F_{\max} = 0.15$, $R = 0.10$ (right); CF Targ1 Advanced, p-2 orbit, $x_{\text{start}} = 0.7$

The advanced CF Targ2 has the form (6.27) and its surfaces are depicted in Figure 6.12 and Figure 6.13.

$$CF_{T2-Adv} = \sum_{1}^{n} \left((NI \cdot SC) + penalization1 + \sum_{t=\tau 1}^{\tau 2} |TS_t - AS_t| \right) \qquad (6.27)$$

where: $x_{initial}$ is from the range 0.05 – 0.95 and uses step 0.1.

TS - target state, AS - actual state

$\tau 1$ - the first min. value of difference between TS and AS

τ_2 – the end of optimalizing interval ($\tau_1 + \tau_s$)

penalization1= 0 if τ_i - $\tau 2 \geq \tau s$

penalization1= 10*(τ_i - $\tau 2$) if τ_i - $\tau 2 < \tau s$ (late stabilization)

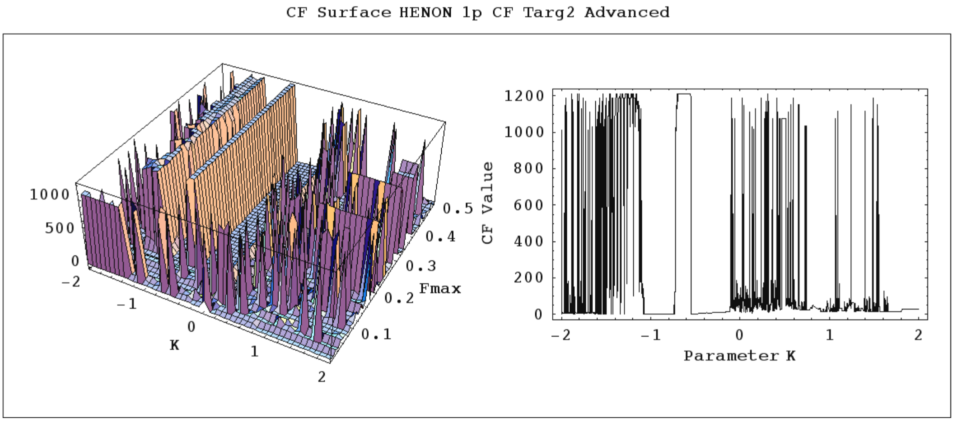


Figure 6.12 Dependence of CF value on parameters K and F_{max} ; $R = 0.21$ (left); and parameter K; $F_{max} = 0.47$, $R = 0.21$ (right); CF Targ2 Advanced, p-1 orbit, $x_{initial} = 0.7$

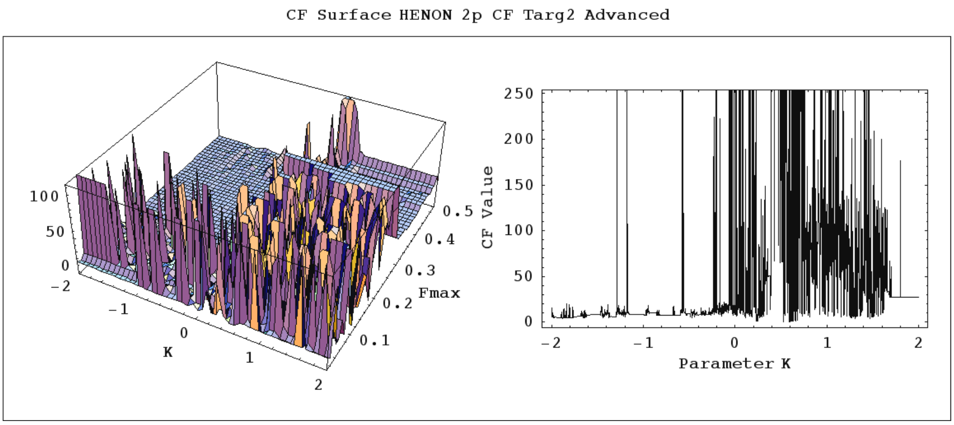


Figure 6.13 Dependence of CF value on parameters K and F_{max} ; $R = 0.35$ (left); and parameter K; $F_{max} = 0.18$, $R = 0.35$ (right); CF Targ2 Advanced, p-2 orbit, $x_{initial} = 0.7$

6.3 Simulation Results – Case study 1: Basic CF

6.3.1 One-Dimensional Example

1p

In this case the objective was to estimate the optimum value of feedback adjustable constants to stabilize p-1 orbit. Each version of SOMA and DE has been applied 100 times due to relatively low cost function evaluation (CFE) demands and thus to obtain more results for successful finding of the actual optimum and to form better statistical comparison of them.

From the presented results given in Table 6.5 and Table 6.6, it is obvious that each SOMA version gave the same result of CF value for the best solution. The same phenomenon occurs also in case of DE. Small divergence between these results is given by robustness of control method together with searching in three-dimensional space, where more combinations of estimated parameters lead to the optimal solution. From the histograms (see Appendix) follows that SOMA ATA and DERand1Bin gave better results from the point of view of CF distribution. On average, around 97 iterations are required for stabilization. See Figure 6.14 and Figure 6.15 for the results of optimization in this case.

Table 6.5 Best individual solutions, LQ, p-1 orbit, CF Basic, SOMA

EA	K	F_{max}	CFVal	AvgCFVal	IStab	AvgIStab
1	-0,5471	0,1944	0	$1,66.10^{-15}$	101	97
2	-0,5471	0,1944	0	$1,29.10^{-15}$	101	97
3	-0,5471	0,1944	0	$1,55.10^{-7}$	101	98
4	-0,5471	0,1944	0	$2,42.10^{-6}$	101	97

Table 6.6 Best individual solutions, LQ, p-1 orbit, CF Basic, DE

EA	K	F_{max}	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.5571	0.1479	0	$1.11.10^{-15}$	101	98
6	-0.5571	0.1479	0	$3.98.10^{-6}$	101	97
7	-0.5571	0.1479	0	$6.40.10^{-6}$	101	96
8	-0.5571	0.1479	0	$4.32.10^{-6}$	101	96
9	-0.5571	0.1479	0	$9.94.10^{-7}$	101	97
10	-0.5571	0.1479	0	$1.97.10^{-6}$	101	97

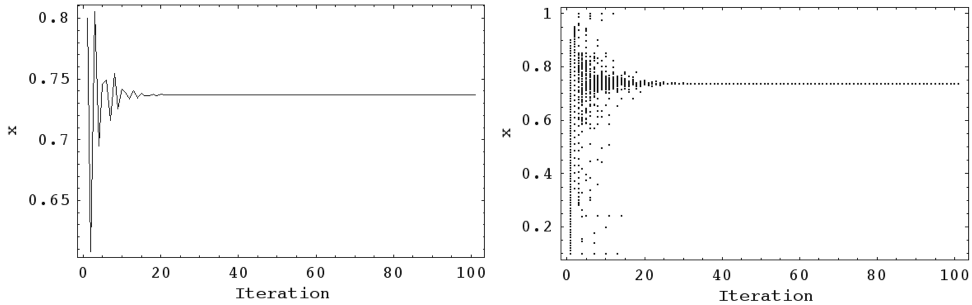


Figure 6.14 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Basic, p-1 orbit, SOMA ATA

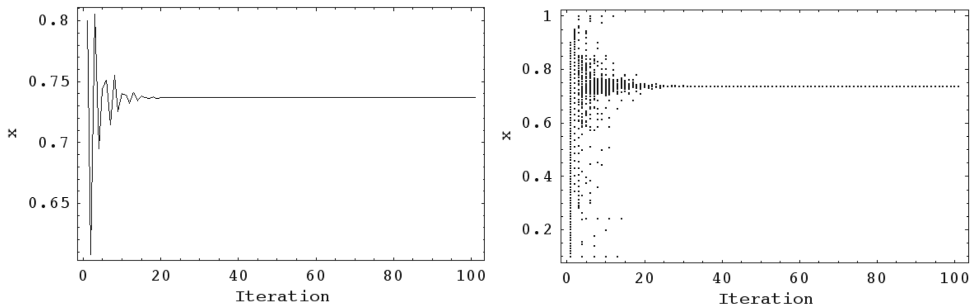


Figure 6.15 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Basic, p-1 orbit, DERand1Bin

2p

The second case is focused on the stabilization of p-2 orbit for logistic equation. Unfortunately, the EA were not able to fully stabilize the unstable p-2 orbit by means of TDAS control method. The EA found the solution where the stabilization is only temporary and of low quality. This may be caused by properties of TDAS method that there is no perturbation when the periodic orbit is achieved. Consequently, the ETDAS method was used. The optimization proceeded with two parameters as in previous case and the third one was added, namely the weight parameter *R*. As can be seen from Table 6.7 and Table 6.8, both EA produced identical results of the best solution; SOMA ATR has found the lowest CF value. As in the previous case, the small divergence between the results is given by searching for the global minimum in higher-dimensional space (four dimensions), where even more combinations of parameters lead to the optimal results. For fully stabilization, on average, about 120 iterations are required. See also Figure 6.16 and Figure 6.17.

Table 6.7 Best individual solutions, LQ, p-2 orbit, CF Basic, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	0.5691	0.2117	0.4996	$1.03.10^{-14}$	0.0003	122	124
2	0.5250	0.2139	0.4483	$5.33.10^{-15}$	0.0003	130	123
3	0.5522	0.1453	0.3944	$4.00.10^{-7}$	0.0003	126	123
4	0.5522	0.0774	0.4945	$1.41.10^{-14}$	0.0002	125	123

Table 6.8 Best individual solutions, LQ, p-2 orbit, CF Basic, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	0.5503	0.0666	0.4901	$1.02.10^{-14}$	0.0003	128	122
6	0.5386	0.0673	0.4726	$1.28.10^{-14}$	0.0003	116	124
7	0.5001	0.0611	0.3871	$3.07.10^{-8}$	0.0003	123	122
8	0.5338	0.0894	0.4524	$1.62.10^{-14}$	0.0002	129	124
9	0.5440	0.0369	0.4767	$7.33.10^{-15}$	0.0003	128	122
10	0.5395	0.1382	0.4733	$7.11.10^{-15}$	0.0003	129	122

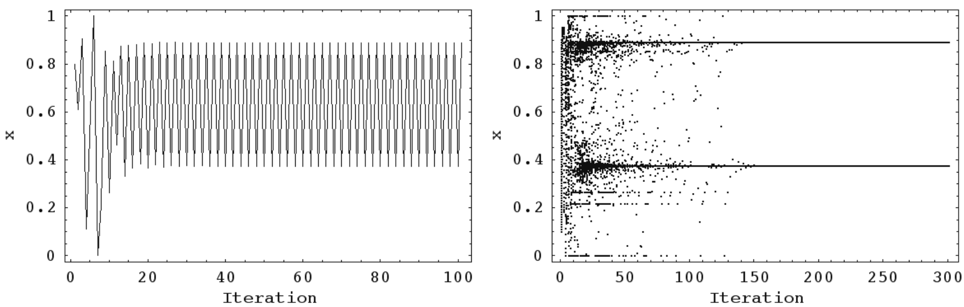


Figure 6.16 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Basic, p-2 orbit, SOMA ATR

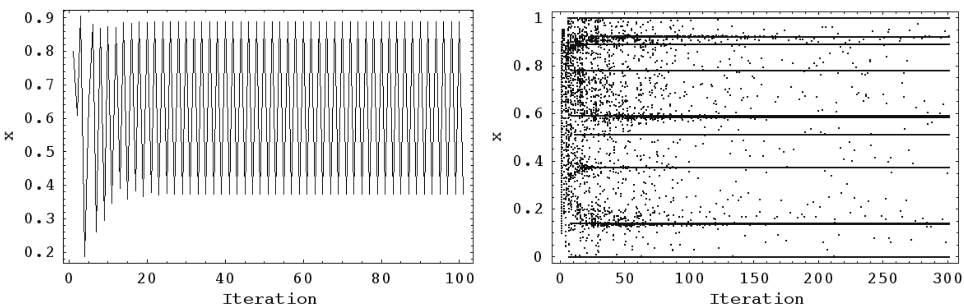


Figure 6.17 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Basic, p-2 orbit, DEBest1JIter

4p

Also in this case TDAS method totally failed and EAs were not able to estimate combination of parameters which will secure at least temporary stabilization. Thus ETDAS method was also used in order to achieve stabilization of desired p-4 orbit. Consequently, this method was used in all following optimizations. For the results see Table 6.9 and Table 6.10. The best solution has been obtained by SOMA ATO due to better final CF value. From Figure 6.18 and Figure 6.19, it follows that both best solutions ensure relatively fast stabilization (on average 210 iterations are required). But on the other hand also here (as in previous case of p-2 orbit, results given by DE) occurs the paradox that the best individual solutions do not give satisfactory results for simulation with initial conditions distributed in the range $0 < x_{initial} < 1$. They secures only the stabilization of almost all samples. However, several samples were stabilized in the fixed point $x_F = 0$. This problem can be solved by additional penalization rule in the CF as it is described in following case studies. From the above mentioned figures and tables it can be clearly seen, that to estimate parameters, which will give good results, it was proven to be a quite a difficult task for EAs. Even the optimization interval had to be increased to $\tau_i = 300$ iterations. To obtain better solution the range of estimated parameter $F_{\max}$ and R was changed as well to $0 \le F_{\max} \le 0.9$ and $0 \le R \le 0.9$. This setting is used in all following case studies for LQ and p-4 orbit.

Table 6.9 Best individual solutions, LQ, p-4 orbit, CF Basic, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.6277	0.2555	0.7110	$3.04.10^{-05}$	0.0809	265	211
2	-0.5184	0.1733	0.6048	$3.23.10^{-05}$	0.0748	263	183
3	-0.6398	0.6378	0.7327	$3.27.10^{-05}$	0.0684	274	202
4	-0.5060	0.0466	0.5691	$3.24.10^{-05}$	0.0638	262	190

Table 6.10 Best individual solutions, LQ, p-4 orbit, CF Basic, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.6076	0.4570	0.7019	$3.13.10^{-05}$	0.0200	221	224
6	-0.5620	0.2567	0.7279	$3.25.10^{-05}$	0.0371	281	213
7	-0.5846	0.3279	0.6700	$3.13.10^{-05}$	0.0358	239	215
8	-0.5395	0.0348	0.6629	$3.18.10^{-05}$	0.0287	273	228
9	-0.5356	0.1214	0.6517	$3.19.10^{-05}$	0.0322	269	220
10	-0.6339	0.3707	0.7187	$3.07.10^{-05}$	0.0428	258	208

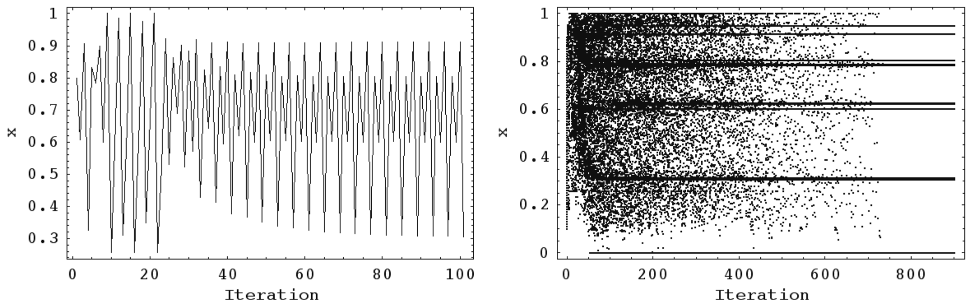


Figure 6.18 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Basic, p-4 orbit, SOMA ATO

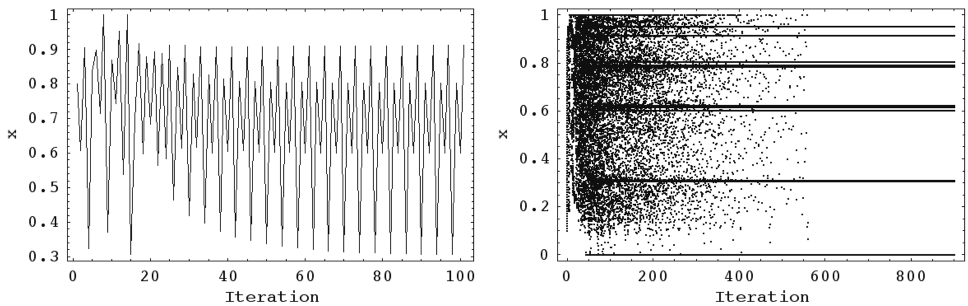


Figure 6.19 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Basic, p-4 orbit, DEBest1JIter

6.3.2 Two-Dimensional Example

1p

This part is focused on the stabilization of p-1 orbit for two dimensional Henon map. The best results of each version of SOMA and DE are shown in Table 6.11 and Table 6.12. The simulation results are depicted in Figure 6.20 and Figure 6.21.

Based on obtained results, it may be stated that the control parameters estimated in the optimizations ensured very fast reaching of desired state and in case of results given by DE (global minimum – CF Value = 0) for any initial conditions. And also all SOMA and DE versions required, on average, only 50 CFE to find a satisfactory solution. These facts show the possibility of using EA for real time chaos control to p-1 orbit (fixed point).

Table 6.11 Best individual solutions, HENON, p-1 orbit, CF Basic, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.9246	0.3400	0.3503	$2.22.10^{-16}$	$2.16.10^{-15}$	61	81
2	-1.2237	0.2745	0.3729	$9.99.10^{-16}$	$2.00.10^{-15}$	78	76
3	-1.2237	0.2745	0.3729	$9.99.10^{-16}$	$2.31.10^{-15}$	78	78
4	-0.8309	0.1379	0.1644	$6.66.10^{-16}$	$2.10.10^{-15}$	44	73

Table 6.12 Best individual solutions, HENON, p-1 orbit, CF Basic, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.8957	0.3865	0.2966	0	$2.07.10^{-15}$	55	74
6	-0.8957	0.3865	0.2966	0	$2.04.10^{-15}$	55	73
7	-1.1644	0.4153	0.2774	$2.22.10^{-16}$	$1.93.10^{-15}$	85	77
8	-0.8957	0.3865	0.2966	0	$2.60.10^{-15}$	55	71
9	-0.8957	0.3865	0.2966	0	$2.74.10^{-15}$	55	77
10	-1.1644	0.4153	0.2774	$2.22.10^{-16}$	$2.18.10^{-15}$	85	80

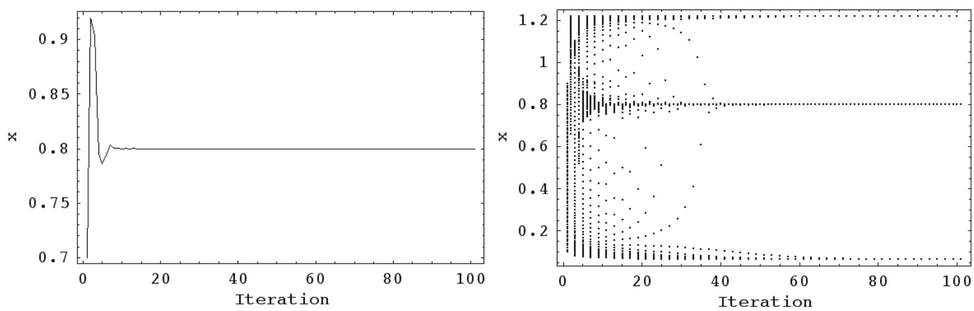


Figure 6.20 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Basic, p-1 orbit, SOMA ATO

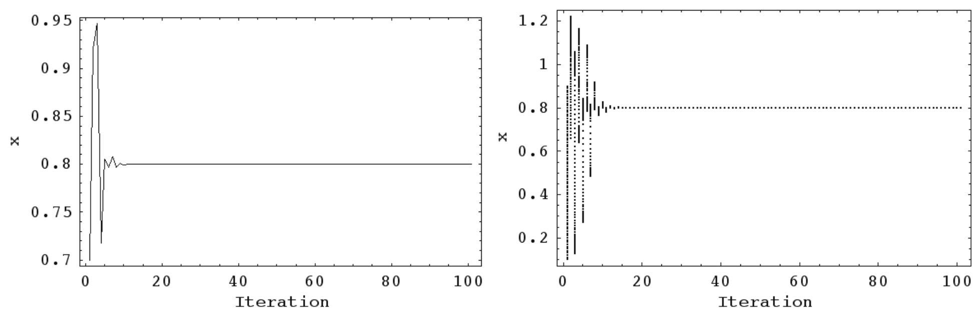


Figure 6.21 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Basic, p-1 orbit, DERand2Bin

2p

The best obtained results for this particular orbit are shown in Table 6.13 and Table 6.14. SOMA ATR has found the lowest value of CF. From Figure 6.22 and Figure 6.23, it follows that the stabilization was achieved relatively quickly and in the case of simulation for distributed initial conditions, the desired periodic orbit was fully reached in first 600 iterations for more than 90% cases.

Table 6.13 Best individual solutions, HENON, p-2 orbit, CF Basic, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	0.3893	0.0986	0.2718	$2.17.10^{-7}$	0.0003	130	123
2	0.4724	0.1559	0.4614	$1.60.10^{-14}$	0.0003	124	124
3	0.5580	0.1526	0.4215	$1.57.10^{-7}$	0.0003	128	125
4	0.5478	0.1534	0.4315	$1.53.10^{-8}$	0.0003	128	125

Table 6.14 Best individual solutions, HENON, p-2 orbit, CF Basic, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	0.5322	0.0799	0.4236	$1.10.10^{-7}$	0.0002	130	125
6	0.4973	0.1586	0.4785	$1.95.10^{-14}$	0.0003	128	122
7	0.4784	0.1494	0.3320	$1.29.10^{-6}$	0.0004	126	125
8	0.5315	0.1614	0.4811	$3.25.10^{-12}$	0.0002	130	125
9	0.4231	0.0912	0.3498	$1.13.10^{-11}$	0.0003	128	126
10	0.5000	0.2101	0.4031	$5.63.10^{-9}$	0.0003	128	124

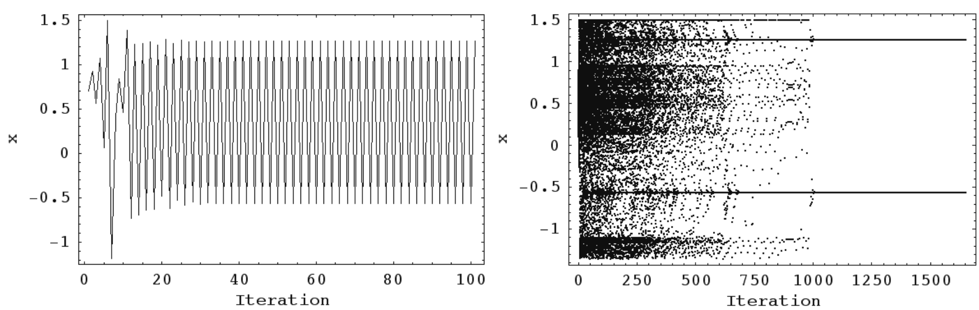


Figure 6.22 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Basic, p-2 orbit, SOMA ATR

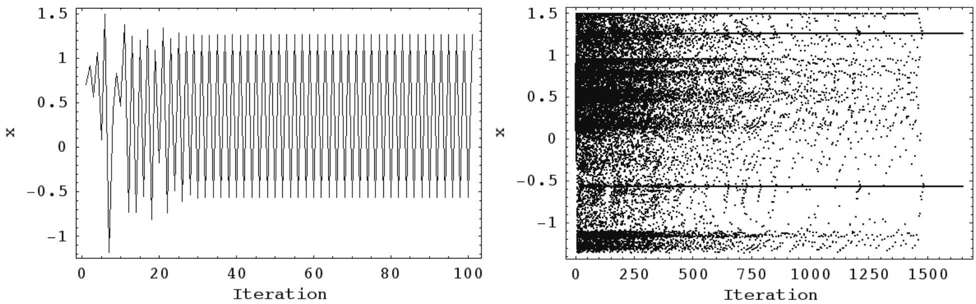


Figure 6.23 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Basic, p-2 orbit, DERand2Bin

4p

The last case is focused on the stabilization of p-4 orbit. For the simulation results please see Table 6.15 and Table 6.16. From Figure 6.24 and Figure 6.25, it is obvious that for the stabilization of p-4 orbit more than 1000 CF evaluations are required. This fact is related to highly increasing complexity of CF surface for higher period orbits. This kind of CF surface contains huge amount of local minimums and a lot of them lead only to successful stabilization only for limited set of initial conditions $x_{initial}$. In the case of the best individual solution the same initial conditions were used for optimization process and for simulation of results, desired state was stabilized very quickly (on average 125 iterations), whereas in the second case for some initial conditions the stabilization is reached after more than 1000 iterations. This problem can be solved by further modification of used CF as written above in case of p-4 orbit and Logistic equation.

Table 6.15 Best individual solutions, HENON, p-4 orbit, CF Basic, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.3833	0.3123	0.4370	$9.57.10^{-8}$	0.0003	125	122
2	-0.4258	0.2830	0.4572	$5.39.10^{-8}$	0.0003	129	122
3	-0.3696	0.1130	0.4103	$9.43.10^{-9}$	0.0003	129	124
4	-0.4124	0.3419	0.4673	$9.90.10^{-8}$	0.0003	121	121

Table 6.16 Best individual solutions, HENON, p-4 orbit, CF Basic, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.3899	0.1191	0.4465	$7.90.10^{-8}$	0.0003	113	123
6	-0.4000	0.2847	0.4221	$1.16.10^{-8}$	0.0003	129	121
7	-0.3651	0.2392	0.3978	$3.19.10^{-8}$	0.0003	129	122
8	-0.3759	0.3171	0.4116	$3.68.10^{-8}$	0.0003	129	122
9	-0.3810	0.1107	0.4197	$9.16.10^{-9}$	0.0003	129	123
10	-0.3720	0.4487	0.4161	$1.01.10^{-8}$	0.0003	129	123

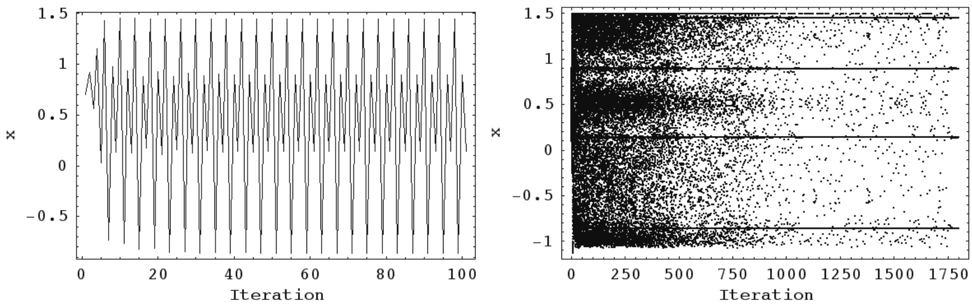


Figure 6.24 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Basic, p-4 orbit, SOMA ATA

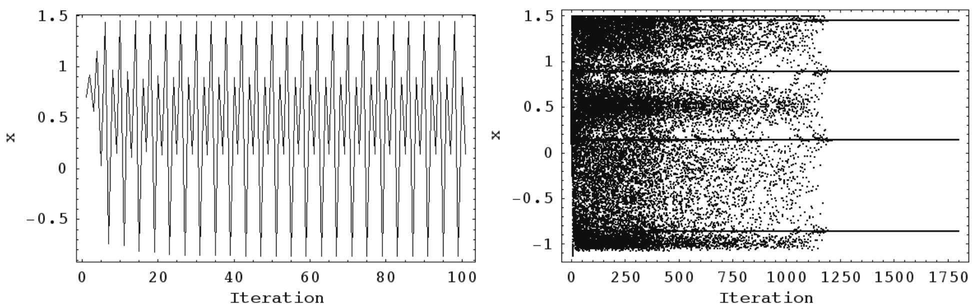


Figure 6.25 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Basic, p-4 orbit, DERand1DIter

6.3.3 Conclusion of Results – Case Study 1

As a conclusion of performed optimizations, it is possible to say that all SOMA and DE versions give almost the same satisfactory results of the best individual solution securing fast and fine stabilization of desired UPOs. On the other hand, EAs has found many solutions that do not give expected good results, especially in the case of simulation for distributed initial condition where the stabilization was only either temporary and of low quality or was achieved after too many iterations. This problem is solved by constructing more complex CFs and is described in details in further case studies 2 - 7. Such a new complex CF should secure faster stabilization not only for initial conditions used in optimization process, but for the whole range of the initial conditions. The question as to why the TDAS was chosen and used in case of LQ and p-1 orbit, although it has proven lower stabilizing performance, has this simple answer. To avoid any long simulations and evolutionary computations it is better to search in lower dimensional space and to work with simpler control algorithm, which is not so demanding for computational time. For the comparison of average number of iterations required for successful stabilization see Table 6.17 and Table 6.18. These numbers were computed from all 50 (100) repeated simulations of all SOMA or DE versions.

Table 6.17 Average IStab values – LQ – Case study1

UPO	**p-1**		**p-2**		**p-4**	
EA Version	SOMA	DE	SOMA	DE	SOMA	DE
CF Basic	97	97	123	123	197	218

Table 6.18 Average IStab values – HENON – Case study1

UPO	**p-1**		**p-2**		**p-4**	
EA Version	SOMA	DE	SOMA	DE	SOMA	DE
CF Basic	77	75	124	125	122	122

6.3.4 Comparison with Classical control Method - OGY

Logistic Equation

Standard OGY method was chosen to compare optimized TDAS or ETDAS with classical control technique. The comparison was done for these two cases: p-1 orbit and p-2 orbit. In the first case TDAS was set up identically as the best solution given by SOMA ATA (see Table 6.5).

As can be seen from Figure 6.26, TDAS method steers the chaotic system very quickly to the desired state. The OGY stabilize 50% of examples in first 100 iterations, but TDAS stabilize more than 50% of examples in first 20 iterations. Thus this supports the theory that TDAS based control method can be simply considered as targeting and stabilizing algorithm.

In the second case, ETDAS was set up identically as the best solution given by SOMA ATR (see Table 6.7). From Figure 6.26 it follows that ETDAS method steers the chaotic system very quickly to close neighborhood of p-2 orbit. To stabilize all of the examples around 150 iterations are required. However, more than 50% of examples oscillate in the close neighborhood after first 50 iterations. Then the stage of reducing of the neighborhood size, caused by progressive converging into p-2 orbit, ensues. This stage can by markedly shortened by targeting CF used in further case studies. The OGY method stabilizes about 50% of examples in first 400 iterations and to stabilize all of the examples about 900 iterations are required.

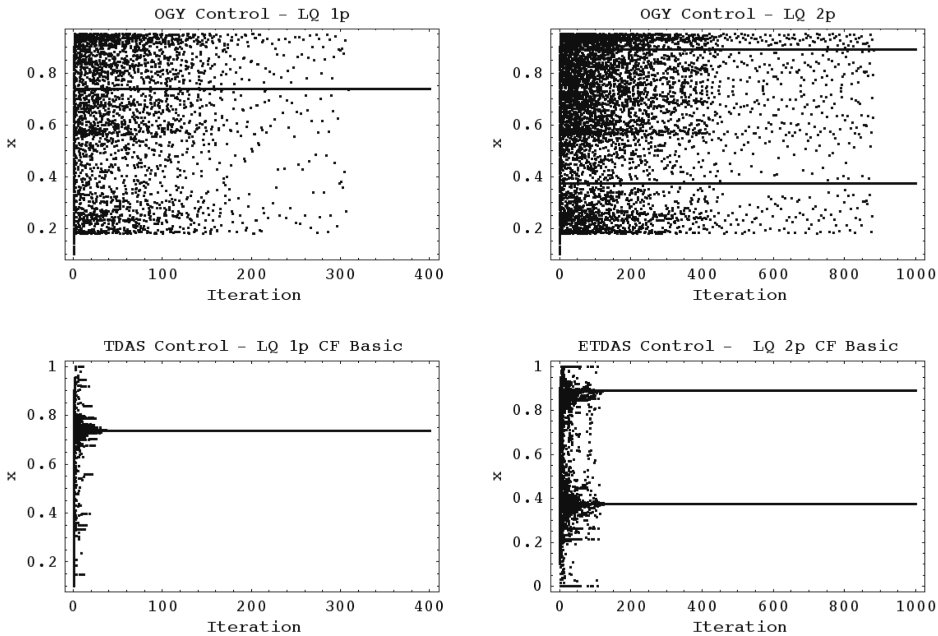


Figure 6.26 Comparison of OGY, optimized TDAS for p-1 orbit (left) and ETDAS for p-2 orbit (right), LQ, CF Basic, $0 < x_{initial} < 1$

Henon map

As with the previous case, optimized ETDAS was compared with standard OGY method. The comparison was done for these two cases: p-1 orbit and p-2 orbit. In the first case, ETDAS was set up identically as the best solution given by DERand2Bin (see Table 6.12.).

From Figure 6.27 it follows, that optimized ETDAS method steers the chaotic system very quickly to the stable state. The difference between two compared methods is very considerable because the OGY stabilize 50% of examples in first 200 iterations, whereas EDAS stabilize more than 50% of examples in first 10 iterations. From the similar comparison for logistic equation given in Figure 6.26 it can be clearly seen the difference between these two control methods increase together with higher dimension or complexity of controlled system. The performance of TDAS or ETDAS is almost the same whereas OGY needs twice more iterations to achieve stabilization. In this case the performance of ETDAS with Henon map is even better then the performance of TDAS with simpler one dimensional equation.

In the second case, ETDAS was set up identically as the best solution given by SOMA ATO (see Table 6.13). From Figure 6.27 it follows that optimized ETDAS method needs approximately 200 iterations to stabilize 50% of chaotic samples system at desired p-2 orbit. To stabilize the rest of the samples around 800 iterations are required. As in previous case the OGY stabilizes about 50% of examples in first 500 iterations and to stabilize all of the examples more than 1000 iterations are required.

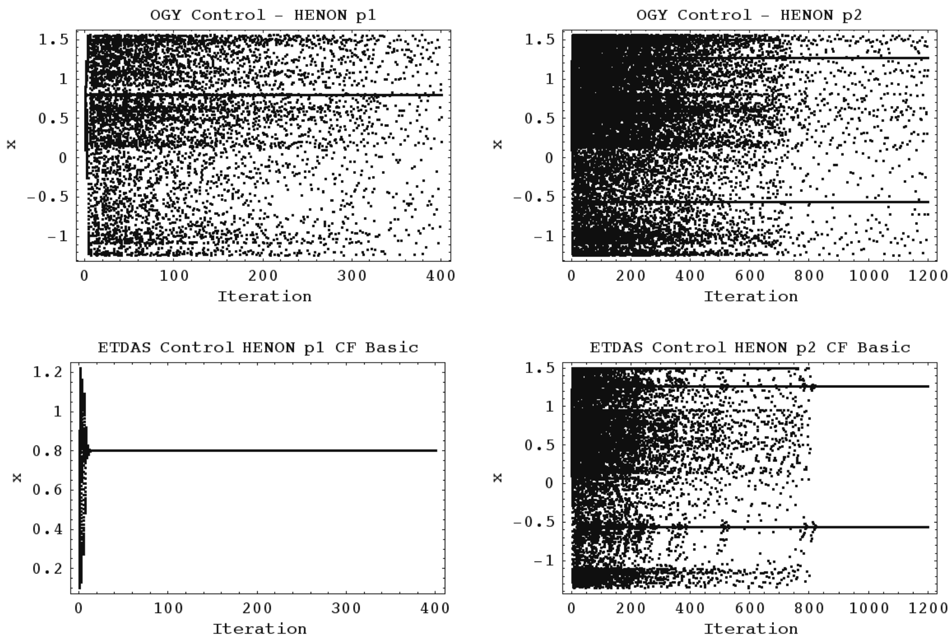


Figure 6.27 Comparison of OGY, optimized ETDAS for p-1 orbit (left) and ETDAS for p-2 orbit (right), HENON, CF Basic, $0 < x_{initial} < 1$

6.4 Simulation Results – Case study 2: Targeting CF Simple

This section presents the new proposal of the simplest targeting CF, which represents the first step in resolving the challenge of faster targeting and stabilizing of the chaotic systems. This CF is suitable only in the task of finding the optimum settings for p-1 orbit. Unlike the pre-mentioned CF Basic, it does not contain any penalization or other extra constraints. Therefore all the following optimizations were easy to implement and without any conspicuous computational-time demands. For the results in case of LQ please refer to in Table 6.19 - Table 6.20 and Figure 6.28 - Figure 6.29. And in case of the Henon map please see Table 6.21 - Table 6.22 and Figure 6.30 - Figure 6.31.

6.4.1 One-Dimensional Example

Based on obtained results, it may be stated that the control parameters estimated in the optimizations ensured fast reaching of a desired state, on average, about 98 iterations are required.

Table 6.19 Best individual solutions, LQ, p-1 orbit, CF Simple, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.5786	0.1030	0.0180	0.3745	0.3763	101	99
2	-0.5781	0.1050	0.0188	0.3760	0.3831	101	98
3	-0.5749	0.1053	0.0192	0.3758	0.3808	98	98
4	-0.5620	0.1044	0.0128	0.3748	0.3791	101	98

Table 6.20 Best individual solutions, LQ, p-1 orbit, CF Simple, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.5722	0.1033	0.0155	0.3745	0.3760	98	99
6	-0.5795	0.1032	0.0184	0.3748	0.3772	101	98
7	-0.5618	0.1043	0.0119	0.3747	0.3762	101	99
8	-0.5603	0.1029	0.0103	0.3738	0.3755	101	99
9	-0.5752	0.1032	0.0165	0.3744	0.3772	97	98
10	-0.5583	0.1043	0.0104	0.3744	0.3759	92	98

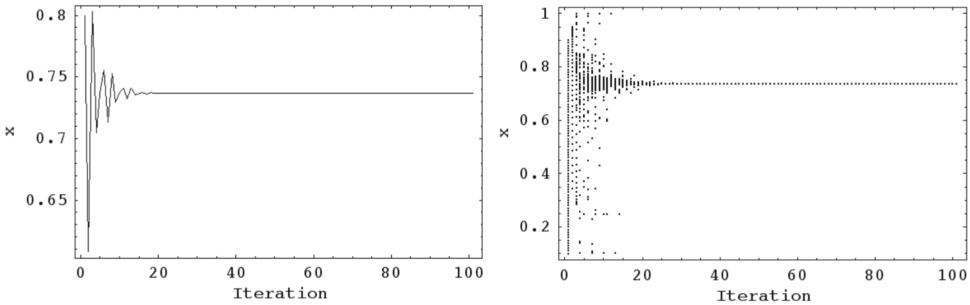


Figure 6.28 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Simple, p-1 orbit, SOMA ATO

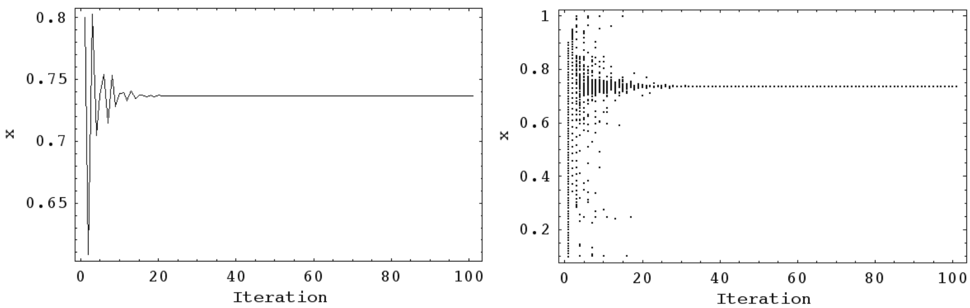


Figure 6.29 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Simple, p-1 orbit, DELocalToBest

6.4.2 Two-Dimensional Example

From the obtained results it follows that as in case of one dimensional example (LQ), this CF design also ensured fast reaching of a desired state, on average, for successful stabilization of chaotic system around 68 iterations were required.

Table 6.21 Best individual solutions, HENON, p-1 orbit, CF Simple, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.7004	0.3024	0.1679	0.2303	0.2310	74	68
2	-0.7002	0.2502	0.1680	0.2304	0.2332	76	63
3	-0.7000	0.3397	0.1683	0.2305	0.2321	72	65
4	-0.7005	0.3459	0.1679	0.2304	0.2316	73	65

Table 6.22 Best individual solutions, HENON, p-1 orbit, CF Simple, DE

EA	K	F_{max}	R	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.7004	0.4232	0.1681	0.2305	0.2314	70	70
6	-0.7002	0.4410	0.1680	0.2304	0.2320	74	69
7	-0.7010	0.2389	0.1672	0.2306	0.2316	72	70
8	-0.7003	0.3544	0.1680	0.2304	0.2312	74	71
9	-0.7005	0.4416	0.1680	0.2305	0.2320	72	70
10	-0.7003	0.2365	0.1680	0.2304	0.2313	73	70

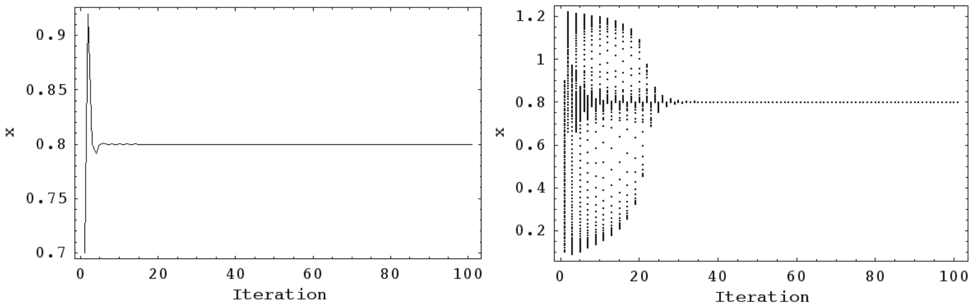


Figure 6.30 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Simple, p-1 orbit, SOMA ATO

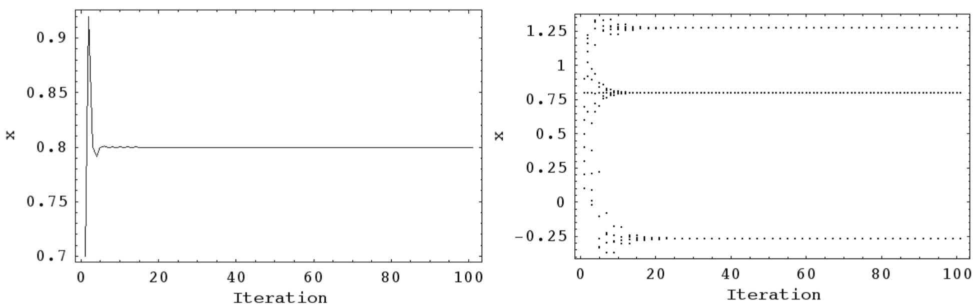


Figure 6.31 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Simple, p-1 orbit, DEBest1JIter

6.4.3 Conclusion of Results – Case Study 2

This partial conclusion and the subsequent ones do not contain any comparison with OGY method, but it is focused on brief description of main differences and problems which arise in the task of optimization of chaos control within this case study and also it is focused on the comparison of main results between all previous case studies.

Based on obtained results it may be stated that this CF design did not give any bigger improvement in the task of faster stabilization. These previous two CFs mostly concentrate on quality of stabilization than its quickness. Thus, the following case studies deals with the development of effective targeting CF. The results obtained in this case can be summarized in the following points:

- In case of Henon there is better average IStab value in comparison with previous case study – CF Basic.
- In case of LQ the results are similar.
- Next case study is focused on better proposal of targeting CF, which is suitable also for higher periodic orbits. Here tested CF Simple can not be used due to devaluation of possible best solution by phase shifting or including the initial chaotic stage into whole CF value.
- For the comparison of average number of iterations required for successful stabilization see Table 6.23 and Table 6.24.

Table 6.23 Comparison of Average IStab values – LQ – Case studies 1-2

UPO	**p-1**		**p-2**		**p-3**	
EA Version	SOMA	DE	SOMA	DE	SOMA	DE
CF Basic	97	97	123	123	197	218
CF Simple	98	99	-	-	-	-

Table 6.24 Comparison of Average IStab values – HENON – Case studies 1-2

UPO	**p-1**		**p-2**		**p-3**	
EA Version	SOMA	DE	SOMA	DE	SOMA	DE
CF Basic	77	75	124	125	122	122
CF Simple	65	70	-	-	-	-

6.5 Simulation Results – Case study 3: Targeting CF NA

This case study presents the new CF with extra constraints, which is an advancement of the first described and tested CF Basic. The optimizations were performed for p-1 and p-2 orbit. This CF can be considered as the second and at the same time significant step towards to time-optimal solutions.

6.5.1 One-Dimensional Example

1p

As can be seen from the best individual solutions for SOMA and DE given in Table 6.25 and Table 6.26, and the simulation results, which are depicted in Figure 6.32 and Figure 6.33, the significant improvement from the point of view of average IStab Value was reached in comparison with previous two CF Basic and CF simple. On average, only 31 iterations are required in the elimination of chaos and stabilization. The best individual solutions were given by SOMA ATO and DEBest1JIter. The *SC* value was set to 1.10^{-16} in this case.

Table 6.25 Best individual solutions, LQ, p-1 orbit, CF NA, SOMA

EA	K	F_{max}	R	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.9335	0.3195	0.4977	$2.80.10^{-15}$	$3.15.10^{-15}$	28	31
2	-0.9254	0.3431	0.4886	$3.00.10^{-15}$	$3.48.10^{-15}$	30	35
3	-0.9335	0.3195	0.4977	$2.80.10^{-15}$	$3.33.10^{-15}$	28	33
4	-0.9318	0.3162	0.4964	$2.90.10^{-15}$	$2.89.10^{-15}$	29	32

Table 6.26 Best individual solutions, LQ, p-1 orbit, CF NA, DE

EA	K	F_{max}	R	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.9344	0.3171	0.4999	$2.80.10^{-15}$	$3.08.10^{-15}$	28	31
6	-0.8984	0.3766	0.4727	$2.90.10^{-15}$	$3.13.10^{-15}$	29	31
7	-0.9344	0.3171	0.4999	$2.80.10^{-15}$	$3.07.10^{-15}$	28	31
8	-0.9344	0.3171	0.4999	$2.80.10^{-15}$	$2.99.10^{-15}$	28	30
9	-0.8984	0.3766	0.4727	$2.90.10^{-15}$	$3.13.10^{-15}$	29	31
10	-0.9319	0.3168	0.4982	$2.70.10^{-15}$	$3.05.10^{-15}$	27	30

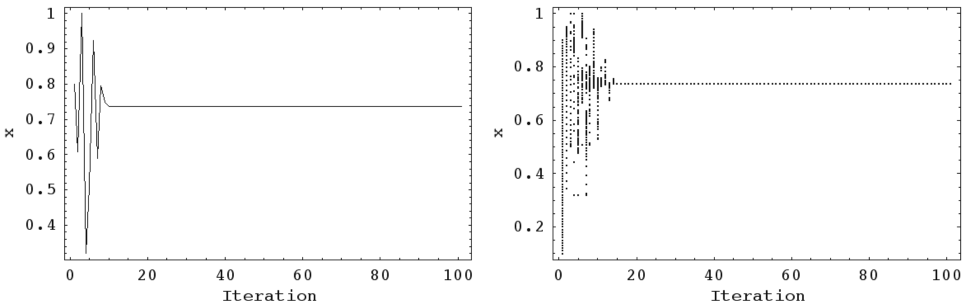


Figure 6.32 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF NA, p-1 orbit, SOMA ATO

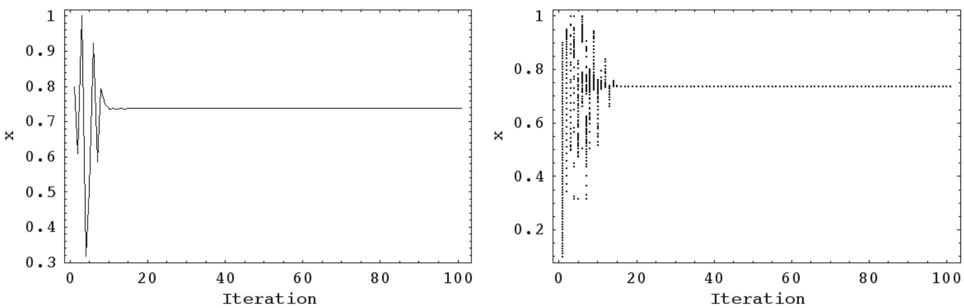


Figure 6.33 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF NA, p-1 orbit, DEBest1JIter

2p

A considerable decrease of IStab value occurs, but only for average value computed from all 50 repeated simulations of EA version, particularly in the case of SOMA. The IStab value for the best individual solution was only slightly reduced. This phenomenon is also discussed in details in the conclusion of this case study.

On average, about 73 (SOMA) or 109 (DE) iterations are required. The best individual solutions were given by SOMA ATR and DEBest1JIter. The *SC* value was set to 1.10^{-8} in this case. For the simulation results please refer to Table 6.27 - Table 6.28, and Figure 6.34 - Figure 6.35.

Table 6.27 Best individual solutions, LQ, p-2 orbit, CF NA, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	0.4469	0.3425	0.2653	$3.95.10^{-4}$	2.0664	124	73
2	0.5435	0.2107	0.4780	$1.14.10^{-6}$	2.3981	114	69
3	0.4389	0.2133	0.2611	$1.60.10^{-4}$	2.5536	117	76
4	0.4611	0.2055	0.3063	$5.31.10^{-6}$	1.7462	130	74

Table 6.28 Best individual solutions, LQ, p-2 orbit, CF NA, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	0.5413	0.0882	0.4693	$1.29.10^{-6}$	0.4573	129	104
6	0.5451	0.0948	0.4758	$1.29.10^{-6}$	0.4300	129	99
7	0.5560	0.0296	0.4883	$1.26.10^{-6}$	0.4270	126	106
8	0.5499	0.0666	0.4923	$1.22.10^{-6}$	0.3217	122	107
9	0.5641	0.1467	0.4441	$1.66.10^{-6}$	0.7630	131	106
10	0.5417	0.0355	0.4756	$1.20.10^{-6}$	0.5368	120	89

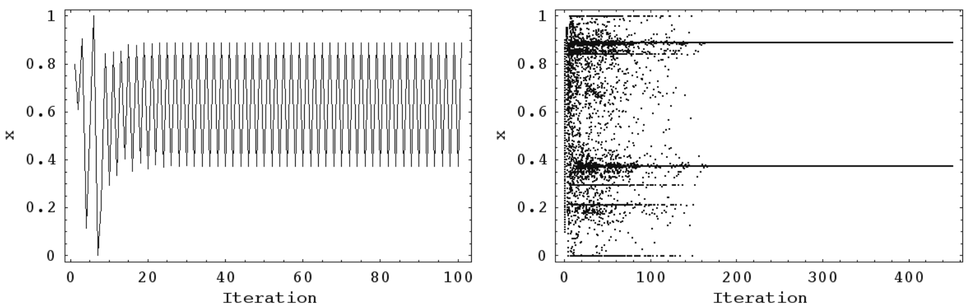


Figure 6.34 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF NA, p-2 orbit, SOMA ATR

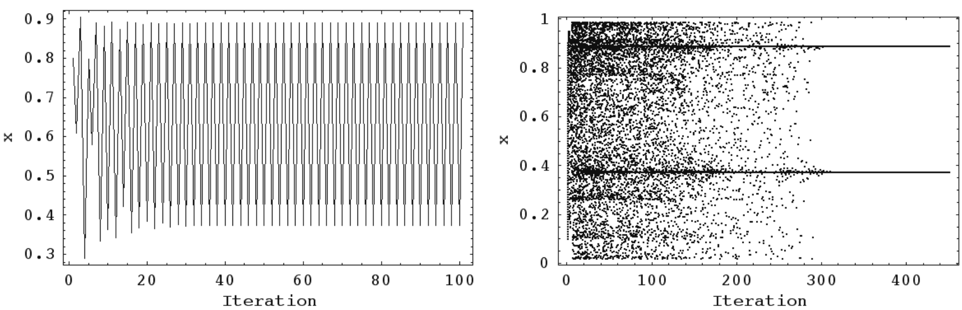


Figure 6.35 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF NA, p-2 orbit, DEBest1JIter

6.5.2 Two-Dimensional Example

1p

From the presented results in Table 6.29 - Table 6.30, and the simulation results depicted in Figure 6.36 and Figure 6.37, it follows that all versions of SOMA and DE have found similar results as the best solution and primarily from comparison of these results with previous two case studies, it can be clearly seen that significant decrease of IStab value occurs as in the case of one dimensional example (LQ).

On average, for stabilization of this UPO only 48 iterations are required. The best individual solutions were given by SOMA ATO and DEBest1JIter. The *SC* value was set to 1.10^{-16} in this case.

Table 6.29 Best individual solutions, HENON, p-1 orbit, CF NA, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.8954	0.2997	0.2612	$4.20.10^{-15}$	$9.58.10^{-15}$	42	49
2	-0.8800	0.1602	0.2330	$4.10.10^{-15}$	$7.04.10^{-15}$	41	51
3	-0.8800	0.1602	0.2330	$4.10.10^{-15}$	$6.21.10^{-15}$	41	49
4	-0.8566	0.2148	0.2031	$4.00.10^{-15}$	$6.57.10^{-15}$	40	46

Table 6.30 Best individual solutions, HENON, p-1 orbit, CF NA, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.8454	0.2282	0.2044	$4.00.10^{-15}$	$4.65.10^{-15}$	40	47
6	-0.8713	0.2571	0.2328	$4.20.10^{-15}$	$4.74.10^{-15}$	42	47
7	-0.8650	0.4728	0.2137	$4.10.10^{-15}$	$4.80.10^{-15}$	41	48
8	-0.8634	0.1699	0.2150	$3.90.10^{-15}$	$4.58.10^{-15}$	39	46
9	-0.8634	0.1699	0.2150	$3.90.10^{-15}$	$4.83.10^{-15}$	39	48
10	-0.8650	0.4728	0.2137	$4.10.10^{-15}$	$5.34.10^{-15}$	41	48

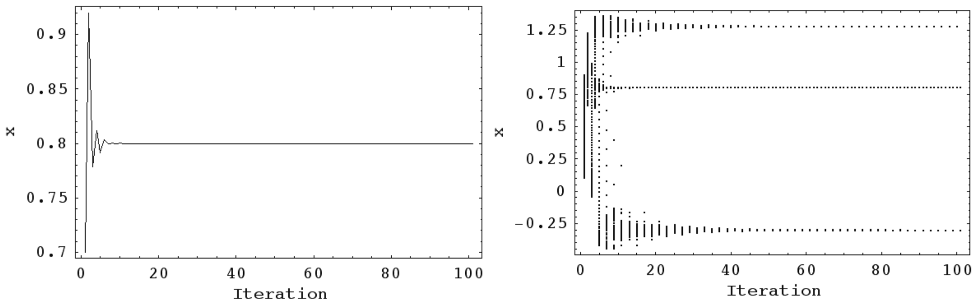


Figure 6.36 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF NA, p-1 orbit, SOMA ATAA

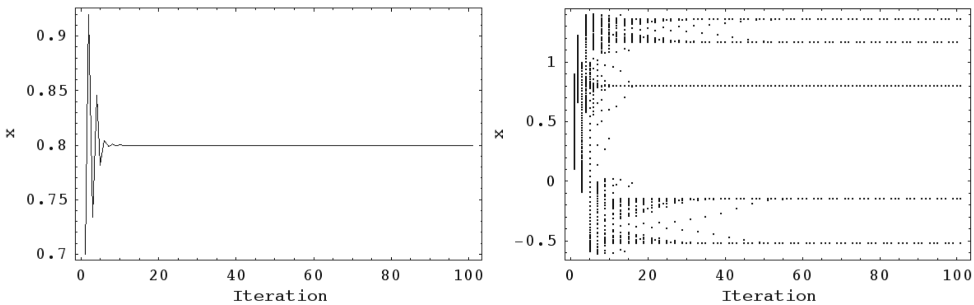


Figure 6.37 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF NA, p-1 orbit, DELocalToBest

2p

As can be see from presented results in Table 6.31 - Table 6.32, and Figure 6.38 - Figure 6.39, all versions of SOMA and DE have found relatively similar results for the best solution, however from comparison with CF basic, it seems that this optimization gives worse results from the point of view of final CF value. This fact is caused by penalizing of CF with parameters *NI* and *SC*. But from the comparison of average needed IStab can be clearly seen also significant improvement as in the case of p-1 orbit. On the other hand, this CF repeatedly give results with very low value of minimum IStab, however with very high CF value and either temporary stabilization or none at all. This is probably caused by highly nonlinear and erratic CF surface and the higher difficulty of stabilization of p-2 orbit for Henon map. Also here occurred the phenomenon of considerable decrease of IStab value only for average value computed from all 50 repeated simulations of EA version, particularly in case of SOMA. The IStab value for the best individual solution was also only slightly reduced.

The parameter settings obtained by means of CF NA required on average about 84 (SOMA) or 110 (DE) iterations for stabilization of this UPO. The best individual solutions were given by SOMA ATA and DERand1DIter. The *SC* value was set to 1.10^{-8} in this case (identically as for LQ).

Table 6.31 Best individual solutions, HENON, p-2 orbit, CF NA, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	0.3756	0.1412	0.2269	$2.51.10^{-5}$	4.2739	128	95
2	0.3984	0.1334	0.2400	$2.12.10^{-5}$	7.3187	124	78
3	0.4656	0.1960	0.4226	$1.31.10^{-6}$	7.1471	131	77
4	0.5191	0.1443	0.4041	$7.55.10^{-5}$	5.3401	130	86

Table 6.32 Best individual solutions, HENON, p-2 orbit, CF NA, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	0.4131	0.1823	0.3167	$2.97.10^{-5}$	0.9755	130	111
6	0.3961	0.1762	0.2743	$5.52.10^{-6}$	2.1411	131	109
7	0.4705	0.1903	0.3387	$6.15.10^{-6}$	2.0737	131	106
8	0.4250	0.1961	0.3081	$8.71.10^{-6}$	1.8350	129	113
9	0.5274	0.0833	0.4380	$1.35.10^{-6}$	1.6335	129	108
10	0.4330	0.1598	0.3080	$1.34.10^{-5}$	1.3303	131	116

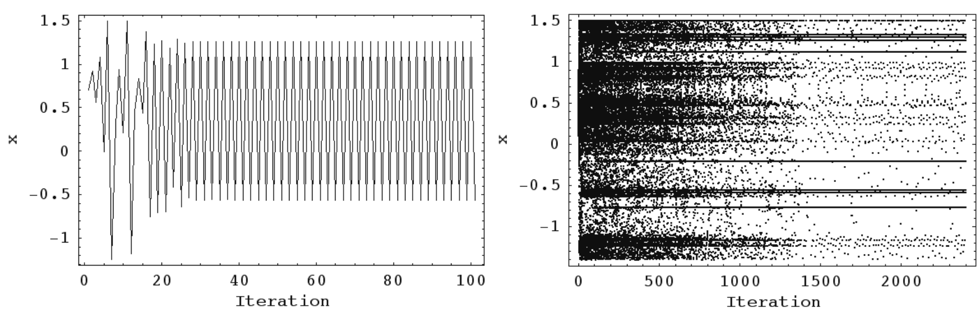


Figure 6.38 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF NA, p-2 orbit, SOMA ATA

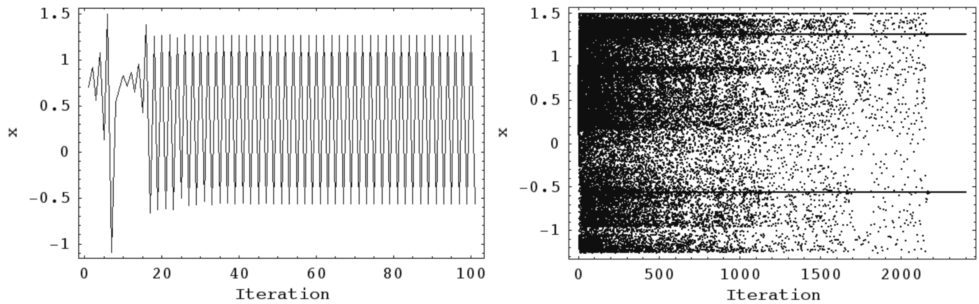


Figure 6.39 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF NA, p-2 orbit, DERand1DIter

6.5.3 Conclusion of Results – Case Study 3

The presented data lends weight to the argument that this CF design gives significant improvement in the task of faster stabilization. On the other hand, this CF gives a lot of inconsistent results with temporary or poor stabilization on desired UPO although with low IStab value. Also the *SC* value was determined on the basis of average results leads to successful stabilization in case of CF Basic. The following two case studies was proposed to solve the problem with determination of *SC* value in advance. Due to the difficulty with correct setting the SC val. for p-4 orbit, this UPO was not included in this case study. The results obtained in this case can be summarized in the following points:

- significant improvement from the point of view of IStab value for both systems, particularly in case of p-1 orbit.
- In case of p-2 orbit there was also improvement, but EA repeatedly give results with very low value of minimum IStab, however with very high CF value and either temporary stabilization or none at all. This can be caused by highly erratic CF surface and included deviation in multi-value optimization by average value *SC*.
- For the comparison of average number of iterations required for successful stabilization see Table 6.33 and Table 6.34. The value in braces represents corrected one, which shows the average IStab value only for solutions, which leads to successful stabilization.

Table 6.33 Comparison of Average IStab values – LQ – Case studies 1-3

UPO	p-1		p-2		p-3	
EA Version	SOMA	DE	SOMA	DE	SOMA	DE
CF Basic	97	97	123	123	197	218
CF Simple	98	99	-	-	-	-
CF NA	33	31	73 (109)	102 (116)	-	-

Table 6.34 Comparison of Average IStab values – HENON – Case studies 1-3

UPO	p-1		p-2		p-3	
EA Version	SOMA	DE	SOMA	DE	SOMA	DE
CF Basic	77	75	124	125	122	122
CF Simple	65	70	-	-	-	-
CF NA	49	47	84 (114)	110 (118)	-	-

6.6 Simulation Results – Case study 4: CF Targ1

This case study is focused on testing of upgraded pre-mentioned CF NA. This improved CF Targ1 is able to firstly, resolve the problem with determination of optimization constant *SC* in advance on the basis of previous results and secondly, allow to use any arbitrary UPO.

6.6.1 One-Dimensional Example

1p

The best individual solutions for SOMA and DE are given in Table 6.35 and Table 6.36, and the simulation results are depicted in Figure 6.40 and Figure 6.41. Also, here it can be seen another slight improvement from the point of view of IStab value for the best individual solutions in comparison with three previous case studies. This CF gives very similar results as previous CF NA, because the automatically computed SC value always reach 1.10^{-16}, i.e. the same as manually set in case of CF NA. The only difference lies in different CF surface, which allowed finding of faster stabilizing solutions, whereas the average IStab value is the same (on average, around 31 iterations). The best individual solutions were given by SOMA ATO and DELocalToBest.

Table 6.35 Best individual solutions, LQ, p-1 orbit, CF Targ1, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.9326	0.1674	0.4975	$2.70.10^{-15}$	$3.10.10^{-15}$	27	31
2	-0.9270	0.3203	0.4910	$2.90.10^{-15}$	$3.53.10^{-15}$	29	35
3	-0.9265	0.3182	0.4942	$2.80.10^{-15}$	$3.39.10^{-15}$	28	34
4	-0.9265	0.3182	0.4942	$2.80.10^{-15}$	$3.26.10^{-15}$	28	33

Table 6.36 Best individual solutions, LQ, p-1 orbit, CF Targ1, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.9252	0.3228	0.4908	$2.90.10^{-15}$	$3.12.10^{-15}$	29	31
6	-0.9252	0.3228	0.4908	$2.90.10^{-15}$	$3.12.10^{-15}$	29	31
7	-0.9296	0.3193	0.4959	$2.80.10^{-15}$	$3.09.10^{-15}$	28	31
8	-0.9296	0.3193	0.4959	$2.80.10^{-15}$	$3.02.10^{-15}$	28	30
9	-0.9296	0.3193	0.4959	$2.80.10^{-15}$	$3.16.10^{-15}$	28	32
10	-0.9296	0.3193	0.4959	$2.80.10^{-15}$	$3.07.10^{-15}$	28	31

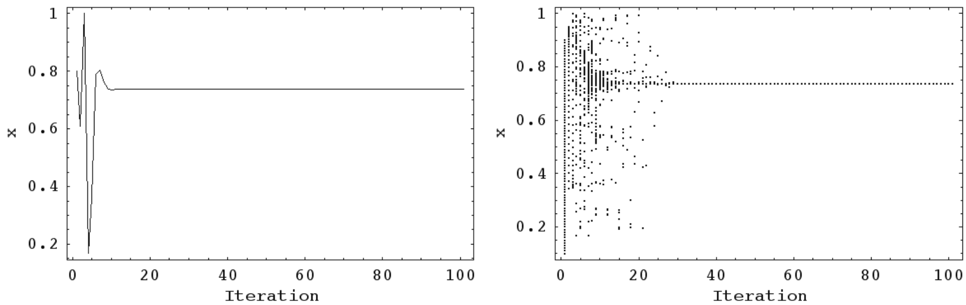


Figure 6.40 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ1, p-1 orbit, SOMA ATO

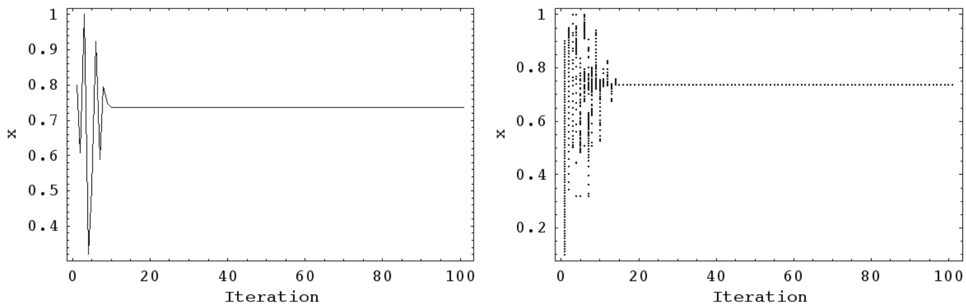


Figure 6.41 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ1, p-1 orbit, DELocalToBest

2p

The simulation results (see Table 6.37 - Table 6.38, and Figure 6.42 - Figure 6.43) also show slight improvement in comparison with CF NA. The small change in CF design allowed to reach significantly lower CF values of the best individual solutions. Thus, it is allowed to reach higher-quality stabilization, because the final CF value was not influenced by the dominant part of sum, which is represented by the multiplication of *SC* and *NI*, whereas it was influenced by the basic part of CF (quality of stabilization), which could by suppressed in case of wrong determination of *SC* value. But also in this case occurs the phenomenon of solutions with very low value of minimum IStab, however with very high CF value and either temporary stabilization or none at all.

Table 6.37 Best individual solutions, LQ, p-2 orbit, CF Targ1, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	0.4867	0.2103	0.3595	$8.69.10^{-9}$	1.9670	130	83
2	0.5351	0.0378	0.4648	$5.37.10^{-13}$	2.3665	126	73
3	0.5995	0.1823	0.4929	$6.63.10^{-9}$	2.3827	122	78
4	0.5449	0.0834	0.4815	$8.30.10^{-13}$	2.0749	127	77

Table 6.38 Best individual solutions, LQ, p-2 orbit, CF Targ1, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	0.5358	0.0374	0.4688	$8.85.10^{-13}$	0.8076	120	100
6	0.5460	0.0776	0.4842	$1.74.10^{-12}$	0.8092	129	102
7	0.5127	0.0965	0.4150	$2.15.10^{-6}$	0.6010	128	102
8	0.5413	0.1752	0.4768	$2.12.10^{-12}$	0.3811	121	116
9	0.4917	0.0429	0.3424	$3.31.10^{-6}$	0.7363	128	97
10	0.5491	0.0316	0.4833	$9.50.10^{-13}$	0.3899	126	108

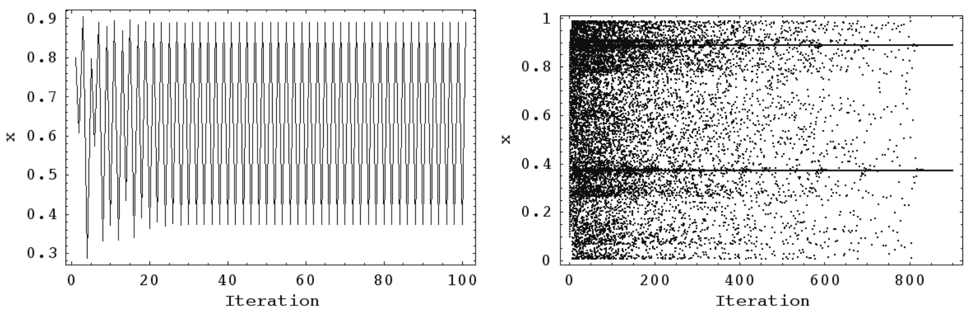


Figure 6.42 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ1, p-2 orbit, SOMA ATR

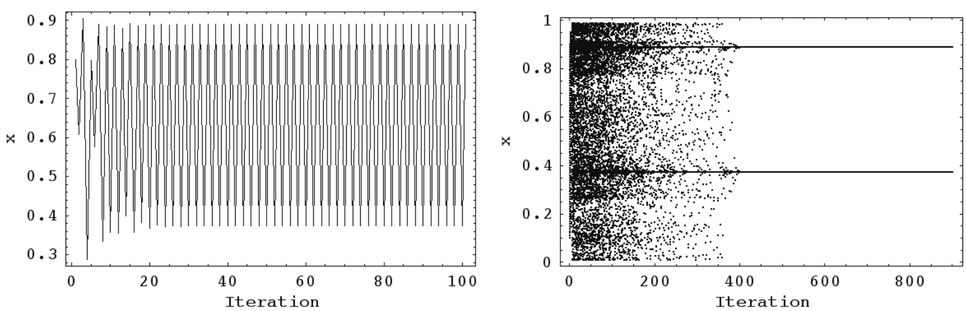


Figure 6.43 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ1, p-2 orbit, DERand1Bin

4p

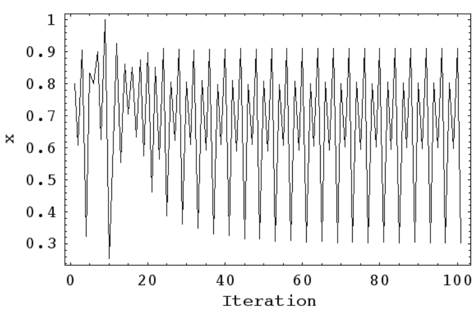
From the optimization results given in Table 6.39 and Table 6.40, it follows that control method reached better performance from the point of view of faster stabilization in comparison with CF Basic. As can be seen in Figure 6.44 and Figure 6.45, even this CF does not give results, which lead to stabilization on desired UPO for simulation with uniformly distributed initial conditions. Also it is not possible to overlook bigger difference in performance of both EAs, in case of proportion of IStab value to Average IStab value. The elimination of chaos required on average around 205 iterations for this problematic UPO.

Table 6.39 Best individual solutions, LQ, p-4 orbit, CF Targ1, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.5408	0.2796	0.6268	0.0068	5.1813	177	72
2	-0.5135	0.2083	0.5835	0.0062	4.5088	181	74
3	-0.5215	0.2948	0.6115	0.0061	4.5811	181	84
4	-0.5218	0.3351	0.6130	0.0070	4.5158	209	78

Table 6.40 Best individual solutions, LQ, p-4 orbit, CF Targ1, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.5238	0.1550	0.6176	0.0059	3.1370	175	110
6	-0.5241	0.6479	0.6165	0.0066	2.3044	181	124
7	-0.5233	0.3899	0.6170	0.0067	2.3346	185	129
8	-0.5481	0.9461	0.6283	0.0067	2.8611	181	117
9	-0.5467	0.3033	0.6256	0.0071	2.2020	189	136
10	-0.5785	0.1602	0.6665	0.0075	2.5950	204	110



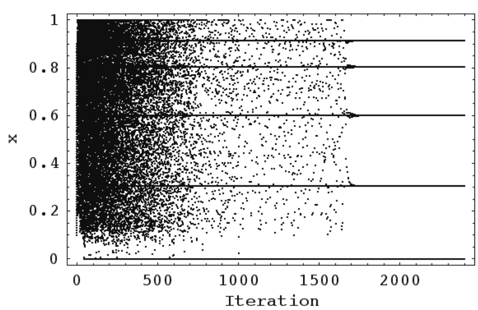


Figure 6.44 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ1, p-4 orbit, SOMA ATR

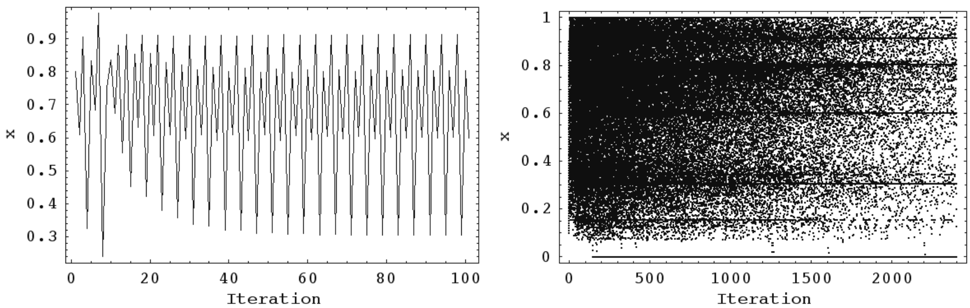


Figure 6.45 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ1, p-4 orbit, DERand1Bin

6.6.2 Two-Dimensional Example

1p

The overview of simulation data given in Table 6.41 and Table 6.42, and the simulation results depicted in Figure 6.46 and Figure 6.47, shows that the system was rapidly targeted and stabilized in p-1 orbit. The performance of this CF design is very similar to previous CF NA. As described above, in the case of LQ and p-1 orbit the automatically computed SC value always reach 1.10^{-16}, i.e. the same as manually set in case of CF NA. The small difference between CF basic and both targeting CF NA and CF Targ1 lies in the unpleasant fact that these CF allowed the finding of faster stabilizing solutions, nevertheless these solutions are not suitable for complex simulation with uniformly distributed initial conditions. For successful stabilization, on average, around 48 iterations are required. The best individual solutions were given by SOMA ATA and DELocalToBest.

Table 6.41 Best individual solutions, HENON, p-1 orbit, CF Targ1, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.8508	0.2127	0.2072	$4.00.10^{-15}$	$9.71.10^{-15}$	40	50
2	-0.8508	0.2127	0.2072	$4.00.10^{-15}$	$7.59.10^{-15}$	40	52
3	-0.8508	0.2127	0.2072	$4.00.10^{-15}$	$7.03.10^{-15}$	40	48
4	-0.8804	0.1967	0.2344	$4.10.10^{-15}$	$6.75.10^{-15}$	41	47

Table 6.42 Best individual solutions, HENON, p-1 orbit, CF Targ1, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.8525	0.2927	0.2165	$4.10.10^{-15}$	$4.67.10^{-15}$	41	47
6	-0.8525	0.2927	0.2165	$4.10.10^{-15}$	$4.72.10^{-15}$	41	47
7	-0.8767	0.2826	0.2418	$4.00.10^{-15}$	$4.74.10^{-15}$	40	47
8	-0.8686	0.2891	0.2349	$3.90.10^{-15}$	$4.61.10^{-15}$	39	46
9	-0.8767	0.2826	0.2418	$4.00.10^{-15}$	$4.76.10^{-15}$	40	48
10	-0.8525	0.2927	0.2165	$4.10.10^{-15}$	$5.22.10^{-15}$	41	48

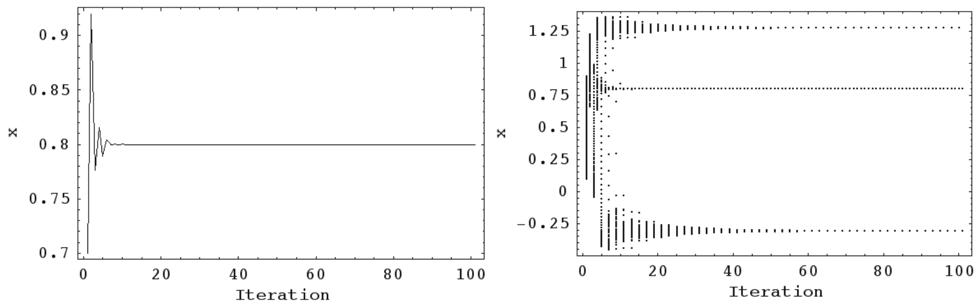


Figure 6.46 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ1, p-1 orbit, SOMA ATA

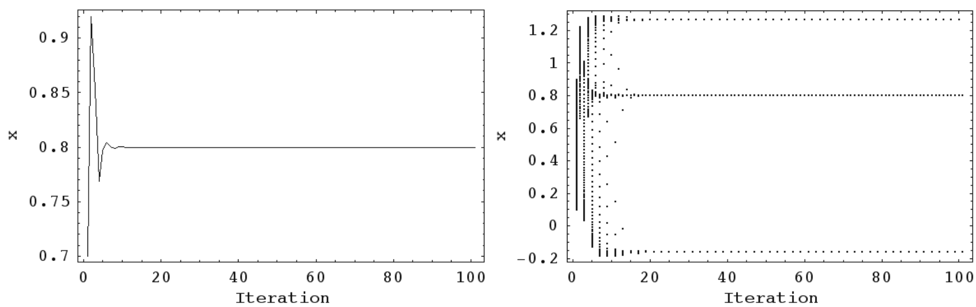


Figure 6.47 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ1, p-1 orbit, DELocalToBest

2p

In comparison with p-1orbit, the control method is not able to reach “exact” stabilization of p-2 orbit. Thus the non-penalized and multiplied basic CF value is always greater than zero. From Table 6.43 and Table 6.44, it follows that all versions of SOMA and DE have found relatively different results for the best solution, from the point of view of CF value. The simulation results are depicted in Figure 6.48 and Figure 6.49. For successful stabilization of p-2 orbit, on average, around 72 (SOMA) or 109 (DE) iterations are required.

Table 6.43 Best individual solutions, HENON, p-2 orbit, CF Targ1, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	0.3528	0.1445	0.1786	$1.39.10^{-4}$	8.0902	124	62
2	0.4074	0.1559	0.2663	$3.46.10^{-5}$	6.5677	122	83
3	0.4188	0.1599	0.2590	$1.01.10^{-4}$	6.5002	129	78
4	0.4018	0.1452	0.2449	$6.72.10^{-5}$	8.7303	126	66

Table 6.44 Best individual solutions, HENON, p-2 orbit, CF Targ1, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	0.4302	0.2067	0.3221	$2.73.10^{-7}$	2.4583	130	109
6	0.5357	0.1754	0.4956	$4.53.10^{-8}$	1.2752	128	112
7	0.3710	0.1562	0.2245	$1.12.10^{-5}$	1.5961	124	106
8	0.4907	0.1914	0.3590	$2.00.10^{-5}$	1.0713	125	113
9	0.4392	0.1415	0.3149	$2.31.10^{-6}$	1.6591	128	104
10	0.3564	0.2087	0.1775	$1.10.10^{-4}$	1.0696	123	112

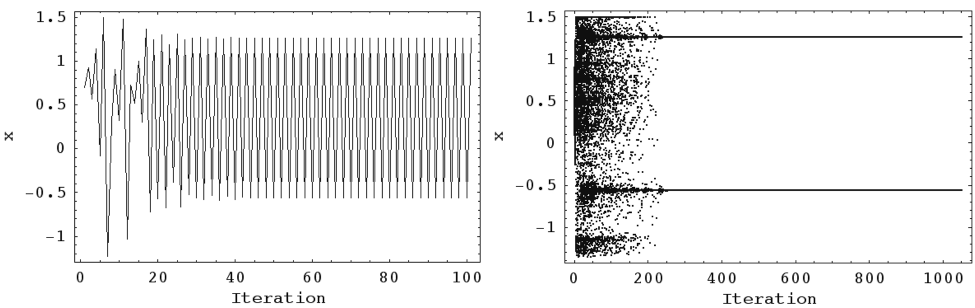


Figure 6.48 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ1, p-2 orbit, SOMA ATR

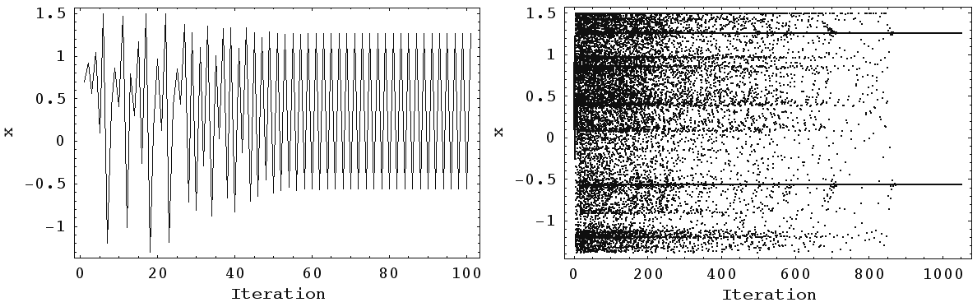


Figure 6.49 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ1, p-2 orbit, DERand2Bin

4p

When comparing the results given by CF Basic and the new results presented in Table 6.45 - Table 6.46, and Figure 6.50 - Figure 6.51, it is obvious, that they are similar without any significant improvement (if corrected results are taken into consideration). For further illustration about exact numbers see the following partial conclusion of this case study.

Table 6.45 Best individual solutions, HENON, p-4 orbit, CF Targ1, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.3784	0.2452	0.4188	$2.15.10^{-6}$	0.2838	129	108
2	-0.3808	0.2254	0.4338	$7.46.10^{-7}$	0.6435	129	109
3	-0.3938	0.1270	0.4572	$2.09.10^{-6}$	0.4052	129	109
4	-0.4153	0.2786	0.4532	$9.46.10^{-6}$	0.3324	125	114

Table 6.46 Best individual solutions, HENON, p-4 orbit, CF Targ1, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.3850	0.1066	0.4348	$2.25.10^{-6}$	0.0281	125	122
6	-0.4131	0.2809	0.4606	$4.82.10^{-6}$	0.1393	129	122
7	-0.3947	0.1044	0.4541	$3.83.10^{-6}$	0.0789	125	120
8	-0.3784	0.2539	0.4329	$5.89.10^{-7}$	0.0902	129	121
9	-0.3830	0.1386	0.4416	$1.64.10^{-6}$	0.1400	129	119
10	-0.3781	0.3154	0.4261	$1.50.10^{-6}$	0.0680	125	124

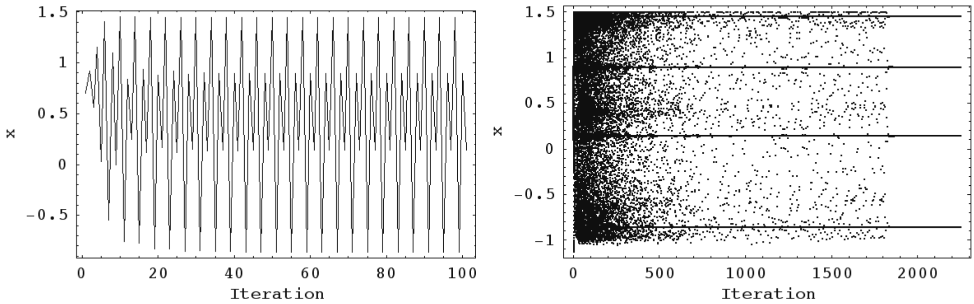


Figure 6.50 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ1, p-4 orbit, SOMA ATR

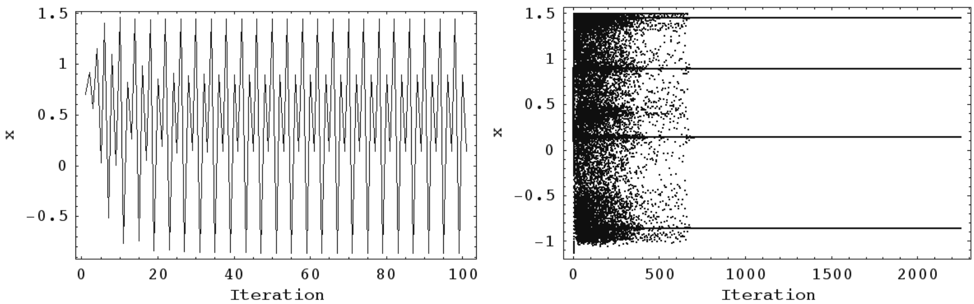


Figure 6.51 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ1, p-4 orbit, DELocalToBest

6.6.3 Conclusion of Results – Case Study 4

From the presented results it follows that this CF design gives only slight improvement in the task of faster stabilization in comparison with previous case study (CF NA). It is necessary to mention, that this CF design mostly allowed to reach significantly lower CF values of the best individual solutions and higher-quality stabilization due to the fact that the final CF value was not influenced by dominant multiplication of *SC* and *NI*, in case of wrong determination of *SC* value. This CF allowed to solve the problem with difficulty of correct setting the SC value for p-4 orbit in advance, because it could be very difficult task to determine the correct value, which does not influence searching in such nonlinear and erratic CF surface in case of p-4 orbit (see Appendix), especially for logistic equation. Also here occurs the phenomenon of difference in performance of both EAs, in case of proportion of IStab value to Average

IStab value, thus the proportion of solutions with either perfect stabilization or temporary or possibly none at all. The next presented case study shows illustratively how slight change in CF design could impact on performance of control technique. The results obtained in this case can be summarized in following main points:

- Slight improvement in comparison with CF NA.
- Problem with determination of *SC* value in advance has been solved.
- Slightly better performance of EA in case of LQ from the point of view of percentage successfulness of giving the solutions leading to stabilization. In case of Henon map, the hard task seems to be p-2 orbit.
- When comparing SOMA and DE, the first one mostly found the lowest CF value, but lots of solutions gives the very fast reaching of desired UPO, but only temporary or un-successful stabilization. Meanwhile DE gives more "stabilization securing" solutions.
- For the comparison of average number of iterations required for successful stabilization see Table 6.47 and Table 6.48. The value in braces represents corrected one, which shows the average IStab value only for solutions, which leads to successful stabilization.

Table 6.47 Comparison of Average IStab values – LQ – Case studies 1-4

UPO	**p-1**		**p-2**		**p-3**	
EA Version	SOMA	DE	SOMA	DE	SOMA	DE
CF Basic	97	97	123	123	197	218
CF Simple	98	99	-	-	-	-
CF NA	33	31	73 (109)	102 (116)	-	-
CF Targ1	33	31	78 (112)	104 (115)	77 (195)	121 (215)

Table 6.48 Comparison of Average IStab values – HENON – Case studies 1-4

UPO	p-1		p-2		p-3	
EA Version	SOMA	DE	SOMA	DE	SOMA	DE
CF Basic	77	75	124	125	122	122
CF Simple	65	70	-	-	-	-
CF NA	49	47	84 (114)	110 (118)	-	-
CF Targ1	49	47	72 (113)	109 (118)	110 (121)	121 (123)

6.7 Simulation Results – Case study 5: CF Targ2

The CF presented in this section shows how can only a slight change in CF design and different approach in the upgrading of previous introduced CF NA (case study 3) positively influenced the obtained results, which are presented in Table 6.49 - Table 6.60, and Figure 6.52 - Figure 6.63. From these results, a general conclusion can be made about the effectiveness in encountering the previous shortcoming of the all pre-mentioned CFs. This new CF is able to firstly, successfully resolve the issue of fast stabilization and secondly, adds more robustness to the execution of the heuristic.

6.7.1 One-Dimensional Example

1p

From the presented results, it can be seen that another slight improvement occurs in the task of faster stabilization. For the stabilization of LQ at the fixed point, on average, around 30 iterations are required. The best individual solutions were given by SOMA ATO and DERand1DIter. For the simulation data please see Table 6.49 - Table 6.50, and Figure 6.52 - Figure 6.53.

Table 6.49 Best individual solutions, LQ, p-1 orbit, CF Targ2, SOMA

EA	K	F_{max}	R	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.9205	0.2485	0.4879	$2.80.10^{-15}$	$2.99.10^{-15}$	28	30
2	-0.9301	0.3207	0.4931	$2.90.10^{-15}$	$3.29.10^{-15}$	29	33
3	-0.9205	0.2485	0.4879	$2.80.10^{-15}$	$3.16.10^{-15}$	28	31
4	-0.9205	0.2485	0.4879	$2.80.10^{-15}$	$3.14.10^{-15}$	28	31

Table 6.50 Best individual solutions, LQ, p-1 orbit, CF Targ2, DE

EA	K	F_{max}	R	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.9321	0.1759	0.4981	$2.80.10^{-15}$	$3.01.10^{-15}$	28	30
6	-0.9346	0.2521	0.4981	$2.90.10^{-15}$	$3.07.10^{-15}$	29	31
7	-0.9321	0.1759	0.4981	$2.80.10^{-15}$	$3.03.10^{-15}$	28	30
8	-0.9321	0.1759	0.4981	$2.80.10^{-15}$	$2.96.10^{-15}$	28	30
9	-0.9353	0.2528	0.4992	$2.70.10^{-15}$	$3.04.10^{-15}$	27	30
10	-0.9321	0.1759	0.4981	$2.80.10^{-15}$	$3.01.10^{-15}$	28	30

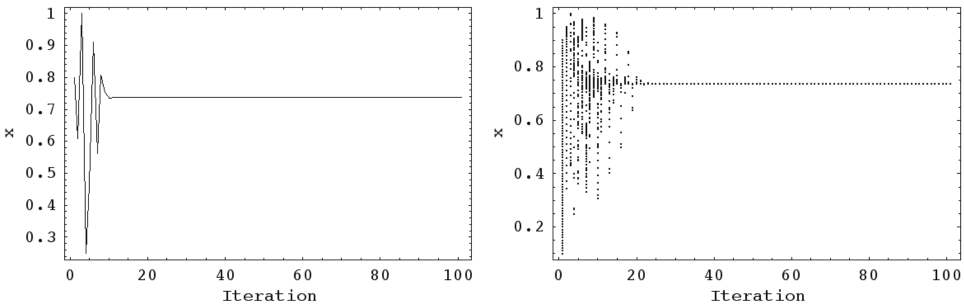


Figure 6.52 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ2, p-1 orbit, SOMA ATO

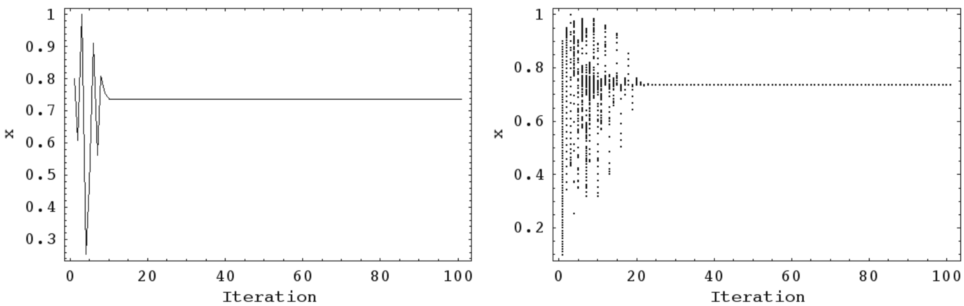


Figure 6.53 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ2, p-1 orbit, DERand1DIter

2p

The obtained results in 2p also lead to slight faster stabilization with comparison with CF Targ1 and other previous case studies. Moreover, it is possible to say that in this case occurs intensifying of the unpleasant fact that this CF Targ2 allowed finding of faster stabilizing solutions, nevertheless this solutions are not suitable for complex simulation with uniformly distributed initial conditions. On average about 95 (SOMA) or 113 (DE) iterations are required. The best individual solutions were given by SOMA ATAA and DERand1DIter. The best individual solutions for SOMA and DE are given in Table 6.51 - Table 6.52, and the simulation results are depicted in Figure 6.54 - Figure 6.55.

Table 6.51 Best individual solutions, LQ, p-2 orbit, CF Targ2, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	0.5395	0.1377	0.4701	$1.60.10^{-14}$	0.0567	127	105
2	0.5810	0.0655	0.4614	$2.06.10^{-10}$	0.1383	131	78
3	0.5493	0.0355	0.4804	$1.29.10^{-13}$	0.0980	118	93
4	0.4439	0.2643	0.2730	$1.34.10^{-6}$	0.0582	120	103

Table 6.52 Best individual solutions, LQ, p-2 orbit, CF Targ2, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	0.5489	0.0870	0.4807	$1.37.10^{-13}$	0.0109	127	120
6	0.5631	0.0894	0.4937	$1.40.10^{-13}$	0.0240	131	105
7	0.4548	0.2532	0.2988	$1.54.10^{-7}$	0.0139	125	110
8	0.5641	0.2122	0.4951	$1.35.10^{-13}$	0.0060	124	118
9	0.5401	0.0394	0.4634	$1.77.10^{-14}$	0.0143	120	108
10	0.5403	0.1807	0.4776	$2.01.10^{-14}$	0.0095	120	115

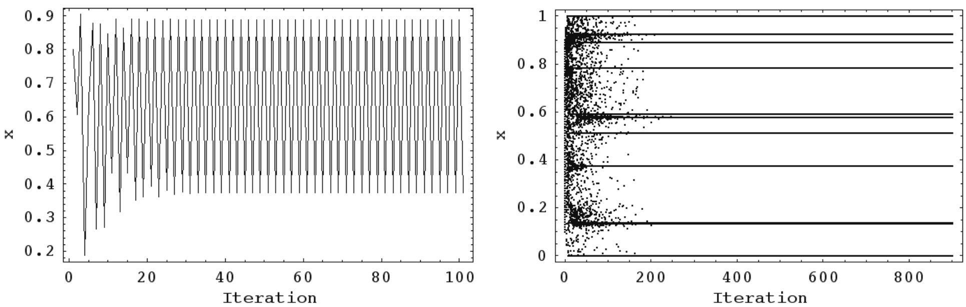


Figure 6.54 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ2, p-2 orbit, SOMA ATO

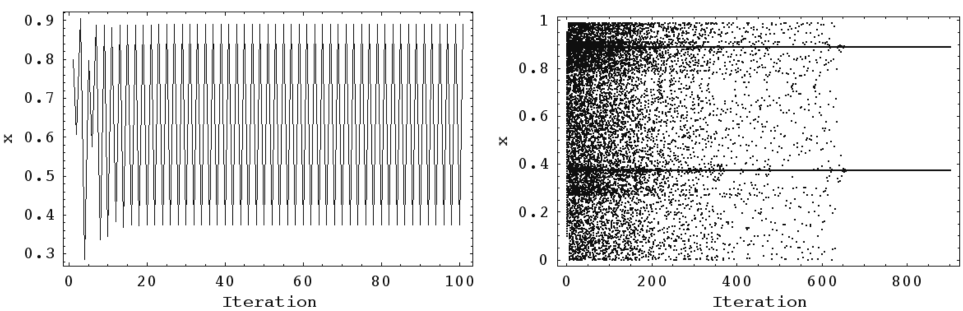


Figure 6.55 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ2, p-2 orbit, DERand1DIter

4p

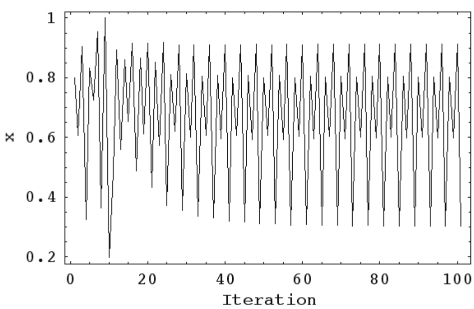
The presented results in Table 6.53 - Table 6.54, and Figure 6.56 - Figure 6.57 show capability of faster reaching of desired UPO for the same initial conditions as in the optimization process. Unfortunately, also in case of complex simulation with uniformly distributed initial conditions any improvement did not came about and the system was not stabilized on p-4 orbit. On average, around 151 (SOMA) or 182 (DE) iterations are required for the stabilization of this particular orbit.

Table 6.53 Best individual solutions, LQ, p-4 orbit, CF Targ2, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.5221	0.1974	0.6096	0.0002	0.2561	149	169
2	-0.5332	0.2122	0.6119	0.0002	0.3596	177	136
3	-0.5190	0.1738	0.5977	0.0002	0.2570	187	167
4	-0.5233	0.5151	0.6140	0.0002	0.3041	149	131

Table 6.54 Best individual solutions, LQ, p-4 orbit, CF Targ2, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.5451	0.2777	0.6329	0.0002	0.1897	185	174
6	-0.5270	0.1993	0.6175	0.0002	0.2176	177	158
7	-0.5218	0.1755	0.6101	0.0002	0.1704	159	183
8	-0.5480	0.2771	0.6313	0.0002	0.0315	173	212
9	-0.5303	0.0883	0.6087	0.0002	0.1275	169	199
10	-0.5794	0.3067	0.6703	0.0002	0.1752	197	167



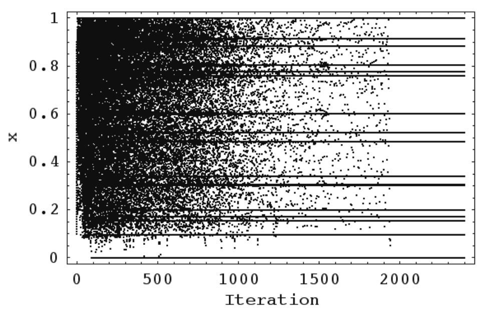


Figure 6.56 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ2, p-4 orbit, SOMA ATO

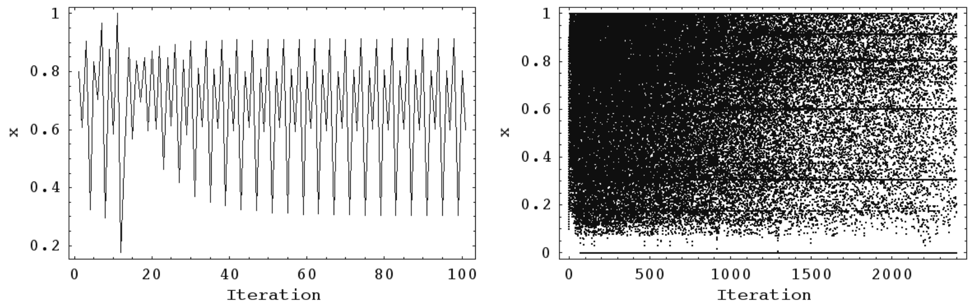


Figure 6.57 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ2, p-4 orbit, DEBest2Bin

6.7.2 Two-Dimensional Example

1p

From the comparison with previous case study 4 (CF Targ1) follows, that the average IStab value is smaller. But on the other hand this CF give the two best solutions (SOMA ATO and ATAA), where the final CF value is not divisible by the *NI* value (or IStab) without remainder Thus it seems, that every subsequent simulation affirms the fact, that this design of CF secures very fast reaching of desired state but with slightly lower quality of stabilization (basic part of CF > 0). For stabilization of Henon map fixed point, on average, around 38 iterations are required. The best individual solutions were given by SOMA ATAA and DELocalToBest. For optimization results please refer to Table 6.55 - Table 6.56, and Figure 6.58 - Figure 6.59.

Table 6.55 Best individual solutions, HENON, p-1 orbit, CF Targ2, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.6912	0.1805	0.0911	$4.00.10^{-15}$	$4.49.10^{-15}$	30	39
2	-0.8857	0.1593	0.2339	$4.00.10^{-15}$	$4.66.10^{-15}$	40	40
3	-0.8856	0.2249	0.2429	$4.10.10^{-15}$	$4.54.10^{-15}$	41	38
4	-0.8649	0.1953	0.2149	$3.90.10^{-15}$	$4.40.10^{-15}$	39	38

Table 6.56 Best individual solutions, HENON, p-1 orbit, CF Targ2, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.8764	0.2805	0.2413	$4.00.10^{-15}$	$4.35.10^{-15}$	40	37
6	-0.8730	0.2516	0.2345	$4.20.10^{-15}$	$4.50.10^{-15}$	42	38
7	-0.8359	0.2716	0.2286	$3.92.10^{-15}$	$4.39.10^{-15}$	37	38
8	-0.8386	0.3089	0.2024	$3.90.10^{-15}$	$4.33.10^{-15}$	39	38
9	-0.8386	0.3089	0.2024	$3.90.10^{-15}$	$4.49.10^{-15}$	39	38
10	-0.7511	0.2076	0.1286	$3.97.10^{-15}$	$4.43.10^{-15}$	33	37

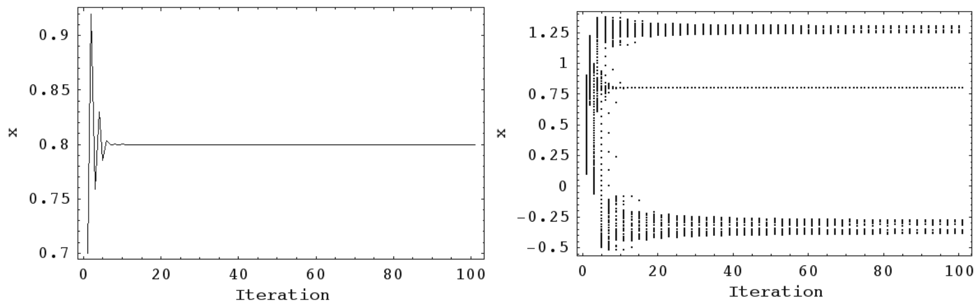


Figure 6.58 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ2, p-1 orbit, SOMA ATAA

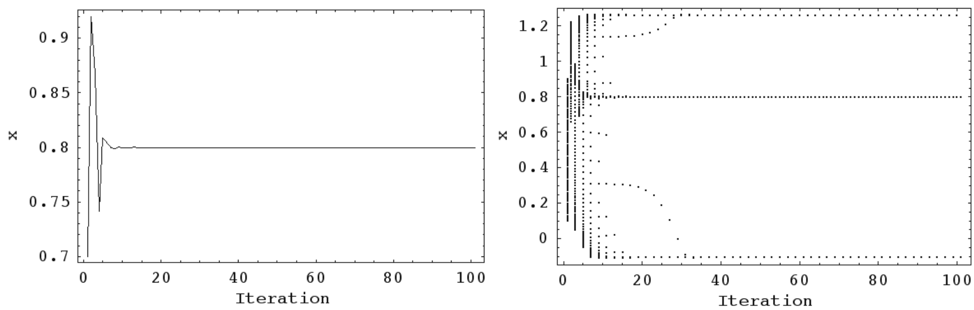


Figure 6.59 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ2, p-1 orbit, DELocalToBest

2p

The best individual solutions for SOMA and DE are given in Table 6.57 and Table 6.58, and the simulation results are depicted in Figure 6.60 and Figure 6.61. In spite of the promising results in case of p-1 orbit, smaller final CF values for the best solutions and less nonlinear CF surface (see Appendix), it appears at the first instance that this optimization by means of CF Targ2 produces worse results than previous optimization (case study 4 - CF Targ1) from the point of view of average IStab value. But the results presented in the following two tables are not corrected and as it was mentioned above in case of the logistic equation, this CF proves better performance in the task of percentage successfulness of stabilization for all 50 repeated simulations of each EA version. The corrected results can be found in the conclusion at the end of this case study For successful stabilization of p-2 orbit in this case, on average, about 91 (SOMA) or 113 (DE) iterations are required.

Table 6.57 Best individual solutions, HENON, p-2 orbit, CF Targ2, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	0.4368	0.2081	0.3534	$2.23.10^{-9}$	0.4029	128	93
2	0.3428	0.1464	0.1372	$1.38.10^{-5}$	0.3775	126	85
3	0.5095	0.1954	0.4075	$1.43.10^{-8}$	0.2856	121	93
4	0.3734	0.1456	0.2230	$2.13.10^{-7}$	0.3179	130	91

Table 6.58 Best individual solutions, HENON, p-2 orbit, CF Targ2, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	0.4524	0.1298	0.3398	$1.62.10^{-6}$	0.0804	131	112
6	0.5122	0.1466	0.3838	$1.40.10^{-6}$	0.0820	128	114
7	0.4496	0.2092	0.3202	$1.97.10^{-7}$	0.0520	126	110
8	0.3661	0.2012	0.2078	$1.34.10^{-6}$	0.0338	117	117
9	0.4871	0.2006	0.4871	$1.73.10^{-13}$	0.0827	121	108
10	0.4988	0.1610	0.4924	$2.12.10^{-14}$	0.0332	128	115

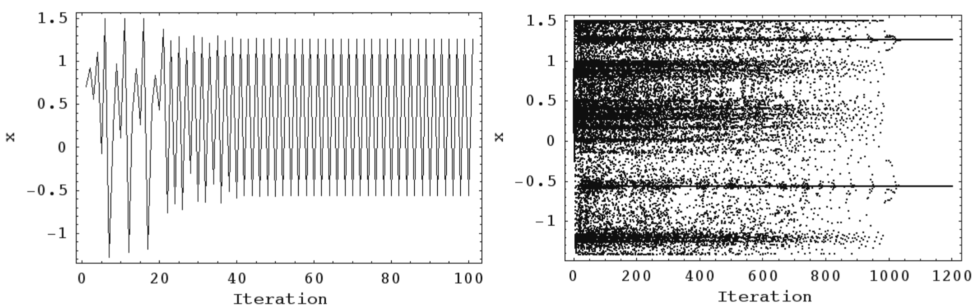


Figure 6.60 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ2, p-2 orbit, SOMA ATO

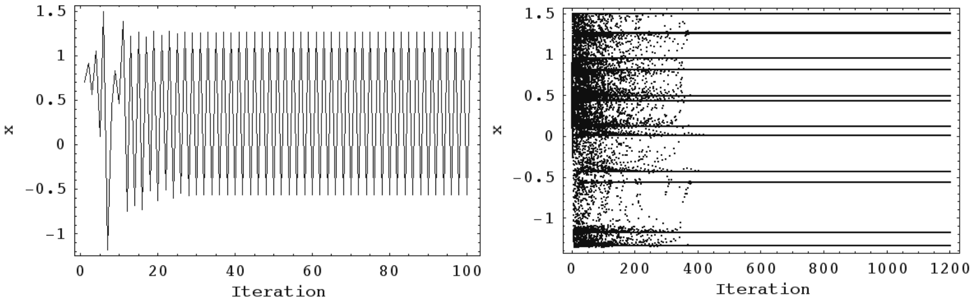


Figure 6.61 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ2, p-2 orbit, DEBest1JIter

4p

The behavior of the control the method is very similar as to the previous two UPOs and show all above mentioned and in detail described features. For successful stabilization of p-4 orbit, on average, about 115 (SOMA) or 123 (DE) iterations are required. For further illustration and the overview of optimization data please see Table 6.59 - Table 6.60, and Figure 6.62 - Figure 6.63.

Table 6.59 Best individual solutions, HENON, p-4 orbit, CF Targ2, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.3741	0.1136	0.4159	$1.37.10^{-7}$	0.0131	125	112
2	-0.3831	0.1107	0.4265	$1.88.10^{-8}$	0.0157	129	115
3	-0.3879	0.2420	0.4437	$2.18.10^{-8}$	0.0076	129	112
4	-0.3702	0.2349	0.4117	$1.41.10^{-7}$	0.0034	129	122

Table 6.60 Best individual solutions, HENON, p-4 orbit, CF Targ2, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.3717	0.2086	0.4125	$1.40.10^{-7}$	0.0013	129	122
6	-0.4176	0.1220	0.4754	$1.69.10^{-7}$	0.0017	129	121
7	-0.3704	0.2031	0.4147	$1.79.10^{-8}$	0.0021	129	120
8	-0.4016	0.2696	0.4642	$1.53.10^{-7}$	0.0002	129	124
9	-0.3768	0.1162	0.4156	$1.45.10^{-7}$	0.0015	129	124
10	-0.3837	0.1136	0.4404	$1.78.10^{-8}$	0.0008	129	126

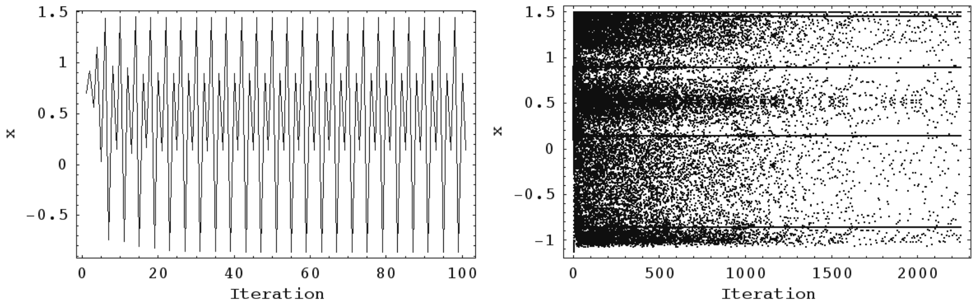


Figure 6.62 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ2, p-4 orbit, SOMA ATR

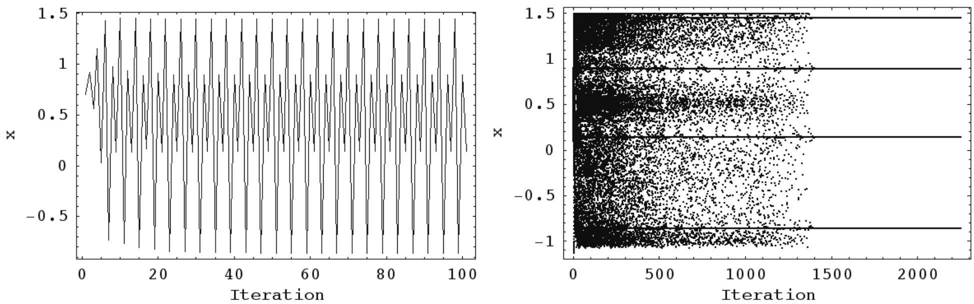


Figure 6.63 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ2, p-4 orbit, DEBest1JIter

6.7.3 Conclusion of Results – Case Study 5

From the presented results it follows that this CF design produces only slight improvement in the task of faster stabilization in comparison with previous case studies (CF Targ1 and CF NA). Also, this CF solved the problem with difficulty of correct setting of the SC value for p-4 orbit in advance and the problem with possible influence of evolutionary searching and final CF value by wrong determination of *SC* in advance. The very important fact is that the phenomenon of difference in performance of both EAs, in case of proportion of IStab value to Average IStab value, thus the proportion of solutions with either perfect stabilization or temporary or possibly none at all was partially suppressed. On the other hand, here occurs the intensifying of the unpleasant fact that this CF Targ2 allowed finding of faster stabilizing solutions, nevertheless this solutions are not suitable for complex simulation with uniformly distributed initial conditions.

When comparing both proposed CF Targ1 and CF Targ2, it appears that the first one gives slightly slowly stabilizing solutions than the second one (the difference lies only in a few iterations). But these solutions are more suitable for wide range of initial conditions. This fact is valid for both chaotic systems and all desired UPOs. Thus the next two presented case studies deals with the change in CF design of both CF Targ1 and CF Targ2, which leads to finding of solutions suitable for simulations with wide range of initial conditions. The results obtained in this case can be summarized in following main points:

- Again slight improvement in comparison with CF NA and CF Targ1
- Problem with the determination of *SC* value in advance has been solved also in this CF design.
- Significant improvement in performance of EA from the point of view of percentage successfulness of giving the solutions leading to stabilization for both systems and all desired UPOs.
- When comparing SOMA and DE also in this case study, DE gives more “stabilization securing” solutions.
- As in the case of CF Targ1, the problem with extreme sensitivity of LQ p-4 orbit and Henon p-1 orbit to proper settings of control algorithm in the task of testing the best solution in simulation with uniformly distributed initial conditions remains.
- For the comparison of average number of iterations required for successful stabilization see Table 6.61 and Table 6.62. The value in braces represents corrected one, which shows the average IStab value only for solutions, which leads to successful stabilization.

Table 6.61 Comparison of Average IStab values – LQ – Case studies 1-5

UPO	**p-1**		**p-2**		**p-4**	
EA Version	SOMA	DE	SOMA	DE	SOMA	DE
CF Basic	97	97	123	123	197	218
CF Simple	98	99	-	-	-	-
CF NA	33	31	73 (109)	102 (116)	-	-
CF Targ1	33	31	78 (112)	104 (115)	77 (195)	121 (215)
CF Targ2	31	30	95 (112)	113 (113)	151 (184)	182 (196)

Table 6.62 Comparison of Average IStab values – HENON – Case studies 1-5

UPO	**p-1**		**p-2**		**p-4**	
EA Version	SOMA	DE	SOMA	DE	SOMA	DE
CF Basic	77	75	124	125	122	122
CF Simple	65	70	-	-	-	-
CF NA	49	47	84 (114)	110 (118)	-	-
CF Targ1	49	47	72 (113)	109 (118)	110 (121)	121 (123)
CF Targ2	39	38	91 (108)	113 (117)	115 (118)	123 (123)

6.8 Simulation Results – Case study 6: CF Targ1 - Advanced

The new advanced design of CF Targ1 is presented in this section. This CF was developed to improve the performance of estimated optimal solutions in the task of complex simulations for wide range of initial conditions.

6.8.1 One-Dimensional Example

1p

The best individual solutions for SOMA and DE are given in. The optimization gives in this case similar results (see Table 6.63 - Table 6.64, and Figure 6.64 - Figure 6.65) as for CF Targ1 from the point of view of IStab value for the best individual solutions. But when comparing the average IStab value the slight worsening came about. This may be the consequence of the endeavor to find "universal solution" for wide range of initial conditions, which seems to be a harder task for EA. Finally it is good to notice that the increase of order of final CF value did not arise because of lower quality of stabilization, but due to addition of several repeated simulations with different initial conditions into CF value. On average, around 35 iterations are required. The best individual solutions were given by SOMA ATO and DELocalToBest.

Table 6.63 Best individual solutions, LQ, p-1 orbit, CF Targ1 Advanced, SOMA

EA	K	F_{max}	R	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.9351	0.4888	0.4990	$2.57.10^{-14}$	$2.97.10^{-14}$	31	34
2	-0.9172	0.3302	0.4857	$2.74.10^{-14}$	$3.56.10^{-14}$	30	40
3	-0.9343	0.1910	0.4962	$2.72.10^{-14}$	$3.13.10^{-14}$	32	36
4	-0.9325	0.4935	0.4985	$2.59.10^{-14}$	$3.08.10^{-14}$	31	36

Table 6.64 Best individual solutions, LQ, p-1 orbit, CF Targ1 Advanced, DE

EA	K	F_{max}	R	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.9187	0.4952	0.4866	$2.67.10^{-14}$	$2.94.10^{-14}$	33	34
6	-0.9089	0.1912	0.4792	$2.79.10^{-14}$	$3.06.10^{-14}$	35	35
7	-0.9385	0.4989	0.4983	$2.66.10^{-14}$	$2.94.10^{-14}$	32	34
8	-0.9323	0.4842	0.4985	$2.56.10^{-14}$	$2.87.10^{-14}$	31	33
9	-0.9385	0.4989	0.4983	$2.66.10^{-14}$	$2.98.10^{-14}$	32	34
10	-0.9326	0.4626	0.4948	$2.68.10^{-14}$	$2.9.10^{-14}$	30	34

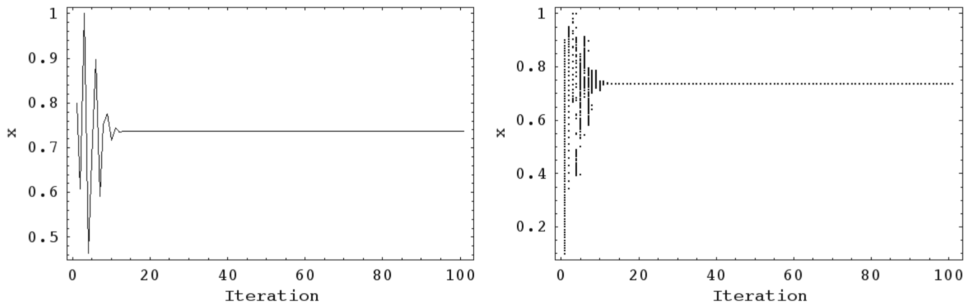


Figure 6.64 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ1 Advanced, p-1 orbit, SOMA ATO

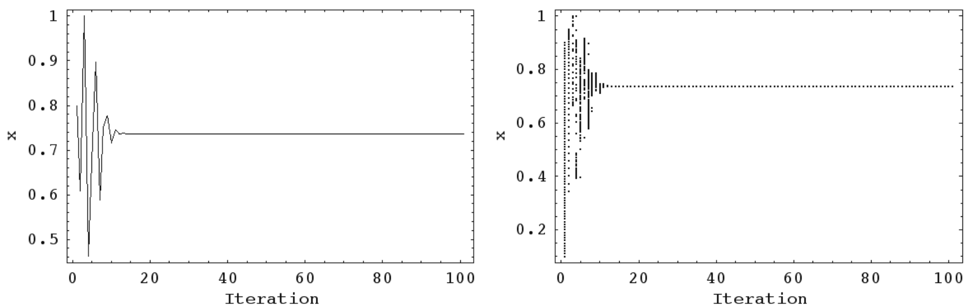


Figure 6.65 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ1 Advanced, p-1 orbit, DELocalToBest

2p

From the results presented in Table 6.65 and Table 6.66, and the behavior of system depicted in Figure 6.66 and Figure 6.67, it follows that the desired UPO was achieved rapidly, nevertheless, this CF design gives very poor performance of proportion of solutions with very low value of minimum IStab, however with very high CF value and either temporary stabilization or none at all. This can be clearly seen from comparison of final CF value presented here and for CF Targ1. Ideally it should be 10 times greater, due to the fact, that 10 repeated simulations for different initial conditions are added into final CF Value. Also a slight period doubling or oscillating in the close neighborhood of desired UPO arose. More about this two mentioned problems is written in the conclusion of this case study. Finally, for the robust stabilization with wide range of initial conditions, on average, around 64 iterations are required. The best individual solutions were given by SOMA ATA and DERand2Bin.

Table 6.65 Best individual solutions, LQ, p-2 orbit, CF Targ1 Advanced, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	0.4125	0.0471	0.2134	44.5190	139.5495	69	11
2	0.4017	0.0510	0.1768	29.9465	133.3694	30	13
3	0.4236	0.2086	0.2421	15.0221	140.0992	56	13
4	0.4219	0.2826	0.2327	31.8591	135.0361	45	12

Table 6.66 Best individual solutions, LQ, p-2 orbit, CF Targ1 Advanced, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	0.4181	0.2645	0.2234	32.9867	124.5893	64	14
6	0.4047	0.4271	0.1819	19.7431	111.3962	35	20
7	0.4105	0.2037	0.2081	30.7959	125.4360	30	12
8	0.4190	0.2029	0.2242	25.5306	120.4480	44	20
9	0.3882	0.2023	0.1576	28.6152	129.9372	68	14
10	0.4105	0.2024	0.2049	22.2266	121.5131	89	14

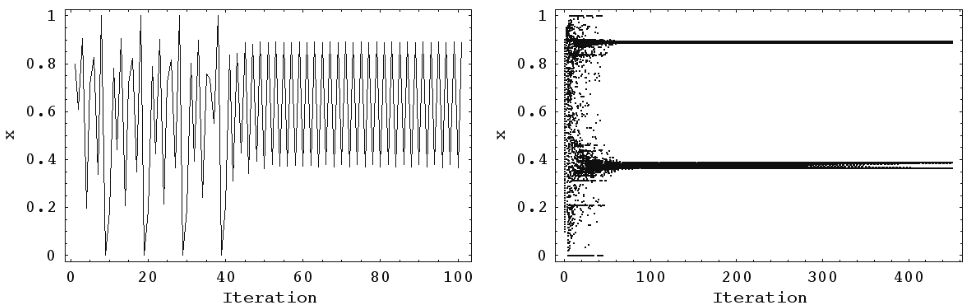


Figure 6.66 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ1 Advanced, p-2 orbit, SOMA ATA

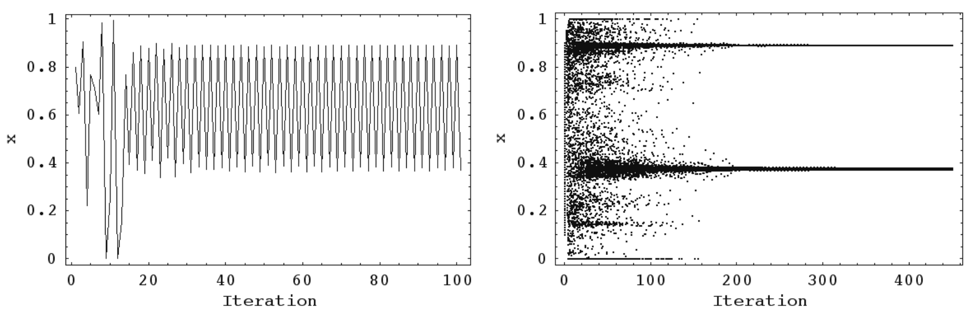


Figure 6.67 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ1 Advanced, p-2 orbit, DERand2Bin

4p

From the optimization results given in Table 6.67 - Table 6.68, and depicted in Figure 6.68 - Figure 6.69, it follows that the control method reached worse performance from the point of view of quickness of the stabilization in comparison with CF Targ1 and other previous Cost Functions. On the other hand, finally this CF gives results, which lead to successful stabilization on p-4 orbit for simulation with uniformly distributed initial conditions. It is necessary to mention, that in this case the length of optimization interval τ_i had to be increased to 500 iterations. The influence of this decision is discussed in conclusion of this case study. Also here occurred the problem with very poor performance of both EAs, in case of proportion of IStab value to Average IStab value. Lastly, on average about 418 (SOMA) or 375 (DE) iterations are required (corrected results). The best individual solutions were given by SOMA ATA and DERand2Bin.

Table 6.67 Best individual solutions, LQ, p-4 orbit, CF Targ1 Advanced, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.6979	0.1269	0.7678	82.5207	161.8815	379	12
2	-0.6394	0.1107	0.7255	0.1229	192.0886	450	31
3	1.8281	0.1884	0.6581	141.3368	174.0108	4	4
4	1.5465	0.1883	0.7033	134.2319	179.2204	4	4

Table 6.68 Best individual solutions, LQ, p-4 orbit, CF Targ1 Advanced, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	1.5747	0.1903	0.7615	138.7033	168.5615	4	4
6	-0.6595	0.0974	0.7236	33.2028	161.4346	274	17
7	-0.6176	0.0906	0.7049	0.6302	146.7834	463	21
8	1.5776	0.0772	0.7046	128.5954	162.6294	4	11
9	-0.7257	0.1319	0.8157	103.5151	158.2030	446	13
10	-0.6001	0.0345	0.8582	79.3885	156.4887	319	10

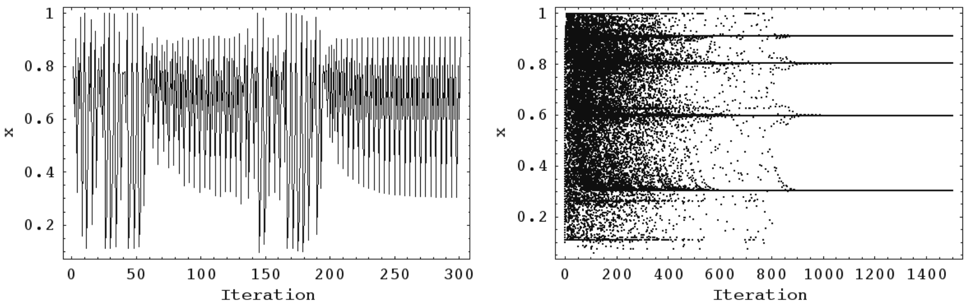


Figure 6.68 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ1 Advanced, p-4 orbit, SOMA ATR

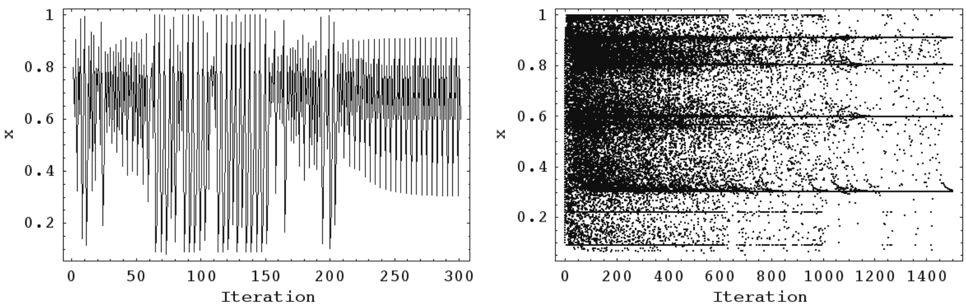


Figure 6.69 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ1 Advanced, p-4 orbit, DEBest2Bin

6.8.2 Two-Dimensional Example

1p

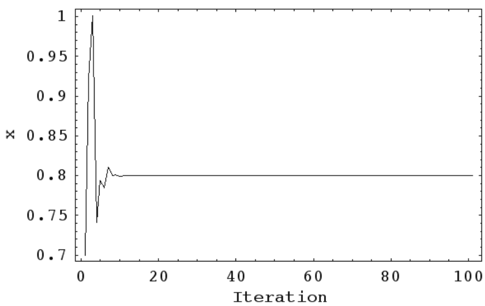
As can be seen from presented optimization results, slightly worse performance from the point of view of quickness of stabilization in comparison with CF Targ1 and CF Targ2 was reached. On the other hand, this CF achieves results, which lead to successful stabilization on p-1 orbit for simulation with uniformly distributed initial conditions, whereas the fast targeting securing CF Targ 1 and CF Targ2 did not produce satisfactory results. On average, around 58 iterations are required for stabilization of the fixed point. The best individual solutions were given by SOMA ATAA and DERand1DIter. For detailed overview of results in this case please refer to Table 6.69 - Table 6.70, and Figure 6.70 - Figure 6.71.

Table 6.69 Best individual solutions, HENON, p-1 orbit, CF Targ1 Advanced, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-1.0148	0.3849	0.4067	$5.2.10^{-14}$	$1.13.10^{-13}$	53	55
2	-1.0355	0.4313	0.4218	$5.25.10^{-14}$	$1.38.10^{-13}$	55	56
3	-0.9977	0.4382	0.3837	$5.01.10^{-14}$	$8.97.10^{-14}$	53	56
4	-0.9977	0.4382	0.3837	$5.01.10^{-14}$	$9.13.10^{-14}$	53	56

Table 6.70 Best individual solutions, HENON, p-1 orbit, CF Targ1 Advanced, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.9769	0.4539	0.3603	$4.97.10^{-14}$	$6.4.10^{-14}$	53	59
6	-0.9966	0.3960	0.3849	$5.1.10^{-14}$	$6.63.10^{-14}$	53	59
7	-0.9716	0.4358	0.3582	$4.99.10^{-14}$	$5.67.10^{-14}$	55	59
8	-1.0028	0.4512	0.3863	$5.01.10^{-14}$	$5.48.10^{-14}$	53	57
9	-0.9242	0.4133	0.3060	$4.73.10^{-14}$	$5.83.10^{-14}$	51	59
10	-0.9832	0.4987	0.3753	$5.03.10^{-14}$	$5.65.10^{-14}$	55	58



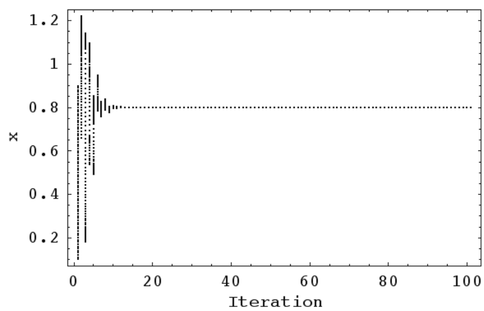
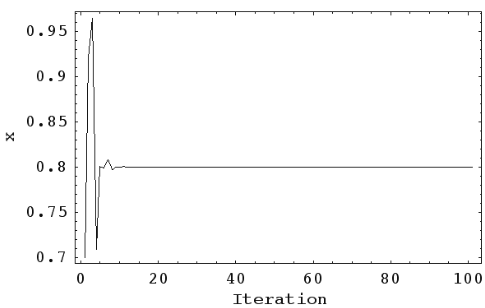


Figure 6.70 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ1 Advanced, p-1 orbit, SOMA ATAA



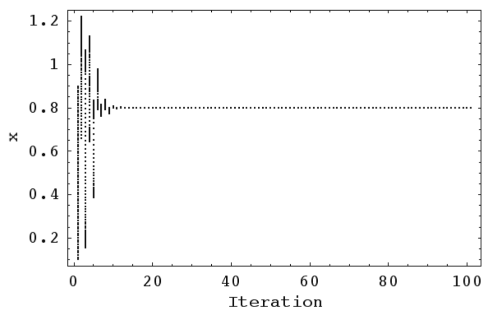


Figure 6.71 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ1 Advanced, p-1 orbit, DERand1DIter

2p

The optimization results given in Table 6.71 - Table 6.72, and depicted in Figure 6.72 - Figure 6.73, shows very similar attributes as in the case of LQ p-2 orbit i.e. rapid achievement of desired UPO (on average, around 51 (SOMA) or 57 (DE) iterations are required), together with very poor performance of proportion of IStab val. and average IStab val. Also, relatively considerable period doubling or oscillating in the close neighborhood of desired UPO arose. The best individual solutions were given by SOMA ATA and DERand1Bin.

Table 6.71 Best individual solutions, HENON, p-2 orbit, CF Targ1 Advanced, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-1.4752	0.4631	0.4936	278.2358	403.2056	9	8
2	-1.4448	0.4083	0.4714	335.9420	422.7275	9	8
3	0.3264	0.1150	0.1342	216.7945	410.7999	51	8
4	-1.4741	0.4355	0.4914	322.6202	414.1843	9	7

Table 6.72 Best individual solutions, HENON, p-2 orbit, CF Targ1 Advanced, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	0.3156	0.1476	0.1034	175.1048	394.1554	44	8
6	-1.4714	0.4448	0.4993	316.6541	403.7943	9	9
7	0.3008	0.1402	0.0703	237.1202	393.1724	39	10
8	0.2987	0.1736	0.0396	275.8986	392.8366	80	9
9	0.3122	0.1536	0.0934	209.5476	393.8310	51	10
10	0.3145	0.1404	0.1113	196.3163	391.5271	39	8

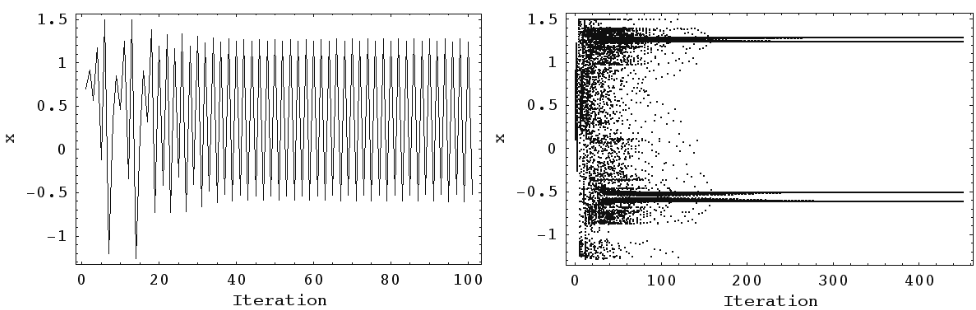


Figure 6.72 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ1 Advanced, p-2 orbit, SOMA ATA

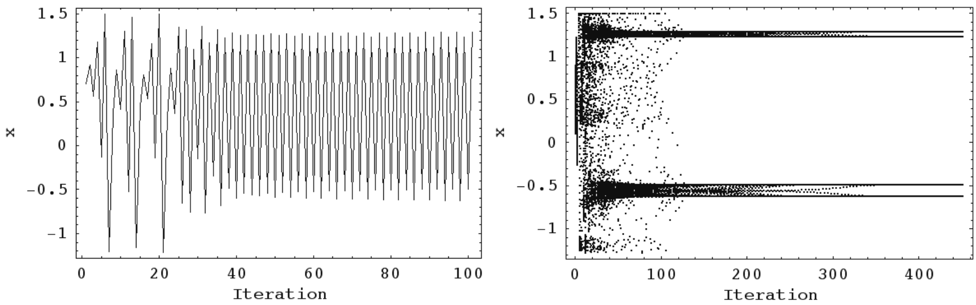


Figure 6.73 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ1 Advanced, p-2 orbit, DERand1Bin

4p

Also in this case occurs the phenomenon, that faster targeting of desired UPO (84 (SOMA) or 82 (DE) iterations) for wide range of initial conditions by means of this CF Targ1 advanced is at the cost of poor performance of EA and period doubling. See the best individual solutions for SOMA and DE in Table 6.73 and Table 6.74, and the simulation results depicted in Figure 6.74 and Figure 6.75. Lastly, the best individual solutions were given by SOMA ATO and DELocalToBest.

Table 6.73 Best individual solutions, HENON, p-4 orbit, CF Targ1 Advanced, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.2873	0.1558	0.1791	211.5812	407.8080	73	24
2	-0.4187	0.4319	0.3426	278.2204	434.3837	73	28
3	-0.4828	0.1904	0.4390	241.1852	416.2615	17	32
4	-0.3436	0.4037	0.4119	234.9283	389.5470	111	29

Table 6.74 Best individual solutions, HENON, p-4 orbit, CF Targ1 Advanced, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.3078	0.1438	0.2432	118.9887	353.5068	125	41
6	-0.6051	0.3869	0.4989	276.3707	380.2217	13	14
7	-0.5454	0.2019	0.4853	94.5564	369.7979	97	29
8	-0.5788	0.4874	0.4890	192.8344	345.0628	101	31
9	-0.5724	0.2739	0.4982	139.5352	362.2666	121	30
10	-0.5481	0.4340	0.4950	132.6462	366.0304	119	32

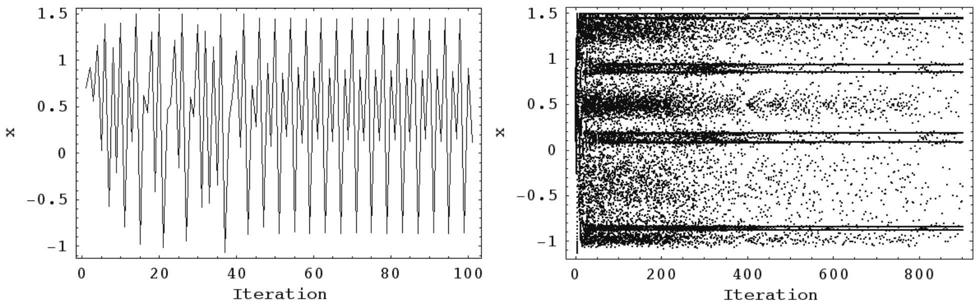


Figure 6.74 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ1 Advanced, p-4 orbit, SOMA ATO

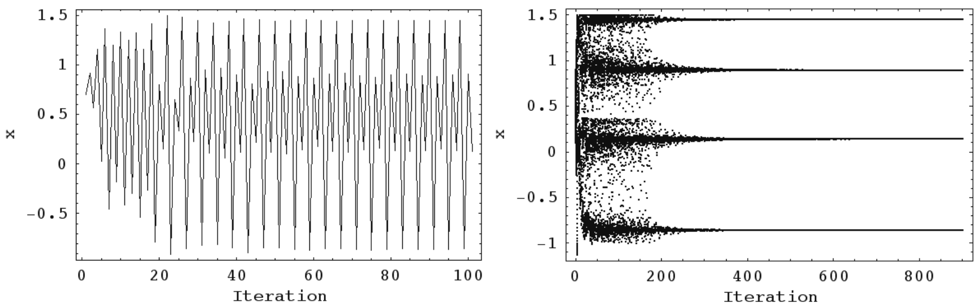


Figure 6.75 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ1 Advanced, p-4 orbit, DELocalToBest

6.8.3 Conclusion of Results – Case Study 6

From presented results in this case study it follows that this CF design gives for the first view significant improvement in the task of faster stabilization in comparison with previous case studies dealing with targeting CFs (CF Targ1 and CF Targ2), on the other hand at the cost of following problems.

The phenomenon of difference in performance of both EAs rapidly increased, in case of higher periodic orbits and proportion of IStab value to Average IStab value, thus the proportion of solutions with either perfect stabilization or temporary or possibly none at all. In some optimizations the performance of EA was very poor, for example Henon p-2 orbit: from all 200 simulations of four SOMA versions only one solution was found. This is probably caused by influence to final CF value by different numbers of iteration required for stabilization in several repeated simulations for different initial conditions during one CF evaluation process. Consequently different durations of initial chaotic stage is included into final CF value. Thereafter these chaotic initial stages are markedly amplified by multiplication of *NI*. The core of this problem can be clearly seen in all Figures, where the simulations for uniformly distributed initial conditions is depicted. It is evident that the extremely sensitive chaotic system shows different behavior for initial conditions used in both optimization and simulation and for wider range of initial conditions. Thus it is very difficult to find suitable CF design, which should secure finding of "universal solution" in case of chaos control.

The next problem which arose as a consequence of this CF design and sensitivity of chaotic system to proper settings is the period doubling or in other words the oscillating around desired higher periodic orbit. This is probably caused by the fact, that different length of stabilization and initial chaotic behavior increases the CF value and at the same time the quality of stabilization (difference between target state and actual state) does not have such a influence to CF value.

Finally, it should be mentioned, that in the case of problematic LQ p-4 orbit the simulation interval had to be increased to 500 iterations. The earlier increased and used value of 300 iterations was not enough to secure finding of good solution. In spite of this fact, it is not possible to compare the performance of this CF with previous results (CF Targ1, CF Targ2 and CF Basic), but the weighty result is that this UPO was finally stabilized at all.

The next and the last presented case study shows again how slight change in CF design could impact on performance of control technique, especially considering the fact, that CF Targ2 gives better and more reasonable results than CF Targ1. The results obtained in this case can be summarized in following main points:

- Marked improvement in comparison with previous case studies from the point of view **corrected** average IStab value for higher periodic orbits for both systems (except LQ p-4). In case of fixed point, the results are slightly worse.
- On the other hand, very considerable worsening in performance of EA from the point of view of percentage successfulness of giving the

solutions leading to stabilization for both systems and all higher periodic UPOs

- Problem with fast targeting not only for initial conditions used in optimization process but for the uniformly distributed initial conditions has been solved but at the cost of slight period doubling in the close neighborhood of desired higher periodic UPO.
- When comparing EAs DE again gives slightly more “stabilization securing” solutions.
- The problem with extreme sensitivity of LQ p-4 orbit and Henon p-1 orbit to proper settings of control algorithm in the task of testing the best solution in simulation with uniformly distributed initial conditions has been solved.
- The final CF value is affected by different numbers of iteration required for stabilization in several runs during one CF evaluation consequently different durations of initial chaotic stage is included into final CF value. Thus the simulation results shows above mentioned very fast but low quality stabilization or tiny period doubling in case of p-2 and p-4 orbit.
- For the comparison of average number of iterations required for successful stabilization see Table 6.75 and Table 6.76. The value in braces represents corrected one, which shows the average IStab value only for solutions, which leads to successful stabilization.

Table 6.75 Comparison of Average IStab values – LQ – Case studies 1-6

UPO	**p-1**		**p-2**		**p-4**	
EA Version	SOMA	DE	SOMA	DE	SOMA	DE
CF Basic	97	97	123	123	197	218
CF Simple	98	99	-	-	-	-
CF NA	33	31	73 (109)	102 (116)	-	-
CF Targ1	33	31	78 (112)	104 (115)	77 (195)	121 (215)
CF Targ2	31	30	95 (112)	113 (113)	151 (184)	182 (196)
CF Targ1 Adv.	37	34	12 (60)	16 (55)	13 (418)	13 (375)

Table 6.76 Comparison of Average IStab values – HENON – Case studies 1-6

UPO	**p-1**		**p-2**		**p-4**	
EA Version	SOMA	DE	SOMA	DE	SOMA	DE
CF Basic	77	75	124	125	122	122
CF Simple	65	70	-	-	-	-
CF NA	49	47	84 (114)	110 (118)	-	-
CF Targ1	49	47	72 (113)	109 (118)	110 (121)	121 (123)
CF Targ2	39	38	91 (108)	113 (117)	115 (118)	123 (123)
CF Targ1 Adv.	56	59	8 (51)	9 (57)	28 (84)	30 (82)

6.9 Simulation Results – Case study 7: CF Targ2 - Advanced

The last section presents the advanced targeting CF developed on the basis of successful CF Targ2 design. The presented data lends weight to the argument, that this CF design is a serious consideration in the robust stabilization of chaotic systems for wide range of initial conditions.

6.9.1 One-Dimensional Example

1p

From the presented results can be seen another slight improvement in comparison with "CF Targ1 advanced" from the point of view of average IStab values, thus the close return to the existing best values given by CF Targ1 and CF Targ2. The best individual solutions for SOMA and DE are given in Table 6.77 and Table 6.78, and the simulation results are depicted in Figure 6.76 and Figure 6.77. On average, about 30 iterations are required. The best individual solutions were given by SOMA ATO and DEBest2Bin.

Table 6.77 Best individual solutions, LQ, p-1 orbit, CF Targ2 Advanced, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.9336	0.4957	0.4994	$2.54.10^{-14}$	$2.75.10^{-14}$	31	32
2	-0.9121	0.4933	0.4816	$2.67.10^{-14}$	$3.24.10^{-14}$	30	36
3	-0.9317	0.4971	0.4967	$2.61.10^{-14}$	$2.93.10^{-14}$	31	33
4	-0.9283	0.4855	0.4955	$2.54.10^{-14}$	$3.02.10^{-14}$	30	35

Table 6.78 Best individual solutions, LQ, p-1 orbit, CF Targ2 Advanced, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.9369	0.4767	0.4973	$2.66.10^{-14}$	$2.83.10^{-14}$	32	33
6	-0.9279	0.4960	0.4940	$2.67.10^{-14}$	$2.89.10^{-14}$	31	33
7	-0.9307	0.4995	0.4973	$2.56.10^{-14}$	$2.77.10^{-14}$	29	32
8	-0.9223	0.4936	0.4911	$2.61.10^{-14}$	$2.73.10^{-14}$	30	32
9	-0.9317	0.1950	0.4980	$2.64.10^{-14}$	$2.86.10^{-14}$	31	33
10	-0.9229	0.4908	0.4898	$2.62.10^{-14}$	$2.73.10^{-14}$	30	32

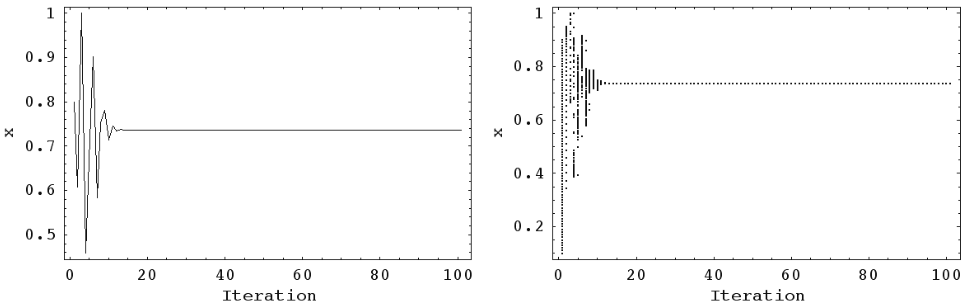


Figure 6.76 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ2 Advanced, p-1 orbit, SOMA ATO

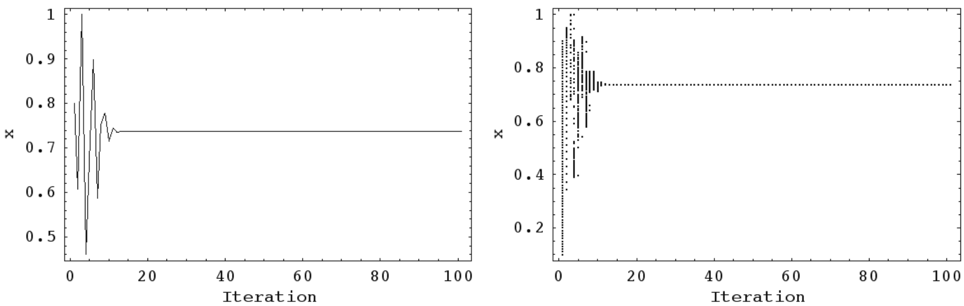


Figure 6.77 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ2 Advanced, p-1 orbit, DEBest2Bin

2p

As in the case of "CF Targ1 advanced", the obtained results lead to faster stabilization with comparison with case studies 1 - 5. Moreover the two main problems discussed in the previous case study 6, i.e. poor performance of EA and period doubling were markedly suppressed here. For robust elimination of chaotic behavior of the system, on average about 66 (SOMA) or 63 (DE) iterations are required. The best individual solutions were given by SOMA ATO and DERand1DIter. See the optimization results in Table 6.79 - Table 6.80, and Figure 6.78 - Figure 6.79.

Table 6.79 Best individual solutions, LQ, p-2 orbit, CF Targ2 Advanced, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	0.4093	0.2842	0.1949	0.6392	11.3885	31	31
2	0.4062	0.2060	0.1904	1.7594	9.9314	40	42
3	0.4299	0.2089	0.2572	1.0202	10.6515	53	39
4	0.3998	0.1892	0.1744	1.4932	11.7007	47	30

Table 6.80 Best individual solutions, LQ, p-2 orbit, CF Targ2 Advanced, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	0.3984	0.1911	0.1696	1.5467	6.9414	75	36
6	0.4139	0.1872	0.2111	1.2938	7.2405	74	42
7	0.4026	0.2009	0.1937	1.0224	5.7534	86	43
8	0.4376	0.2026	0.1798	0.6683	4.7281	114	35
9	0.4092	0.2794	0.1847	0.2353	7.1450	105	38
10	0.4106	0.2020	0.1992	0.8455	6.5165	54	33

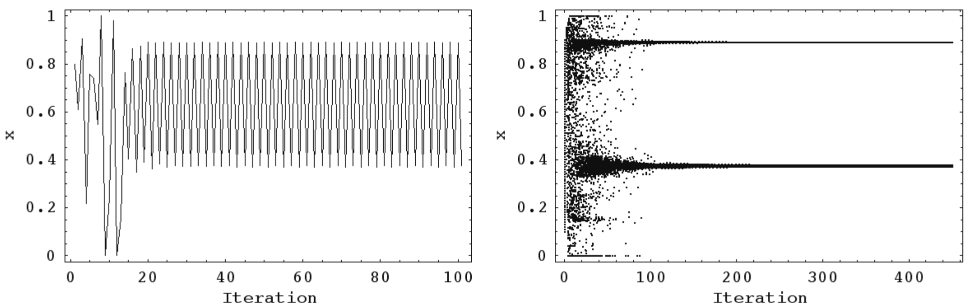


Figure 6.78 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ2 Advanced, p-2 orbit, SOMA ATO

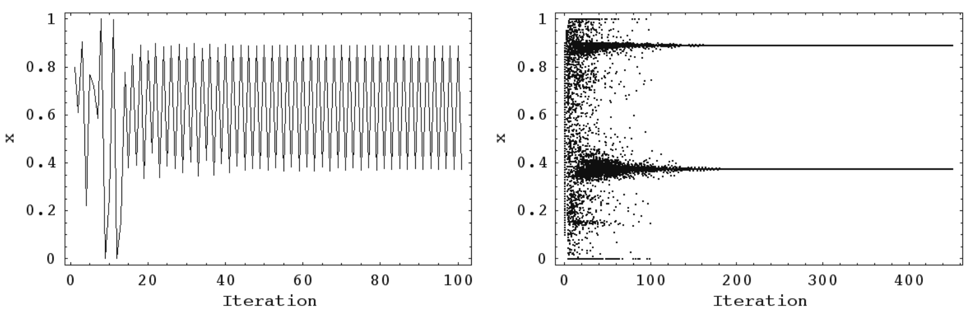


Figure 6.79 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ2 Advanced, p-2 orbit, DERand1DIter

4p

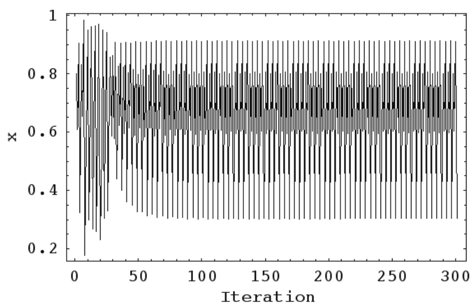
For the positive results of optimization in this case, please refer to Table 6.81 - Table 6.82, and Figure 6.80 - Figure 6.81. As in the case of p-1 orbit the presented results show the close return to the existing best values given by CF Targ1 and CF Targ2, but unlike these two CF designs, the problematic p-4 UPO was fast reached also in the case of complex simulation with uniformly distributed initial conditions. Also the performance of EAs became better. The stabilization of problematic p-4 orbit required, on average, around 194 (SOMA) or 199 (DE) iterations.

Table 6.81 Best individual solutions, LQ, p-4 orbit, CF Targ2 Advanced, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.6307	0.0931	0.6851	2.3172	33.1160	335	60
2	-0.6684	0.0922	0.7272	3.2511	34.9924	126	34
3	-0.5362	0.0966	0.6706	6.0179	34.9690	98	36
4	-0.5132	0.1249	0.5960	2.1906	32.7929	73	44

Table 6.82 Best individual solutions, LQ, p-4 orbit, CF Targ2 Advanced, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.5355	0.1267	0.6508	0.0031	29.9159	248	78
6	-0.6428	0.1059	0.7040	2.6696	30.1628	404	46
7	-0.7593	0.2580	0.8193	6.2542	29.1275	193	50
8	-0.5144	0.0938	0.5932	0.0033	29.9832	413	59
9	-0.6571	0.1091	0.7072	2.9633	31.7237	400	54
10	-0.7768	0.0344	0.8596	3.4763	29.5741	328	58



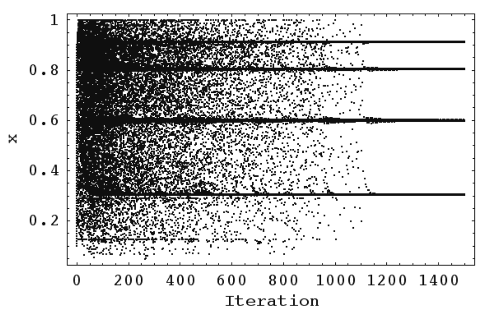


Figure 6.80 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ2 Advanced, p-4 orbit, SOMA ATAA

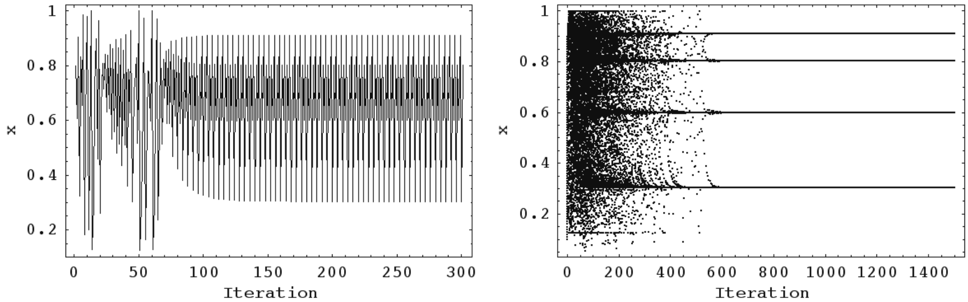


Figure 6.81 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) LQ, CF Targ2 Advanced, p-4 orbit, DERand1Bin

6.9.2 Two-Dimensional Example

1p

From Table 6.83 and Table 6.84, and the simulation results depicted in Figure 6.82 and Figure 6.83 it follows that this CF gives similar performance from the point of view of quickness of stabilization in comparison with CF Targ1 and CF Targ2. As described in the case of LQ, this CF secures the approach to the existing best results given by three targeting CFs (CF NA, CF Targ1 and CF Targ2), and secures the successful stabilization on p-1 orbit for simulation with uniformly distributed initial conditions as well, whereas the above mentioned three fast targeting CFs did not give satisfactory results in this task. For fast and robust stabilization, on average, about 40 iterations are required. The best individual solutions were given by SOMA ATAA and DELocalToBest.

Table 6.83 Best individual solutions, HENON, p-1 orbit, CF Targ2 Advanced, SOMA

EA	K	F_{max}	R	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.7513	0.4154	0.1280	$3.93.10^{-14}$	$4.29.10^{-14}$	33	40
2	-0.8880	0.4551	0.2454	$4.12.10^{-14}$	$4.45.10^{-14}$	45	42
3	-0.7548	0.4543	0.1316	4.10^{-14}	$4.38.10^{-14}$	34	40
4	-0.8575	0.4688	0.2148	$3.86.10^{-14}$	$4.3.10^{-14}$	41	41

Table 6.84 Best individual solutions, HENON, p-1 orbit, CF Targ2 Advanced, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.8456	0.4526	0.2017	$3.92.10^{-14}$	$4.23.10^{-14}$	47	41
6	-0.8591	0.4419	0.2167	$3.94.10^{-14}$	$4.3.10^{-14}$	41	40
7	-0.7545	0.4982	0.1317	$3.9.10^{-14}$	$4.23.10^{-14}$	34	39
8	-0.8500	0.4076	0.2022	$3.9.10^{-14}$	$4.18.10^{-14}$	41	39
9	-0.7152	0.4549	0.1038	$4.07.10^{-14}$	$4.38.10^{-14}$	36	40
10	-0.8663	0.4884	0.2143	$3.91.10^{-14}$	$4.29.10^{-14}$	42	40

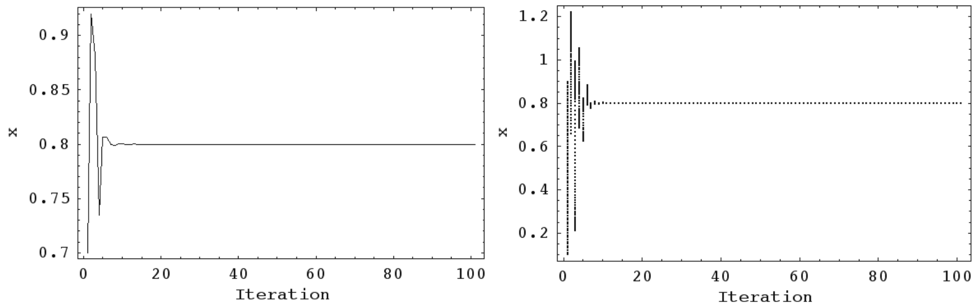


Figure 6.82 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ2 Advanced, p-1 orbit, SOMA ATAA

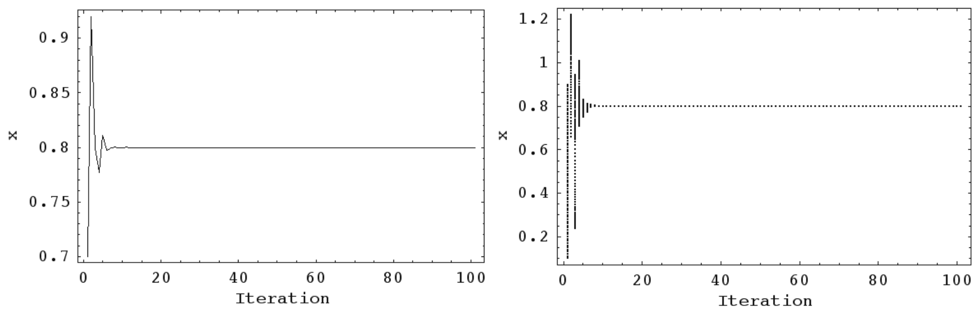


Figure 6.83 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ2 Advanced, p-1 orbit, DELocalToBest

2p

The results given by optimization in this case shows satisfactory results. The two main problems with period doubling (i.e. low-quality stabilization) and very poor performance of EAs in finding the stabilizing securing solutions were noticeably suppressed. See the optimization results given in Table 6.85 and Table 6.86, and the graphical output depicted in Figure 6.84 and Figure 6.85 as well. For successful stabilization of p-2 orbit in this case, on average, about 133 (SOMA) or 130 (DE) iterations are required.

Table 6.85 Best individual solutions, HENON, p-2 orbit, CF Targ2 Advanced, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	0.4097	0.1185	0.2830	$3.97.10^{-6}$	27.7130	125	71
2	0.4834	0.1695	0.4733	$1.97.10^{-8}$	20.0447	150	94
3	0.4208	0.1767	0.3451	$5.81.10^{-9}$	18.9881	145	87
4	0.4960	0.1689	0.4546	$3.94.10^{-8}$	30.5318	150	61

Table 6.86 Best individual solutions, HENON, p-2 orbit, CF Targ2 Advanced, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	0.4360	0.1768	0.3117	$5.87.10^{-7}$	20.6972	147	84
6	0.3882	0.1380	0.2324	$2.5.10^{-5}$	21.3478	150	76
7	0.4013	0.1151	0.2627	$2.9.10^{-5}$	17.3035	151	87
8	0.4244	0.1773	0.3565	$2.45.10^{-8}$	14.4064	151	103
9	0.4177	0.1774	0.2727	$4.35.10^{-5}$	18.6511	147	92
10	0.4264	0.1769	0.3536	$3.93.10^{-9}$	19.7192	151	83

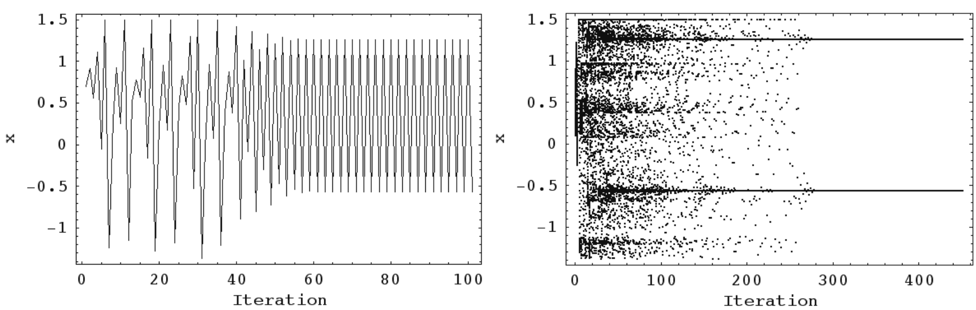


Figure 6.84 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ2 Advanced, p-2 orbit, SOMA ATA

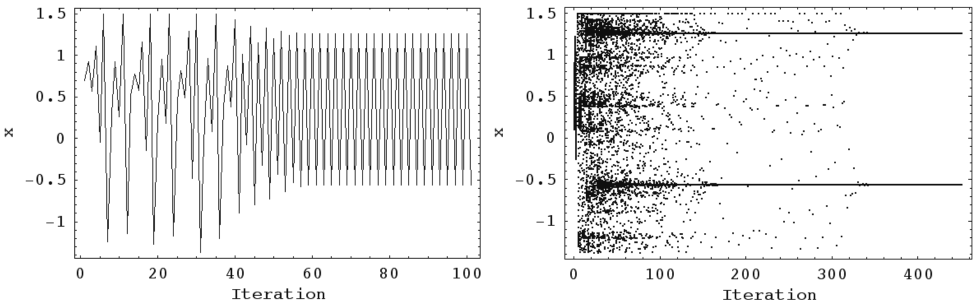


Figure 6.85 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ2 Advanced, p-2 orbit, DERand1JIter

4p

For the results in the last optimization please refer to Table 6.87 - Table 6.88, and Figure 6.86 - Figure 6.87. The presented results show positive features as in case of p-2 orbit. For successful stabilization of p-4 orbit, on average, around 143 (SOMA) or 147 (DE) iterations are required.

Table 6.87 Best individual solutions, HENON, p-4 orbit, CF Targ2 Advanced, SOMA

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
1	-0.4154	0.2808	0.4969	$2.85.10^{-6}$	0.0060	149	145
2	-0.4160	0.2289	0.4810	$2.23.10^{-5}$	0.3374	149	139
3	-0.4167	0.2272	0.4844	$4.1.10^{-6}$	0.1369	149	145
4	-0.4144	0.2906	0.4998	$5.15.10^{-6}$	0.0976	149	143

Table 6.88 Best individual solutions, HENON, p-4 orbit, CF Targ2 Advanced, DE

EA	*K*	F_{max}	*R*	CFVal	AvgCFVal	IStab	AvgIStab
5	-0.4022	0.2150	0.4875	$2.07.10^{-6}$	0.0035	149	147
6	-0.4114	0.3764	0.4986	$3.01.10^{-6}$	0.0090	149	144
7	-0.4122	0.2137	0.4748	$2.5.10^{-6}$	0.0094	137	146
8	-0.4128	0.2170	0.4834	$2.45.10^{-6}$	0.0022	149	147
9	-0.4201	0.2147	0.4933	$1.52.10^{-6}$	0.0050	149	147
10	-0.3934	0.3491	0.4763	$8.25.10^{-7}$	0.0009	149	148

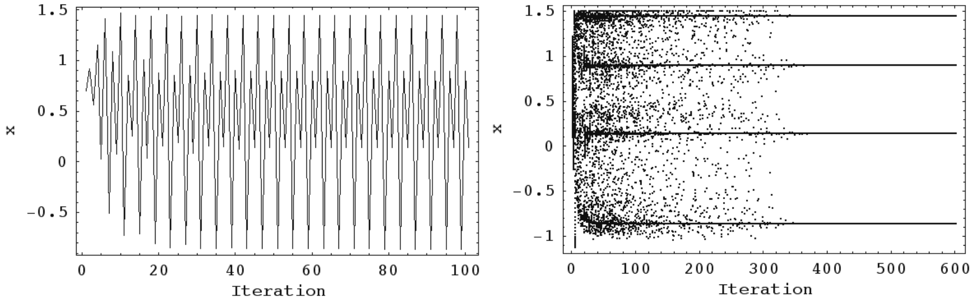


Figure 6.86 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ2 Advanced, p-4 orbit, SOMA ATO

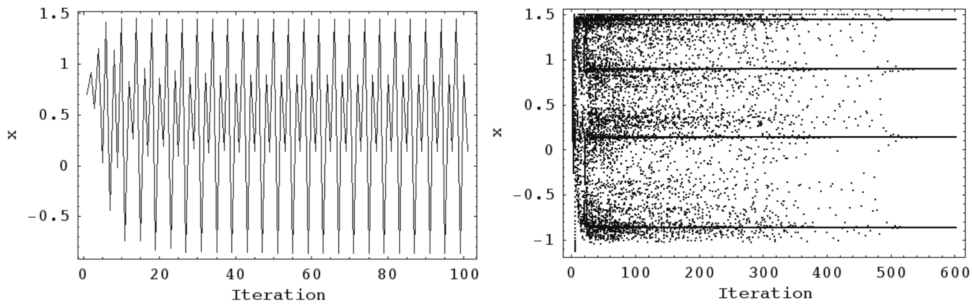


Figure 6.87 Best individual solution (left); Simulation with distributed initial conditions $0 < x_{initial} < 1$, 100 samples (right) HENON, CF Targ2 Advanced, p-4 orbit, DERand1JIter

6.9.3 Conclusion of Results – Case Study 7

The presented results in this case study lend weight to the argument that this CF design produces significant improvement in the task of solving the problems which occurred in previous case studies dealing with targeting CF Targ1 advanced.

The momentous problem with poor performance of both EAs, in case of higher periodic orbits and proportion of IStab value to Average IStab value, was mostly suppressed (more successfully for Henon map, the LQ seems to be harder task, although it is "only" one dimensional system).

The next problem which arose as a consequence of this CF Targ1 design and sensitivity of chaotic system to proper settings is the period doubling or in other words the oscillating around desired higher periodic orbit. This problem was mostly suppressed as well. This improvement is a consequence of slight change in CF design. Only the different numbers of iteration required for stabilization in several repeated simulations of different initial conditions during one CF evaluation process are amplified by multiplication of *NI*, whereas in previous case study different length of stabilization and initial chaotic behavior increases the basic CF value (difference between target state and actual state) and thereafter this increased basic CF value is markedly amplified by multiplication of *NI*. Thus this CF is more focused on quality and fast stabilization and is not influenced by several repeated simulations for wide range of initial conditions and comprise the most universal CF design, which should secure finding of "universal solution" in case of chaos control.

It should also be mentioned, that in the case of the problematic LQ p-4 orbit, the simulation interval had to be increased to 500 iterations as well. In spite of this fact, it is not possible to compare the performance of this CF with previous results (CF Targ1, CF Targ2 and CF Basic), but the most important fact is that this UPO was finally stabilized at all.

Finally, it is possible to say, that CF Targ2 advanced gives better and more reasonable results than CF Targ1 advanced and these results proves the return to the best existing given by CF Targ2. The results obtained in this case can be summarized in following main points:

- When comparing with previous case study 6, the average IStab values (both corrected and not-corrected) are higher but on the other hand here occurs the considerable improvement in performance of EA from the point of view of percentage successfulness of giving the solutions leading to stabilization for both systems and all desired UPOs
- Marked improvement in comparison with previous CF Basic, CF NA, CF Targ1 and 2 from the point of view **corrected** average IStab value for LQ p-2 orbit.
- Problem with fast targeting not only for initial conditions used in optimization process but for the uniformly distributed initial conditions has been solved also here for both systems and for all desired UPOs.

- When comparing EAs, DE again gives slightly more “stabilization securing” solutions.
- The two main problems with affected final CF value by different numbers of iteration required for stabilization in several runs during one CF evaluation and the tiny period doubling in case of p-2 and p-4 orbit was mostly suppressed.
- This CF design in general gives the best results of all when considering all described and solved problems and compare the results with previous CF designs.

For the comparison of average number of iterations required for successful stabilization see final conclusion of all case studies (Table 6.89 and Table 6.90).

6.10 Conclusion of All Results

This dissertation covers seven case studies with different used CF as presented. An apt statistical comparison of all case studies is depicted in the following tables and figures. All previous partial conclusions of all case studies gives the following piece of knowledge. (For detailed description of each case study please see the corresponding partial conclusion.)

- The first proposed CF Basic gives satisfactory results and can be used wherever the good quality of stabilization is expected and the speed of stabilization and "universality of this solution" for wider range of initial conditions are not decisive. This CF does not require any special experiences and knowledge about the system.
- The second CF simple represents the simplest example of targeting CF suitable only for stabilization of fixed point. In comparison with CF Basic it gives similar results in the case of LQ and slightly better results in the case of Henon map.
- The next proposal of CF NA represents the progressive targeting CF suitable for p-1 and p-2 orbit, which gives very good results in the task of shortening of the initial chaotic stage. The results for p-1 orbit are significantly better than in the previous two CFs, on the other hand the slightly better results for p-2 orbit were achieved at the cost of arising of problem with worse performance of EAs and obtaining of solutions with only temporary stabilization or none at all. Moreover this CF requires some knowledge about results achieved in the case of CF Basic due to proper setting of *SC* value.
- The fourth CF Targ1 brings the advantage of automatically computed *SC* value, thus it is suitable for any desired UPO. The obtained results were similar as in case of CF NA.
- In the next proposal of CF Targ2 there were only slight changes in CF design, but from the presented results it can be seen, how can such a small change influence the performance of a controlled system, especially when it is an extremely sensitive chaotic system. Here, another improvement from the point of view of quickness of stabilization was achieved and furthermore the performance of EAs was increased, thus the proportion of non-stabilizing and stabilizing securing solutions. This seems to be the best

choice, when very good solution from the close neighborhood of initial conditions is expected.

- Penultimate CF Targ1 advanced is the example of an upgraded CF Targ1 design for the purpose of improving the behavior of controlled chaotic system for wide range of initial conditions. The results were for the first view satisfactory, even the stabilization of very problematic p-4 orbit for Logistic equation was achieved, but two very momentous problems arose – period doubling and very poor performance of EAs. These problems uncovered hiden non-optimal structure of CF Targ1.
- Last proposal of CF Targ2 advanced gives excellent results for simulations with wide range of initial conditions and seem to be the choice for the task of finding of “universal and robust solution”. The problems with poor EA performance and period doubling were mostly suppressed here. The only disadvantage of this proposal is relatively big computational-time demands.

For the comparison of average number of iterations required for successful stabilization, see Table 6.89 and Table 6.90. The value in braces represents corrected one, which shows the average IStab value only for solutions, which leads to successful stabilization.

For the comparison of SOMA and DE from the point of view of CF Value of best individual solutions see Table 6.91 and Table 6.92.

Lastly, the comparison of performance of both EAs in the case of higher periodic orbits and proportion of IStab value to Average IStab value, thus the proportion of solutions with either perfect stabilization or temporary or possibly none at all is depicted in Table 6.93 and Table 6.94.

From these three pair of tables it shows that it is complex to answer as to which algorithm is better or worse. In case of LQ, a SOMA seems to be the better choice, and in the case of Henon, DE seems to be the “winner” when the final CF value of best individual solution is taken into account. But from the point of view of the behavior of each EA during optimizations, it is a difficult question. The SOMA rapidly heads towards global optimal solution, whereas DE slowly searches in the erratic CF surface. And this is the reason for the phenomenon, where DE gives slightly more “stabilization securing” solutions, whereas SOMA got stuck in one of huge amount of local minimums. Eventually both EAs gives satisfactory results.

Table 6.89 Comparison of Average IStab values – LQ – Case studies 1-7

UPO	p-1		p-2		p-4	
EA Version	**SOMA**	**DE**	**SOMA**	**DE**	**SOMA**	**DE**
CF Basic	97	97	123	123	197	218
CF Simple	98	99	-	-	-	-
CF NA	33	31	73 (109)	102 (116)	-	-
CF Targ1	33	31	78 (112)	104 (115)	77 (195)	121 (215)
CF Targ2	31	30	95 (112)	113 (113)	151 (184)	182 (196)
CF Targ1 Adv.	37	34	12 (60)	16 (55)	13 (418)	13 (375)
CF Targ2 Adv.	34	33	36 (66)	38 (63)	44 (194)	58 (199)

Table 6.90 Comparison of Average IStab values – HENON – Case studies 1-7

UPO	p-1		p-2		p-4	
EA Version	**SOMA**	**DE**	**SOMA**	**DE**	**SOMA**	**DE**
CF Basic	77	75	124	125	122	122
CF Simple	65	70	-	-	-	-
CF NA	49	47	84 (114)	110 (118)	-	-
CF Targ1	49	47	72 (113)	109 (118)	110 (121)	121 (123)
CF Targ2	39	38	91 (108)	113 (117)	115 (118)	123 (123)
CF Targ1 Adv.	56	59	8 (51)	9 (57)	28 (84)	30 (82)
CF Targ2 Adv.	41	40	78 (133)	88 (130)	143 (143)	146 (147)

Table 6.91 Comparison of CF values – LQ – Case studies 1-7

UPO	p-1		p-2		p-4	
EA Version	SOMA	DE	SOMA	DE	SOMA	DE
CF Basic	0	0	$5.33.10^{-15}$	$7.11.10^{-15}$	$3.04.10^{-05}$	$3.07.10^{-05}$
CF Simple	0.3745	0.3738	-	-	-	-
CF NA	$2.80.10^{-15}$	$2.70.10^{-15}$	$1.14.10^{-6}$	$1.20.10^{-6}$	-	-
CF Targ1	$2.70.10^{-15}$	$2.80.10^{-15}$	$5.37.10^{-13}$	$8.85.10^{-13}$	0.0061	0.0059
CF Targ2	$2.80.10^{-15}$	$2.70.10^{-15}$	$1.60.10^{-14}$	$1.77.10^{-14}$	0.0002	0.0002
CF Targ1 Adv.	$2.57.10^{-14}$	$2.56.10^{-14}$	15.0221	19.7431	0.1229	0.6302
CF Targ2 Adv.	$2.54.10^{-14}$	$2.56.10^{-14}$	0.6392	0.2353	2.1906	0.0031

Table 6.92 Comparison of CF values – HENON – Case studies 1-7

UPO	p-1		p-2		p-4	
EA Version	SOMA	DE	SOMA	DE	SOMA	DE
CF Basic	$2.22.10^{-16}$	0	$1.60.10^{-14}$	$1.95.10^{-14}$	$9.43.10^{-9}$	$9.16.10^{-9}$
CF Simple	0.2303	0.2304	-	-	-	-
CF NA	$4.00.10^{-15}$	$3.90.10^{-15}$	$1.31.10^{-6}$	$1.35.10^{-6}$	-	-
CF Targ1	$4.00.10^{-15}$	$3.90.10^{-15}$	$3.46.10^{-5}$	$4.53.10^{-8}$	$7.46.10^{-7}$	$5.89.10^{-7}$
CF Targ2	$3.90.10^{-15}$	$3.90.10^{-15}$	$2.23.10^{-9}$	$2.12.10^{-14}$	$1.88.10^{-8}$	$1.78.10^{-8}$
CF Targ1 Adv.	$5.01.10^{-14}$	$4.73.10^{-14}$	216.7945	175.1048	211.5812	192.8344
CF Targ2 Adv.	$3.86.10^{-14}$	$3.90.10^{-14}$	$5.81.10^{-9}$	$3.93.10^{-9}$	$2.85.10^{-6}$	$8.25.10^{-7}$

Table 6.93 Comparison of EA performance - LQ

UPO	p-2		p-4	
EA Version	**SOMA**	**DE**	**SOMA**	**DE**
CF NA	63%	85%	-	-
CF Targ1	65%	89%	38%	55%
CF Targ2	81%	92%	79%	92%
CF Targ1 Adv.	6%	10%	2%	2%
CF Targ2 Adv.	32%	34%	18%	26%

Table 6.94 Comparison of EA performance - HENON

UPO	p-2		p-4	
EA Version	**SOMA**	**DE**	**SOMA**	**DE**
CF NA	70%	92%	-	-
CF Targ1	59%	91%	89%	98%
CF Targ2	80%	95%	97%	99%
CF Targ1 Adv.	1%	3%	23%	23%
CF Targ2 Adv.	53%	63%	100%	100%

For the comparison of performance of best solution in the task of simulation for uniformly distributed initial conditions see Figure 6.88 - Figure 6.93. Here it can be clearly seen all described differences in performance of each CF design for both chaotic systems and all desired UPOs.

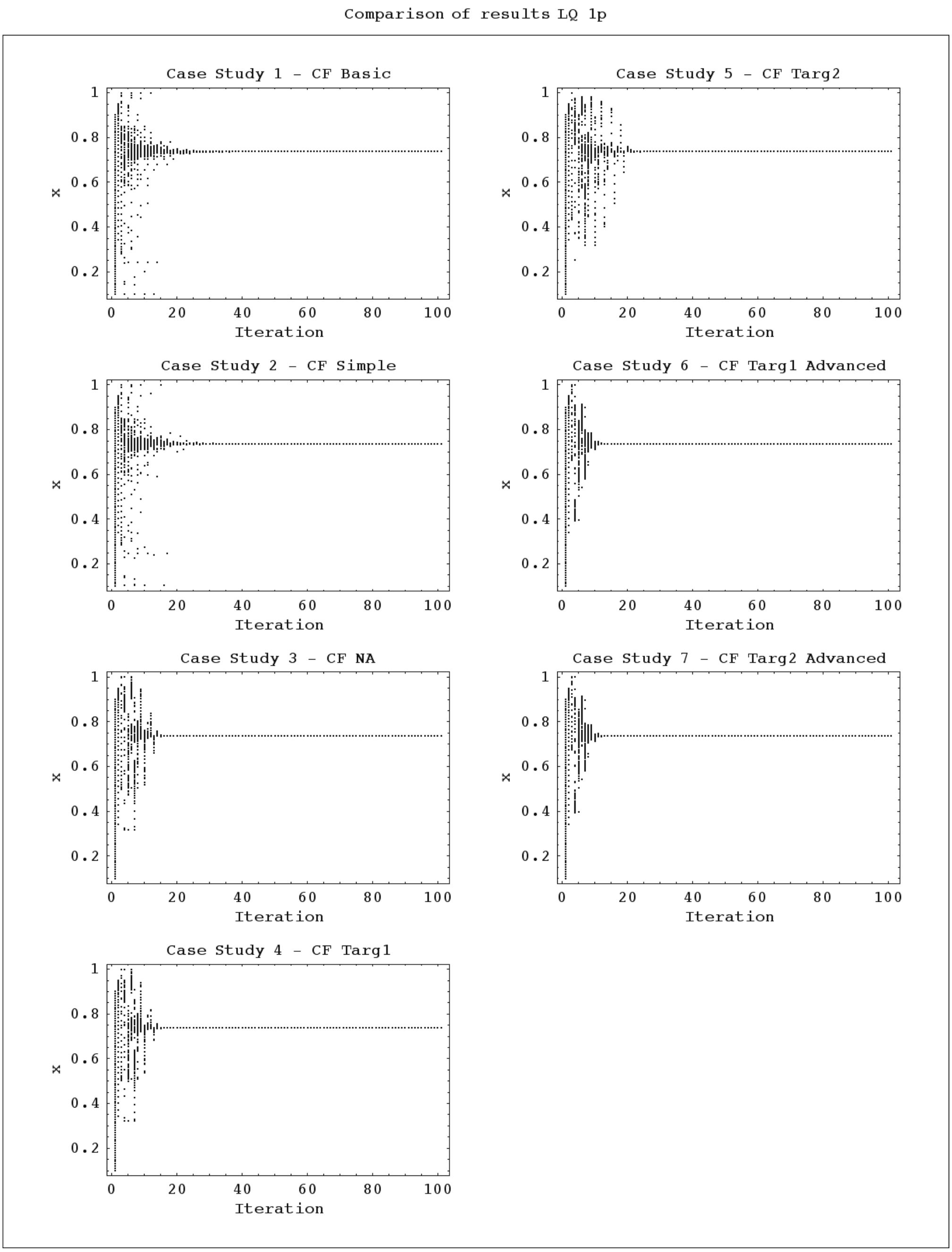


Figure 6.88 Comparison of results for LQ – 1p, simulations with distributed initial conditions $0 < x_{initial} < 1$, 100 samples; 1-SOMA ATA, 2-DELocalToBest, 3-DEBest1JIter, 4-DERand1Bin, 5-DERand1DIter, 6-DELocalToBest, 7-SOMA ATO

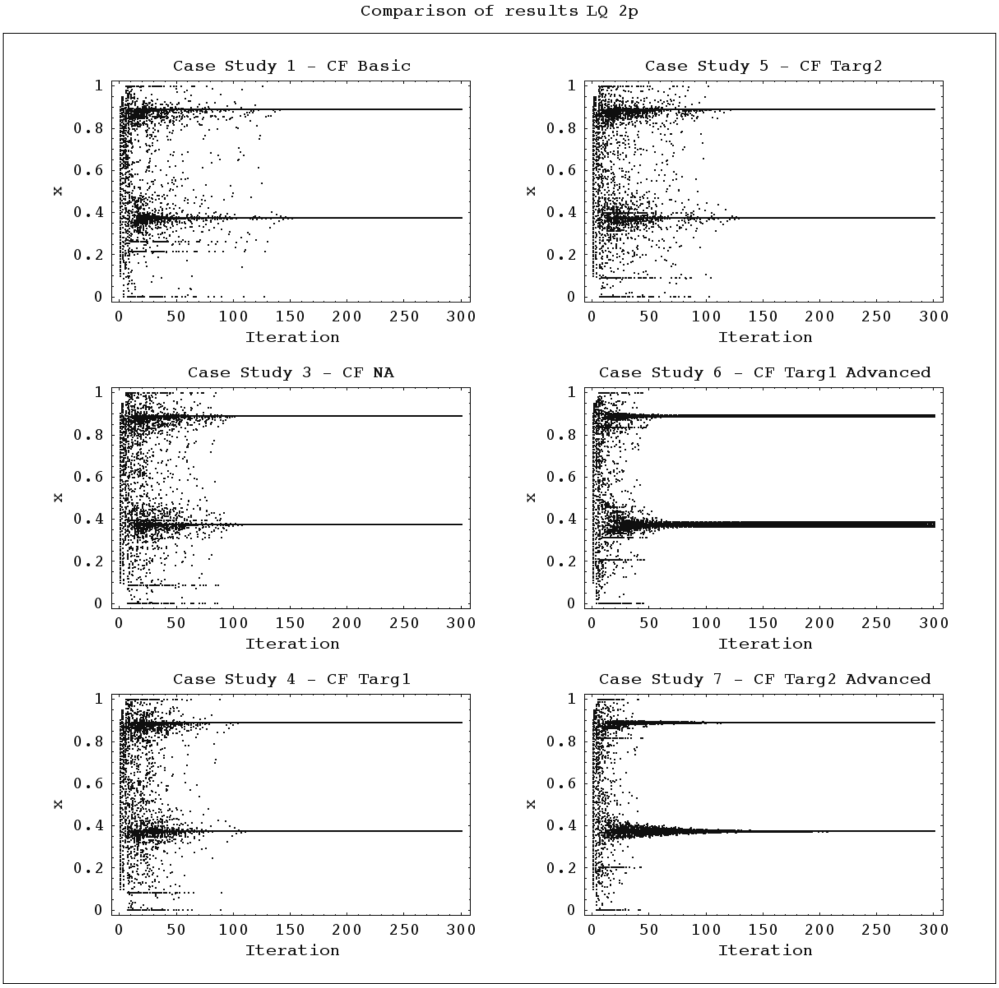


Figure 6.89 Comparison of results for LQ – 2p, simulations with distributed initial conditions $0 < x_{initial} < 1$, 100 samples; 1-SOMA ATR, 3- DERand1Bin, 4-SOMA ATAA, 5-DERand2Bin, 6-SOMA ATA, 7- DELocalToBest

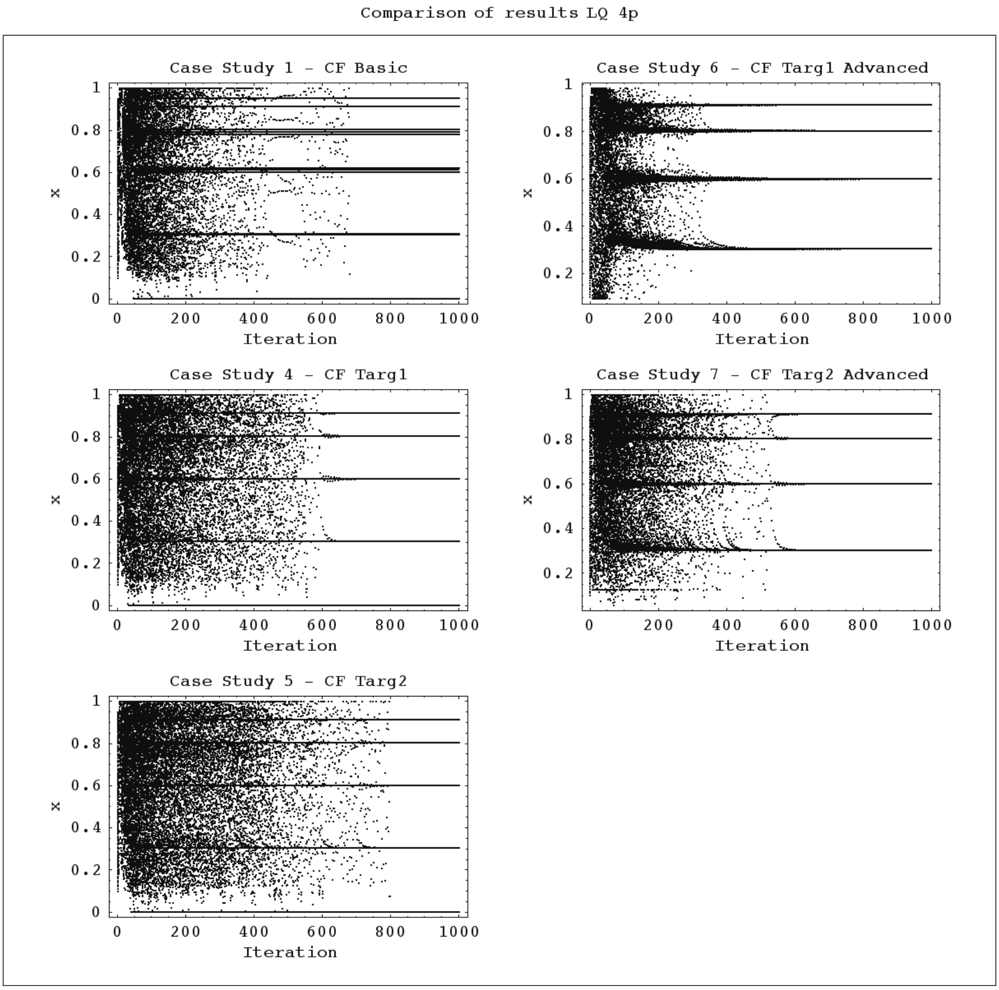


Figure 6.90 Comparison of results for LQ – 4p, simulations with distributed initial conditions $0 < x_{initial} < 1$, 100 samples; 1-SOMA ATA, 4-DEBest2Bin, 5- DELocalToBest, 6- DEBest1JIter, 7- DELocalToBest

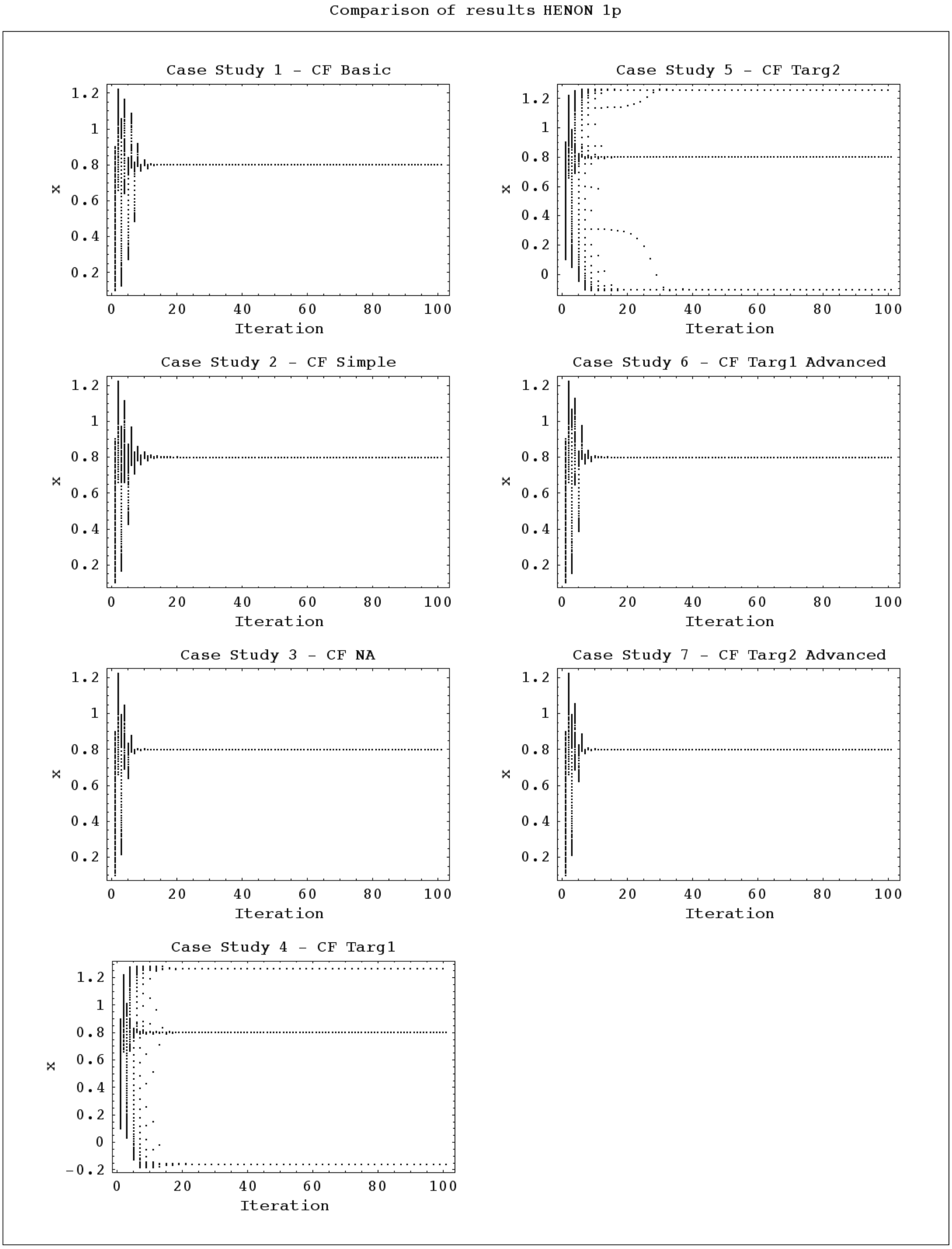


Figure 6.91 Comparison of results for HENON – 1p, simulations with distributed initial conditions $0 < x_{initial} < 1$, 100 samples; 1-DERand2Bin, 2-DERand1Bin, 3-DEBest2Bin, 4-DELocalToBest, 5-DELocalToBest, 6-DERand1DIter, 7-SOMA ATAA

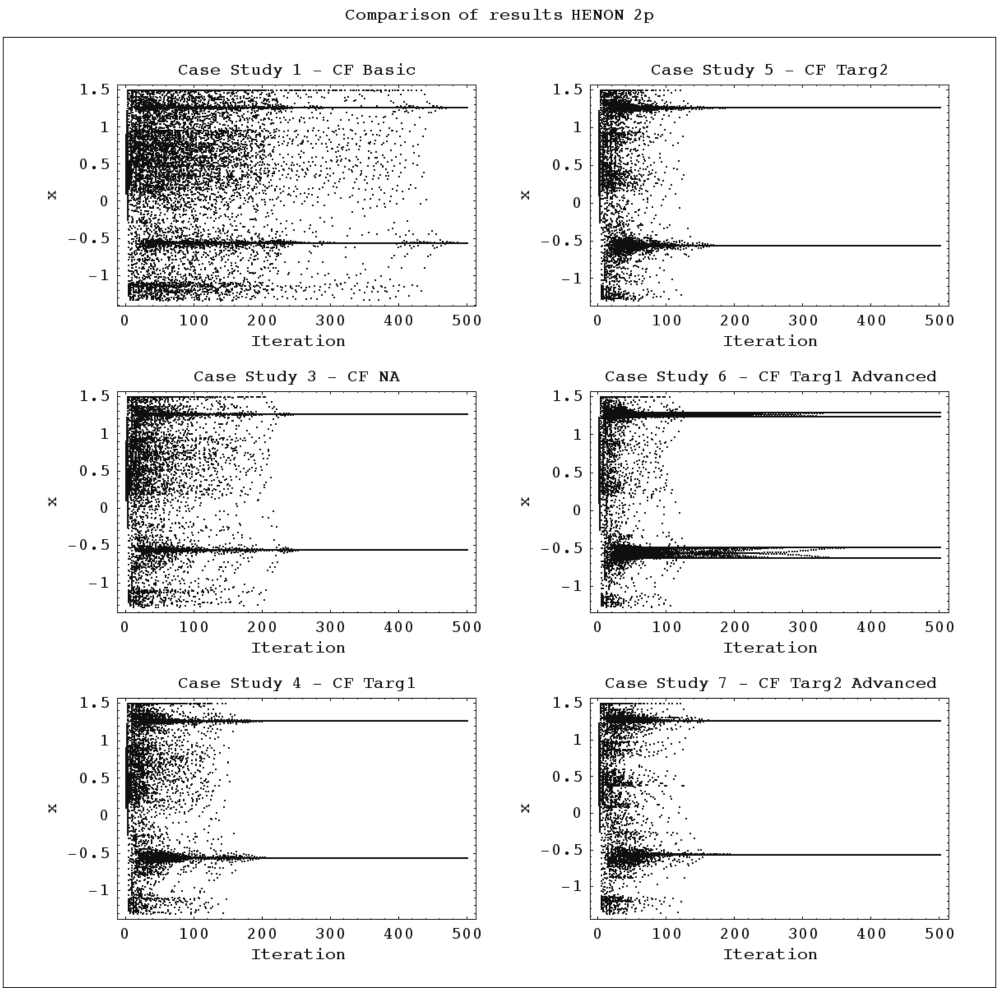


Figure 6.92 Comparison of results for HENON – 2p, simulations with distributed initial conditions $0 < x_{initial} < 1$, 100 samples; 1-DEBest2Bin, 3-SOMA ATO, 4-SOMA ATO, 5-SOMA ATR, 6-DERand1Bin, 7-DERand1DIter

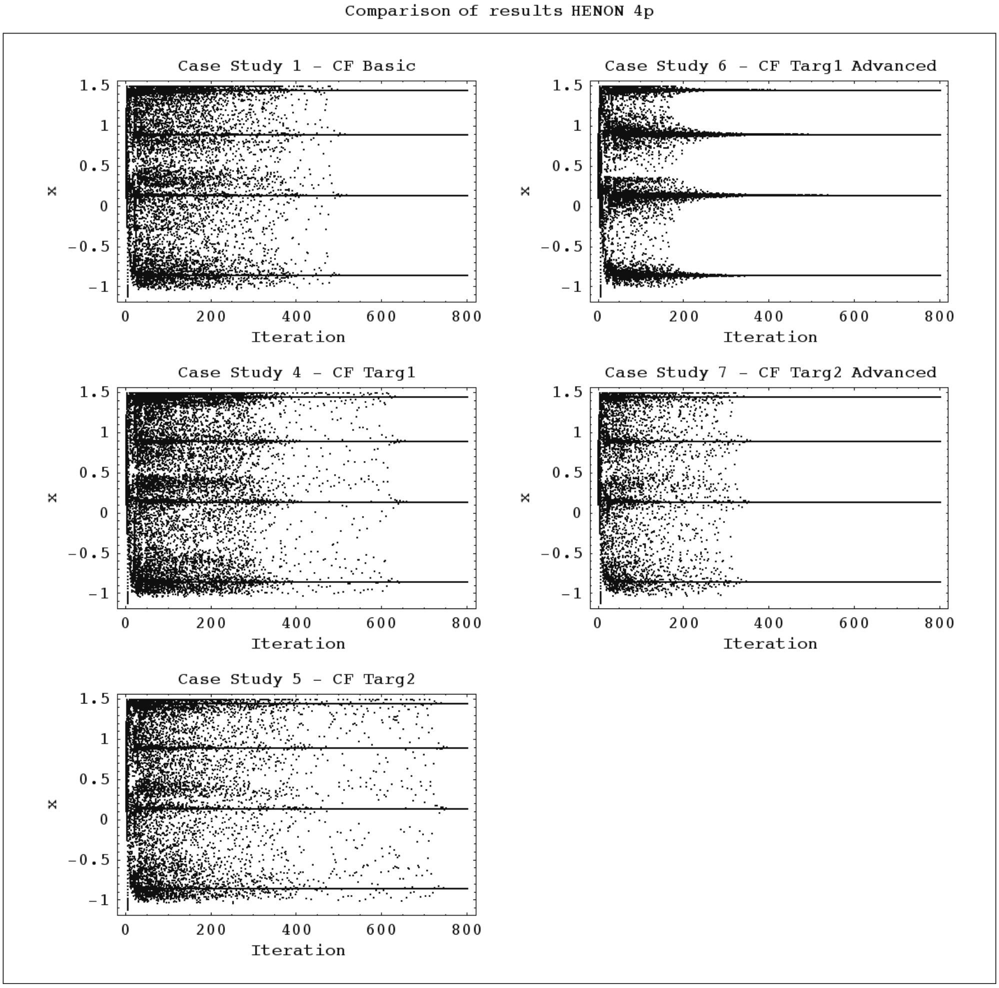


Figure 6.93 Comparison of results for HENON – 4p, simulations with distributed initial conditions $0 < x_{initial} < 1$, 100 samples; 1-SOMA ATAA, 4-DEBest1JIter, 5-DELocalToBest, 6-DELocalToBest, 7-SOMA ATO

6.11 Suggestion of technique for optimization of time-continuous systems

This last and the smallest chapter deals with the proposal of method and Cost Functions in the task of optimization of control of time-continuous systems (i.e. Lorenz, Rössler).

In this case, it is very hard task, because the EAs have to estimate not only three adjustable parameters K, F_{max}, R, but the length of time delay (τ in equations 5, 6) and as a consequence of this length of step during numerical solving of set of differential equations and of course the simulation interval as well.

Of course, there is also problem with Cost Function design because it is not possible to use the basic idea of minimizing the difference between target state and actual output from system, due to enormous computational demands for analyzing of system and exact numerical determination of UPO. The cost function have to be able to recognize the behavior of system and returns the optimal solutions not for exact target state but for selected behavior of system. For example in the case of discrete-time system and p-2 orbit, the difference between n and n+2 iteration has to be minimal.

Some research in this field has been already done, but the simulations are very time-demanding, thus it is still not possible to obtain sufficient amount of results. Nevertheless, the future research will be mainly focused to this field.

7 CONCLUSIONS AND DISCUSSIONS

The optimization of chaos control described here is relatively simple and easy to implement. Based on obtained results, it may be claimed that all simulations give satisfactory results and thus EAs are capable of solving this class of difficult problems and the quality of results does not depend only on the problem being solved but also on the proper definition of the CF.

From the comparison with classical control technique – OGY follows that TDAS or ETDAS based control method can be simply considered as targeting and stabilizing algorithm and their performance is much better than OGY.

As can be seen from the optimization results presented here, they are extremely sensitive to the construction of used CF, for example the problem with fast stabilization not only for initial conditions used in optimization process, but for the whole range of the initial conditions or other continuously described problems in all case studies. From the presented results follows, that any small change in the design of CF can cause radical improvement of system behavior (as in case of CF Targ2), but of course on the other hand can cause worsening of observed parameters and behavior of chaotic as well. This negative change can be seen from the comparison between CF Targ1 advanced and CF Targ2 advanced.

This work consists of seven case studies. Each one deals with different proposal of cost function used in optimizations. The results obtained in each case study are discussed and compared with previous cases continuously and at the end of each case study as well.

From all partial conclusions follows, that it is hard task to propose a CF, which gives excellent results, especially "universal results" suitable for simulation with wide range of initial conditions. As repeatedly mentioned the chaotic systems are extremely sensitive to proper settings of control algorithm and of course they are very sensitive to even very tiny change in any parameter. This extreme sensitivity is transferred into complexity of CF surface thus it is also hard task for EAs to find good solution. It is also difficult to determine the conditions for optimizations and subsequent simulations. For example the specification of correct length of optimization interval τ_i is very difficult and it can be stated that it is alike to balancing at the edge of knife when considering this fact, that the difference in final CF value of size 1.10^{-4} and subsequent very tiny change in combination of estimated parameters can cause improvement or worsening of system behavior. And this small difference can be caused by change in CF design or just only by adding another 50 iterations (if the system is not absolutely stabilized, the difference between ideal target state and actual output of system can be relatively appreciable).

As a consequence of these facts it is possible to say that all presented CFs gives good results and each one is more or less suitable depending on concrete demands for quickness or quality of stabilization, computational-time demands, order of UPO, whether it should be the solution only for limited circle of initial conditions or for wider range etc.

Lastly, as a conclusion it seems that CF Targ2 (case study 5) is the best choice for optimizations and simulations with limited circle of initial conditions, while its upgraded version CF Targ2 advanced (case study 7) gives excellent results for wide range of initial solutions, thus gives the "universal" solutions, which contradict with words chaos or chaotic system.

Of course, there can be the question, why all results are depicted and described in this work, when to show these best two case studies could be enough, but as mentioned above, each case study brings results suitable for concrete demands and the most important fact is that from all case studies can be seen progressive development of CF design and or parameters of optimizations and mainly the demandingness of this interesting task, which the optimization of chaos control is.

There is no problem for the future research in defining much more complex CF comprising as subcriteria control of stability, costs, time-optimality, controllability, or any of their arbitrary combinations. Furthermore parameter settings for EA were based on heuristic approach; therefore there is also possibility for the future research. According to all results showed here it is planned that the main activities will be focused on testing of evolutionary deterministic chaos control in continuous-time and high-order systems and finally testing of evolutionary real-time chaos control.

All simulation and statistical results together with complete overview of CF surfaces for both chaotic systems and all desired UPOs are depicted in the Appendix.

The total amount of optimizations was 36 000. The optimization took from one minute in the case of p-1 orbit, CF Basic and TDAS control method to one hour in case of CF Targ2, Logistic equation and p-4 orbit. All simulations were performed in Wolfram Mathematica environment.

The total number of cost function evaluations (CFE) for all presented results in this dissertation thesis was 200 millions.

All five main goals of this thesis are stated at the very beginning and the fulfillment of them could be discussed in the following points.

1. **To prove that EAs are able to find optimal solution in case of chaos control.**
   The experimental part of this thesis show in several case studies that EAs are capable and can be in reality used for optimization of chaos control with excellent results. This point can be considered as the main goal of this work.

2. **To test several examples of chaotic systems (one and higher dimensional).**
   In this thesis, two discrete-time systems were used for all optimizations and subsequent simulations. The one dimensional Logistic equation and two-dimensional Henon map.

3. **To test a stabilization for various states (stable state – a fixed point) or higher dimensional periodic orbits.**
   Three desired UPOs were used in presented case studies - only with two exceptions: case study 2 (p-2 and p-4 orbit are missing) and case study 3 (p-4 orbit was not used). These periodic orbits were selected as the basic test benchmark for optimizations from these following reasons. The Fixed point (p-1 orbit) is relatively easy to reach and stabilizes the system without any major problems. This UPO gives great chance to test each designed CF and compare both EAs without possibility of influence on final results and statistic by problems with stabilization and extremely sensitivity of chaotic systems to tiny changes in CF design, initial conditions or control method set up. The next p-2 orbit was chosen as the representative example of higher periodic orbit to compare

how the EAs in each case study can deal with this task. The last p-4 orbit was selected due to the fact, that with growing dimension of UPO, the sensitivity of system and control algorithm to proper settings increases, thus also rapidly increase demands for EA to find satisfactory solution. Consequently this point was also fulfilled.

4. **To compare the results between different EAs**
   This goal has also been reached in this work. All optimizations were performed at the same time with identical settings for following two evolutionary algorithms: SOMA (Self Organizing Migrating Algorithm) and DE (Differential Evolution).

5. **To try a various designs of cost functions and compare their performance (faster stabilization – targeting to close neighborhood of desired UPO, solving the small problems described in the individual case study).**
   The whole practical part describes this point in detail. It consists of seven case studies. Each one is focused on testing of a new cost function design for both systems and all desired UPOs. The results, which fulfill this goal can be clearly seen in Tables within Chapter 6 and particularly in the Appendix, where the comparison of all case studies from the point of view simulation results and CF surfaces is depicted.

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8 APPENDIX

This chapter is divided into two parts. The first one deals with the complete overview of CF surfaces for all case studies and all desired periodic orbits in both controlled systems. All figures are depicted in this way so that it is possible to compare the difference between all 7 designed Cost Functions for selected system and desired UPO. This part consists of six sections 8.1 – 8.6. All CF surfaces are created for the best solution given by SOMA or DE in each concrete case.

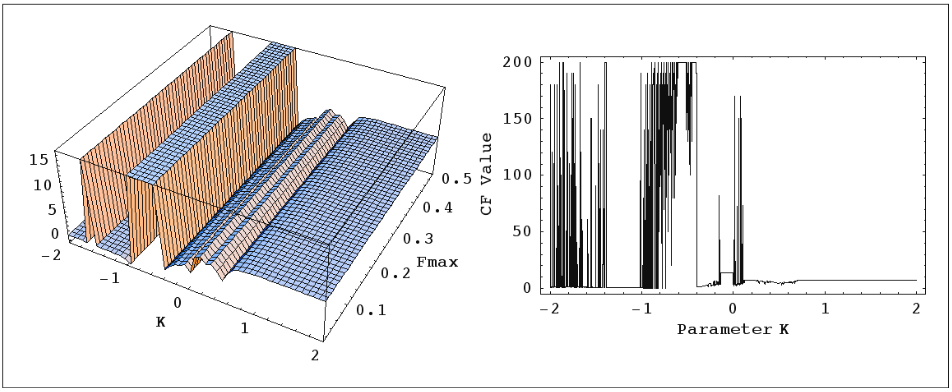
The second part (sections 8.7 – 8.20) is focused on simulation of the best individual solutions and detailed statistics of optimization results for all case studies as in previous part. All figures are depicted in following way.

Firstly, the best individual solutions given by SOMA are depicted followed by simulations of these best solutions for distributed initial conditions from the range 0 – 1. Then the simulations of six best solutions given by DE for the initial conditions used in optimizations and for the distributed initial conditions ensues as in case of SOMA. All these four comparisons are made for three desired UPOs.

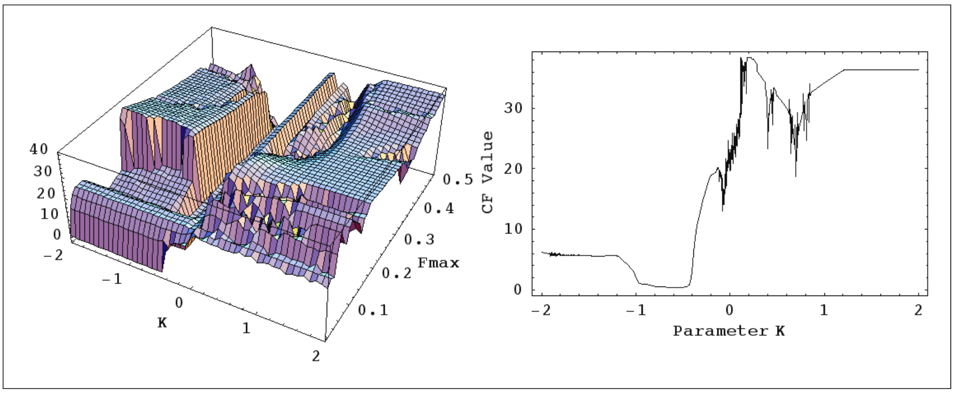
After that the statistical comparison of SOMA and DE follows. This comprises histograms of CF Value and comparison from the point of view of estimated parameters K, F_{max}, R (in case of ETDAS method) This is depicted in diagrams, which show the variance from min to max of observed parameter and the small rectangular mark represents average value.

8.1 Comparison of CFs, LQ, p-1 orbit

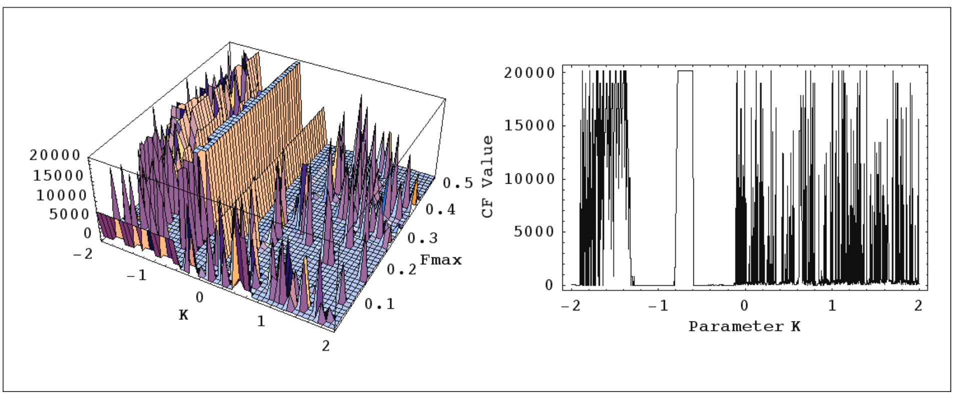
CF Surface LQ 1p CF Basic



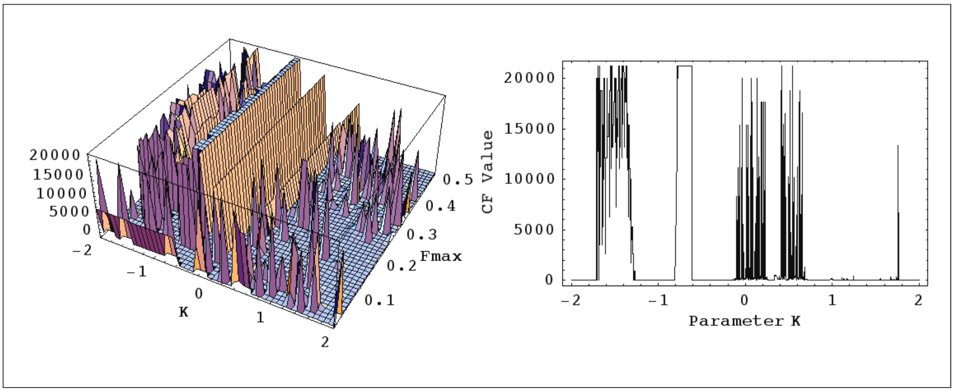
CF Surface LQ 1p CF Simple



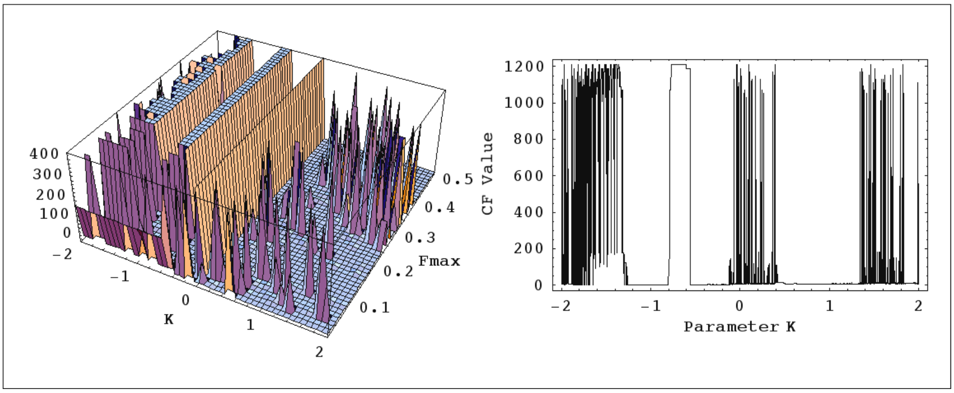
CF Surface LQ 1p CF Non-Auto



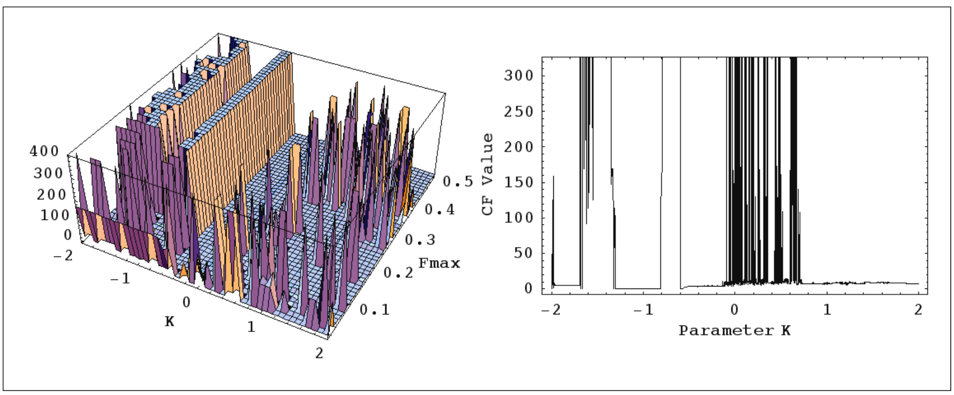
CF Surface LQ 1p CF Targ1



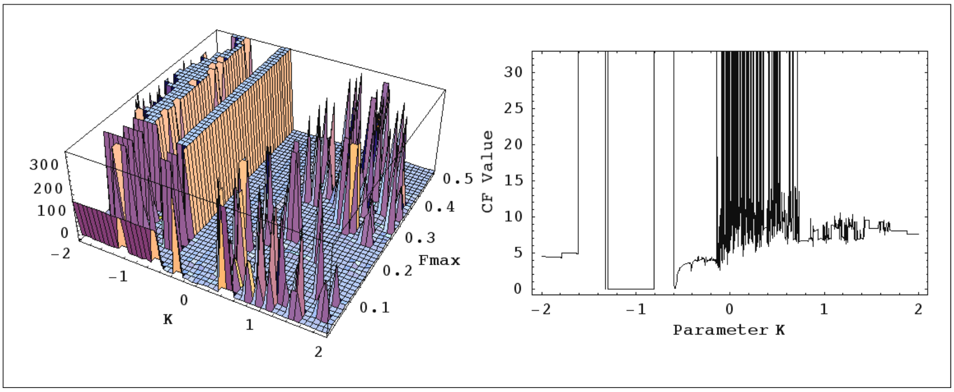
CF Surface LQ 1p CF Targ2



CF Surface LQ 1p CF Targ1 Advanced

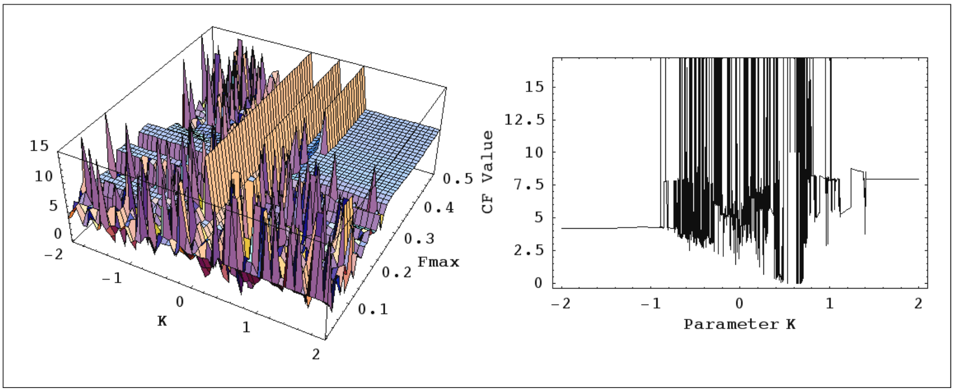


CF Surface LQ 1p CF Targ2 Advanced

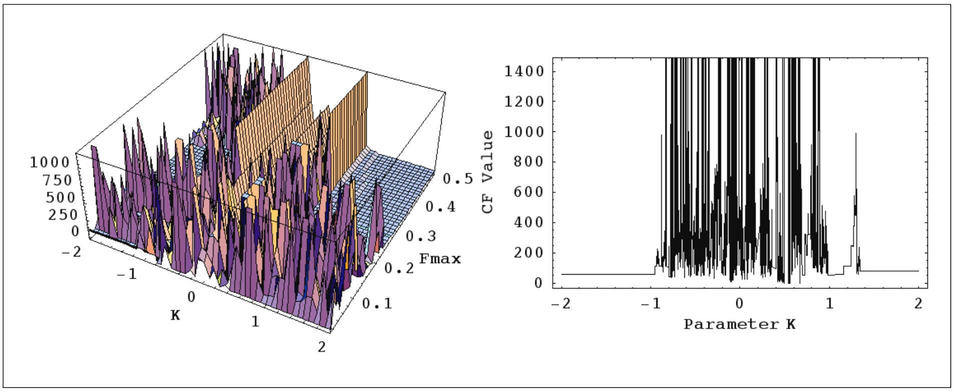


8.2 Comparison of CFs, LQ, p-2 orbit

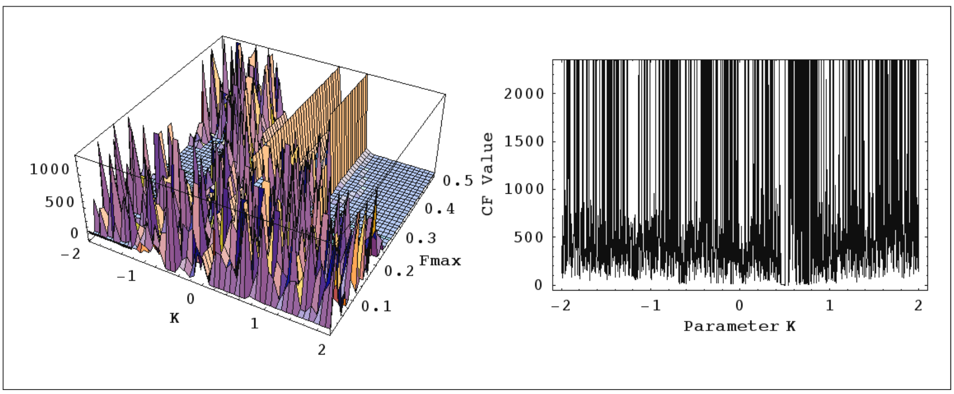
CF Surface LQ 2p CF Basic



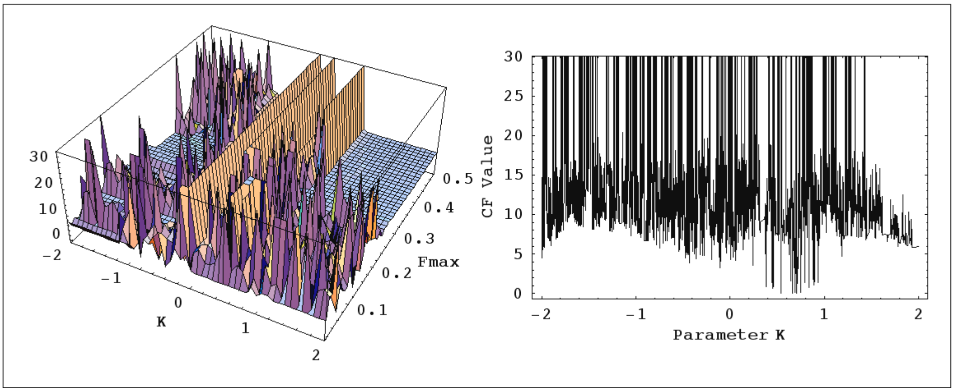
CF Surface LQ 2p CF Non-Auto



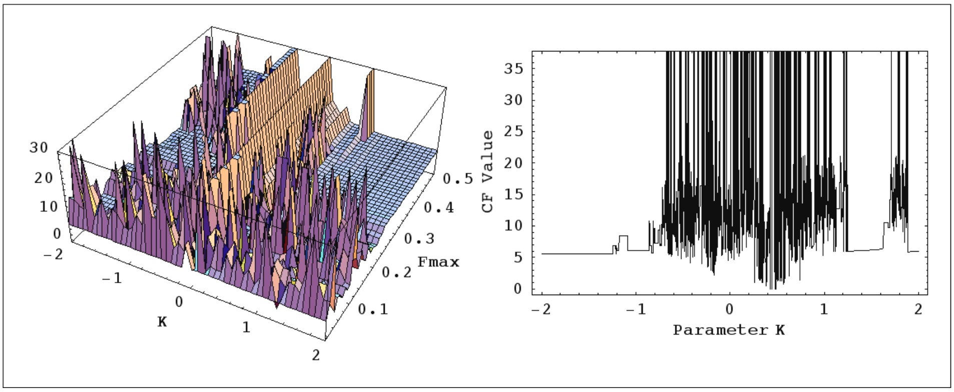
CF Surface LQ 2p CF Targ1



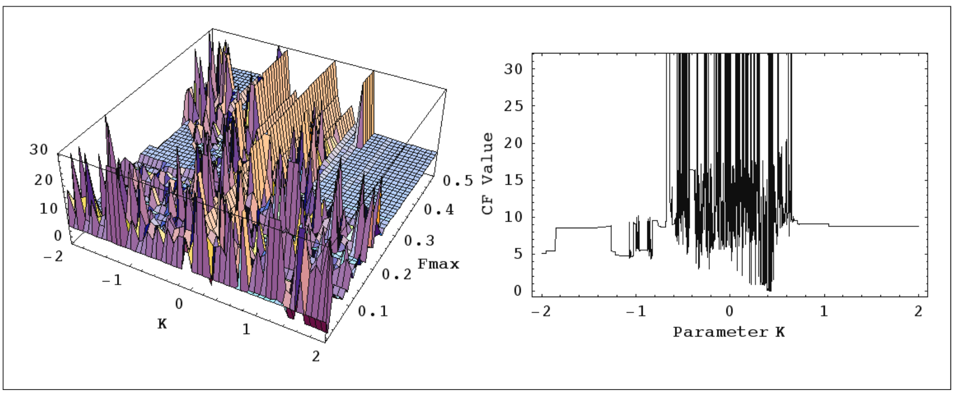
CF Surface LQ 2p CF Targ2



CF Surface LQ 2p CF Targ1 Advanced

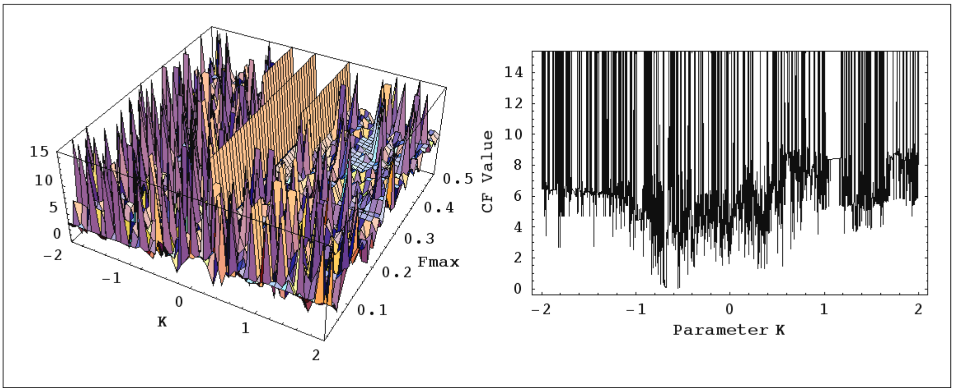


CF Surface LQ 2p CF Targ2 Advanced

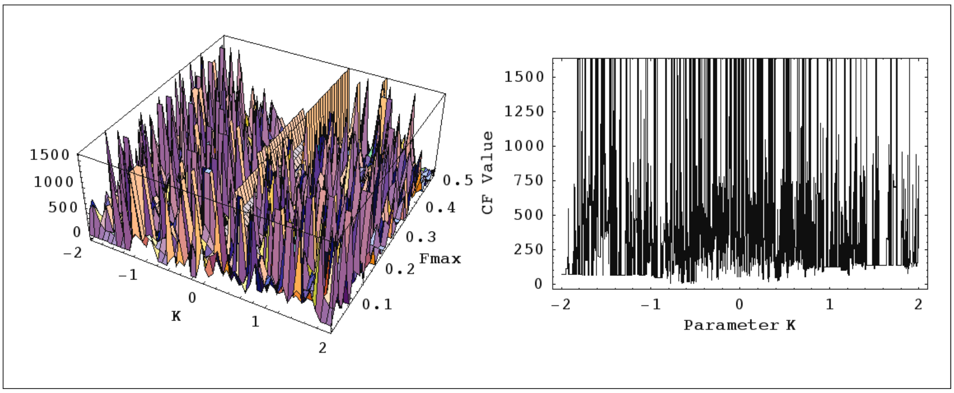


8.3 Comparison of CFs, LQ, p-4 orbit

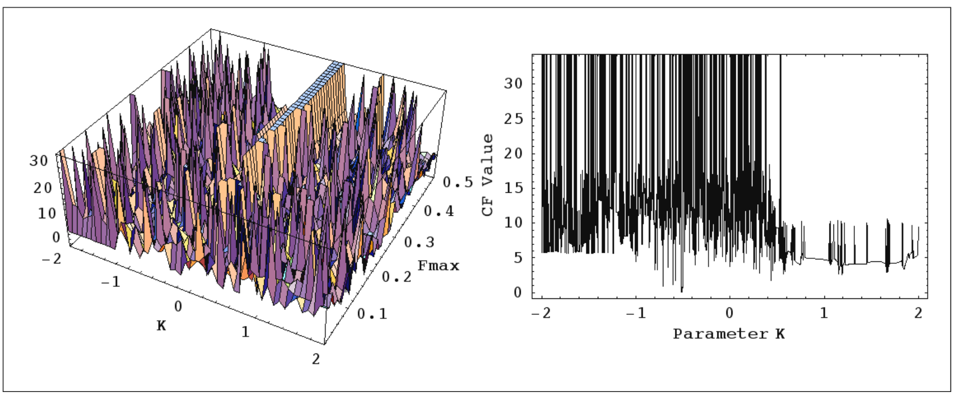
CF Surface LQ 4p CF Basic



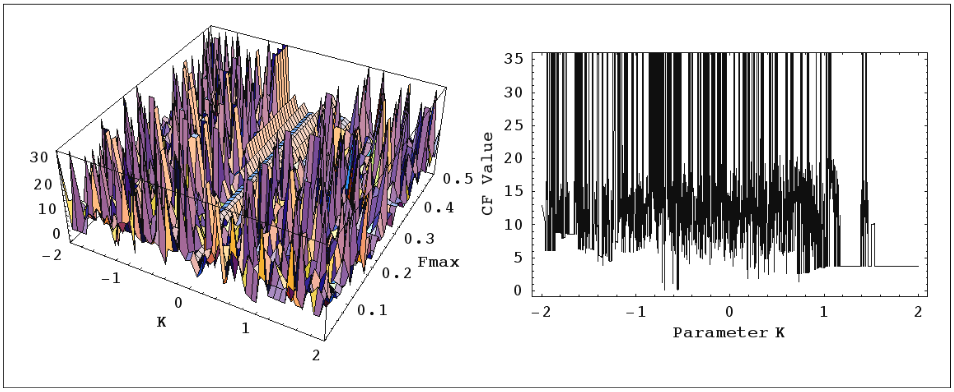
CF Surface LQ 4p CF Targ1



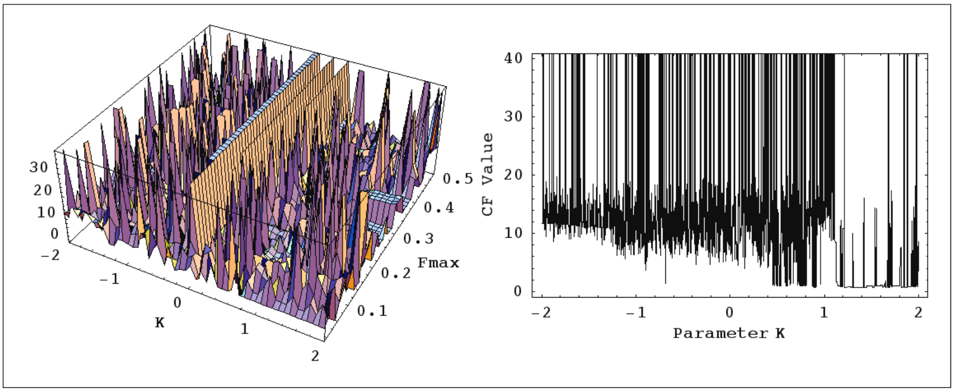
CF Surface LQ 4p CF Targ2



CF Surface LQ 4p CF Targ1 Advanced

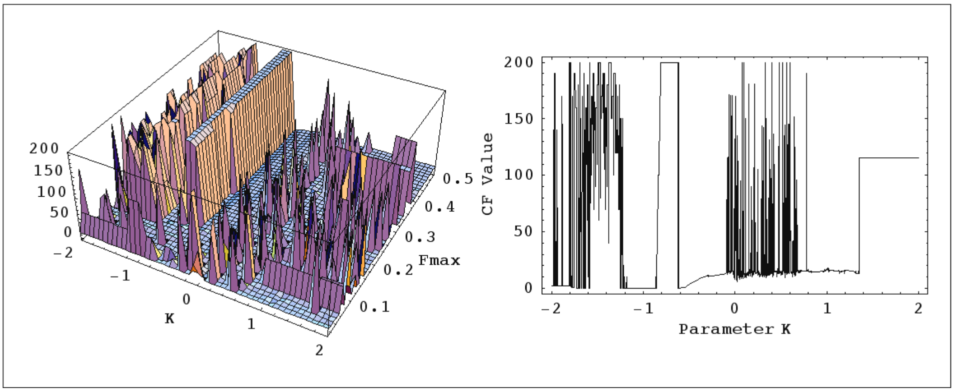


CF Surface LQ 4p CF Targ2 Advanced

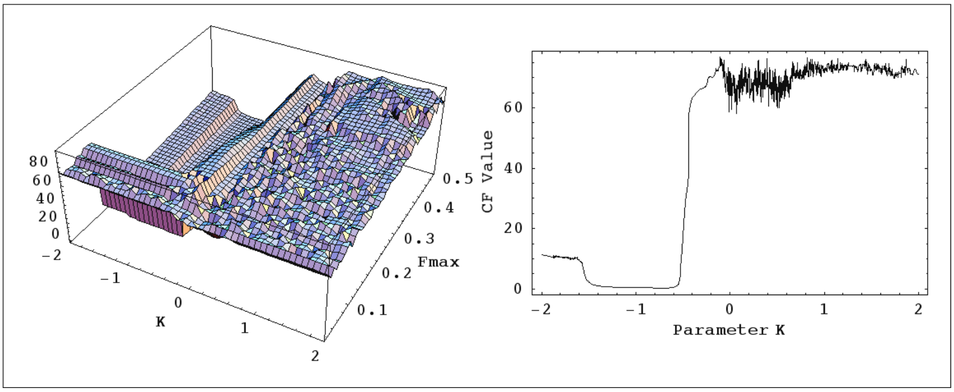


8.4 Comparison of CFs, HENON, p-1 orbit

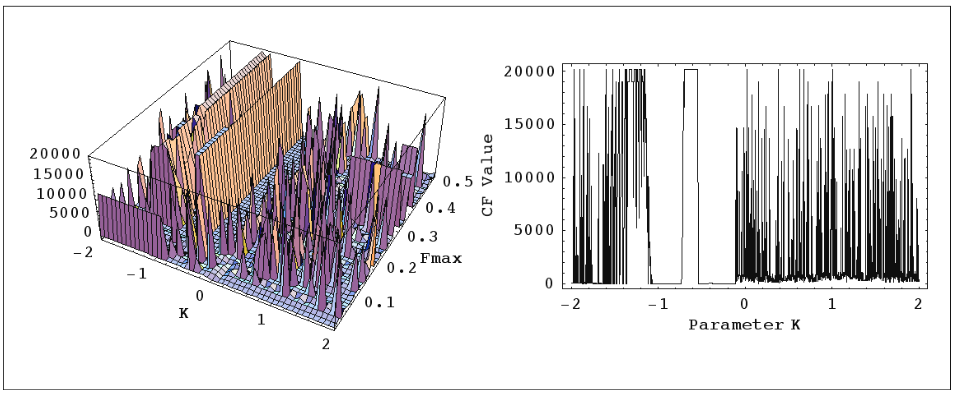
CF Surface Henon 1p CF Basic



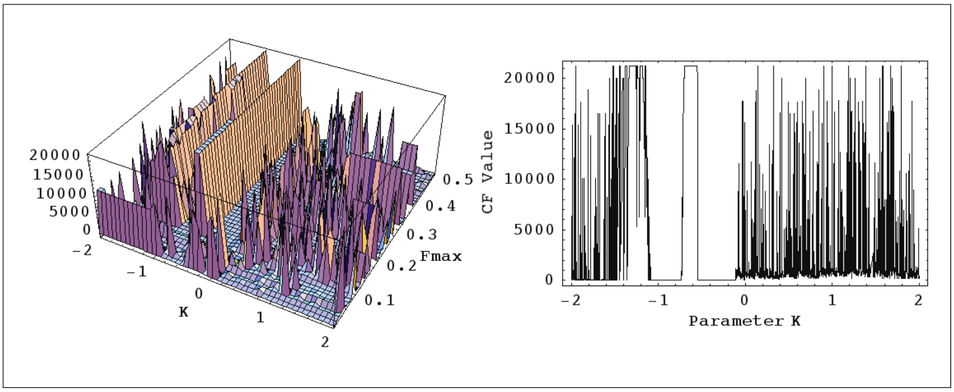
CF Surface HENON 1p CF Simple



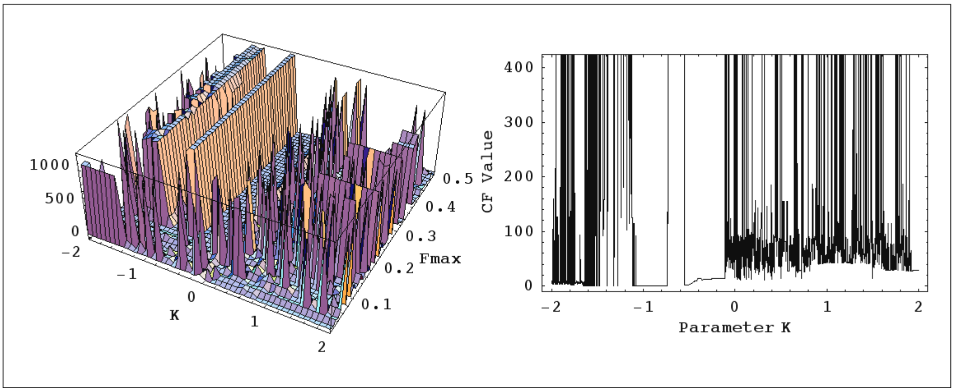
CF Surface HENON 1p CF Non-Auto



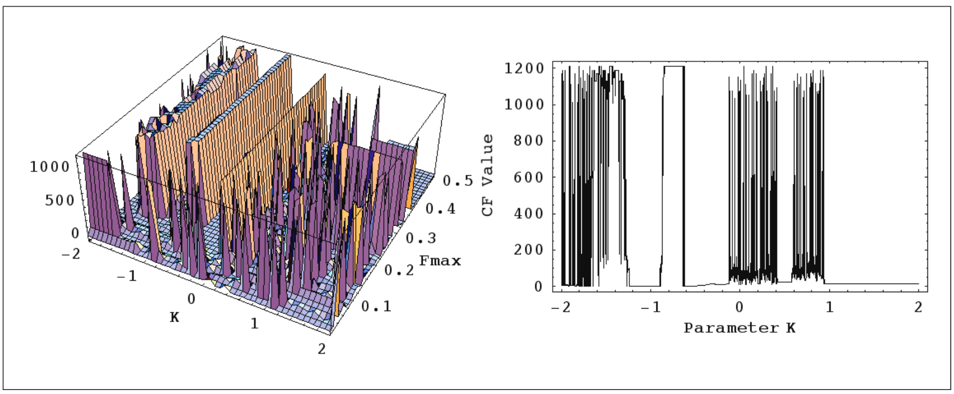
CF Surface HENON 1p CF Targ1



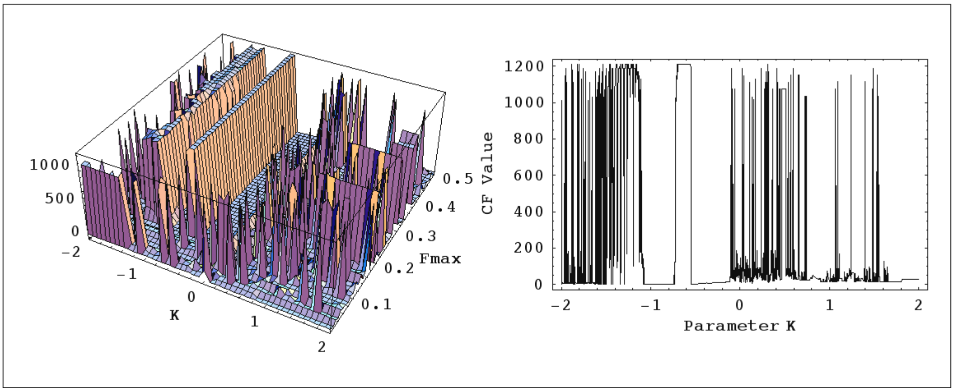
CF Surface HENON 1p CF Targ2



CF Surface HENON 1p CF Targ1 Advanced

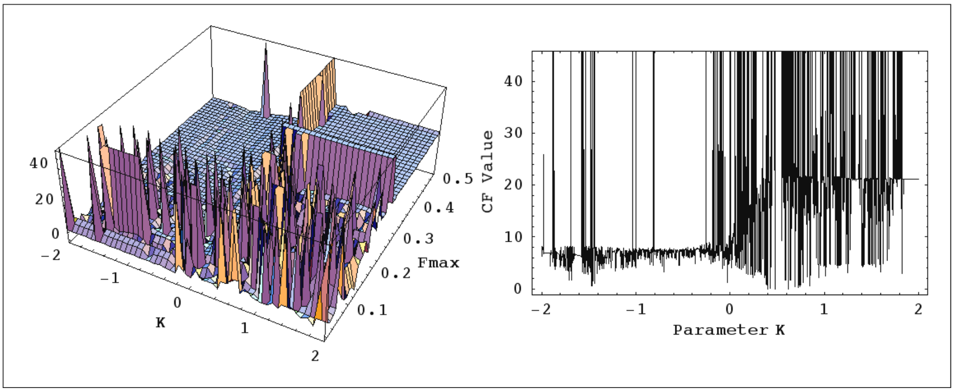


CF Surface HENON 1p CF Targ2 Advanced

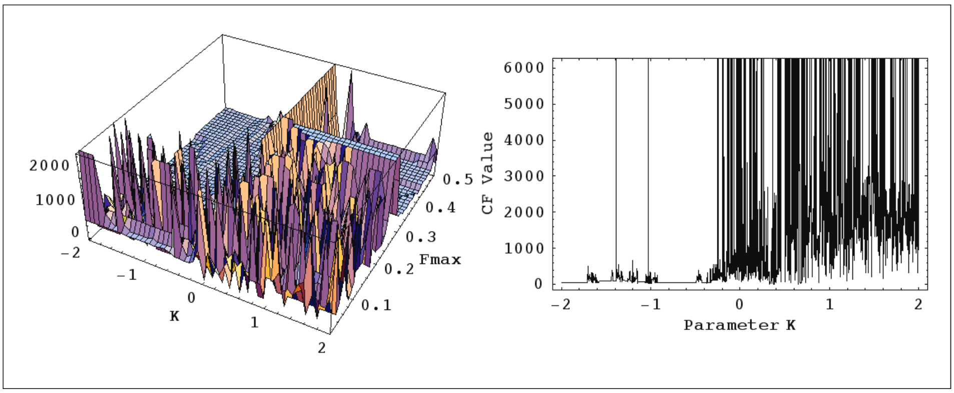


8.5 Comparison of CFs, HENON, p-2 orbit

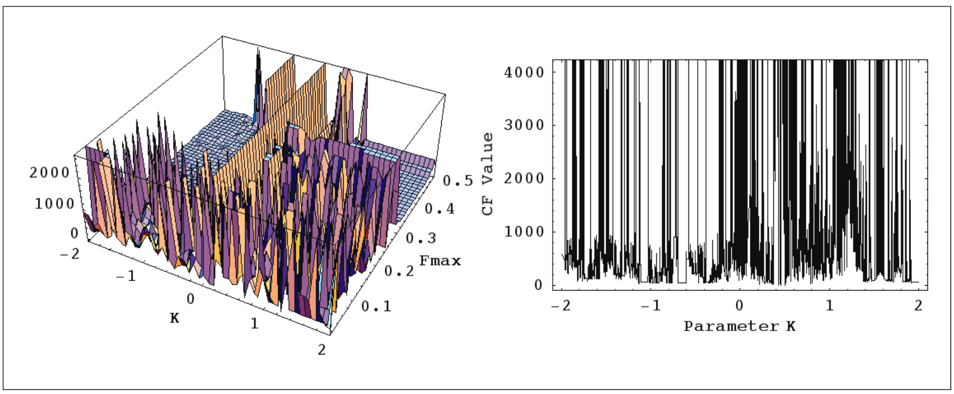
CF Surface HENON 2p CF Basic



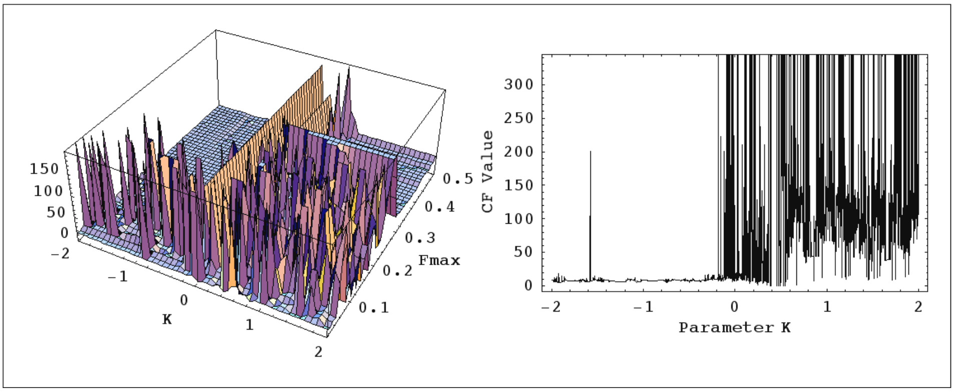
CF Surface HENON 2p CF Non-Auto



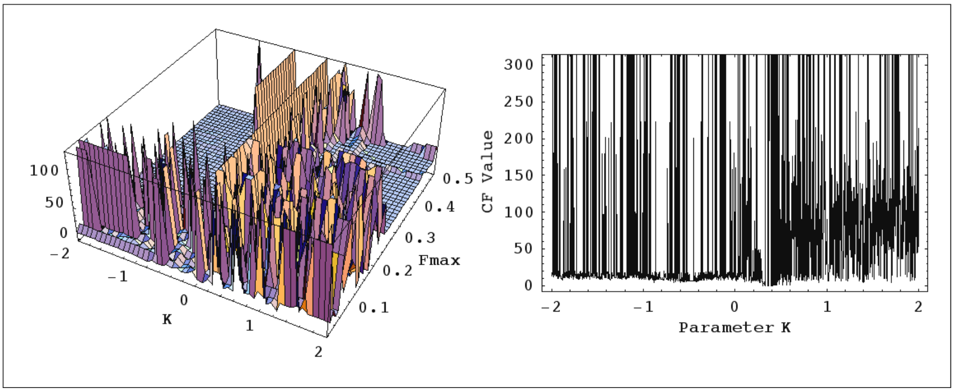
CF Surface HENON 2p CF Targ1



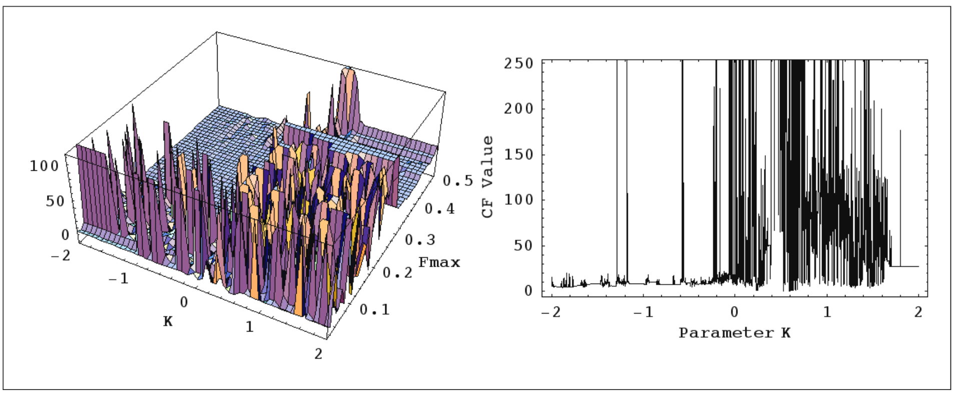
CF Surface HENON 2p CF Targ2



CF Surface HENON 2p CF Targ1 Advanced

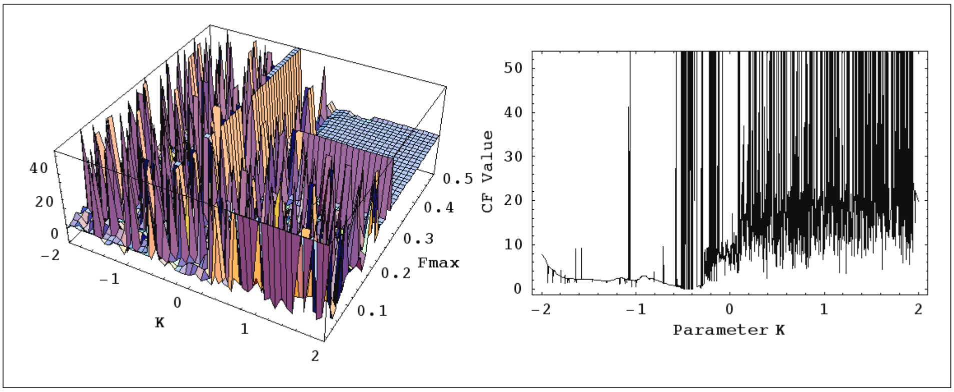


CF Surface HENON 2p CF Targ2 Advanced

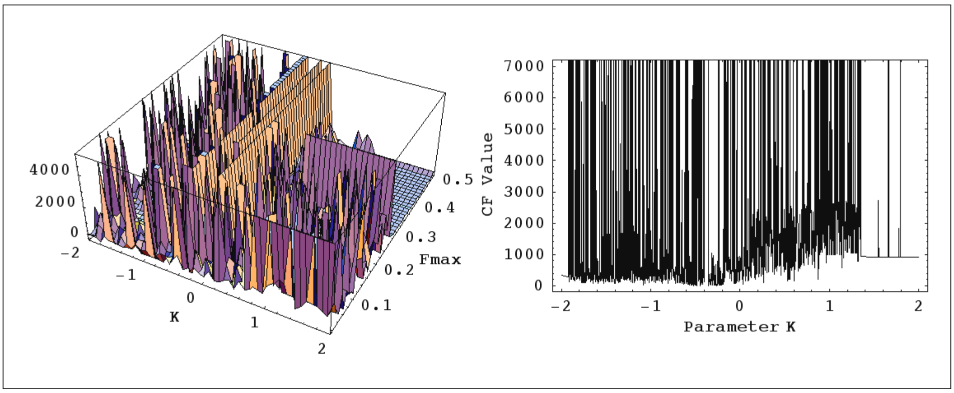


8.6 Comparison of CFs, HENON, p-4 orbit

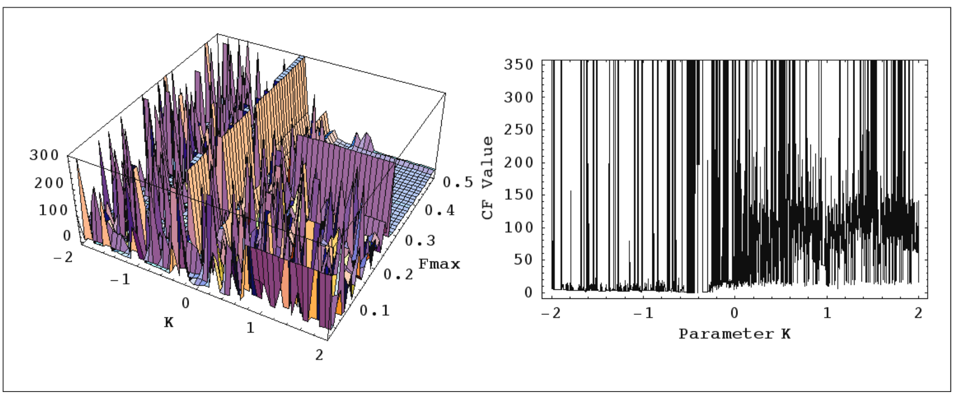
CF Surface HENON 4p CF Basic



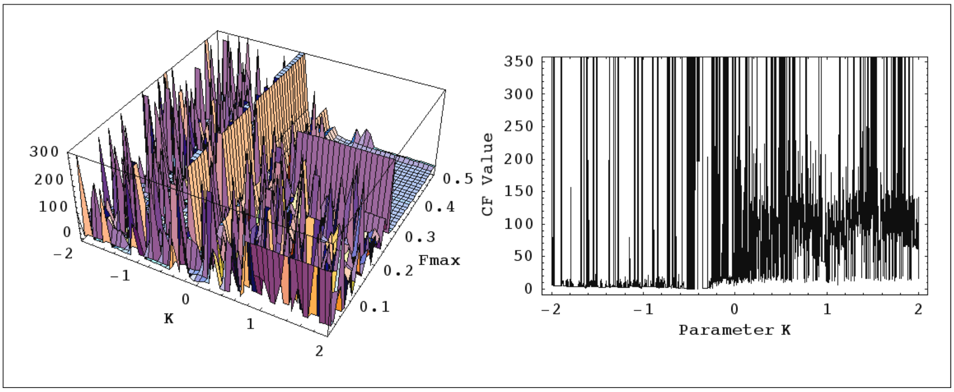
CF Surface HENON 4p CF Targ1



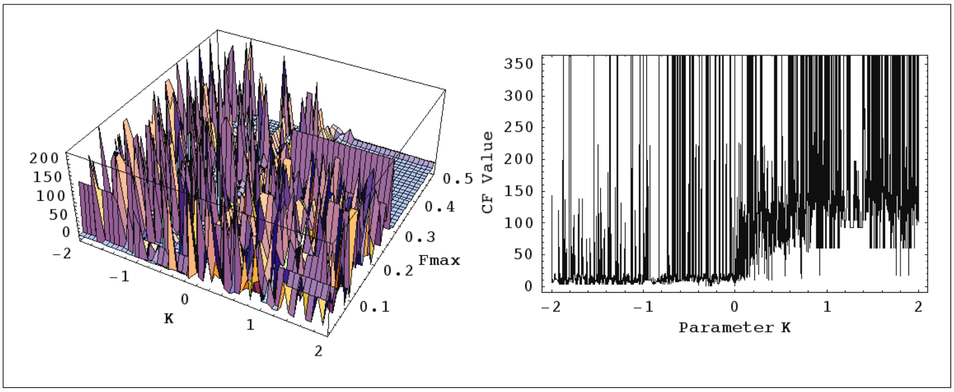
CF Surface HENON 4p CF Targ2



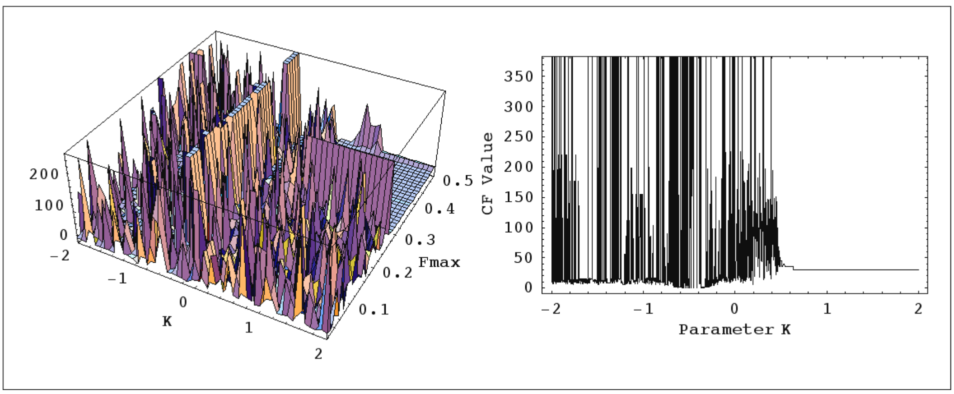
CF Surface HENON 4p CF Targ2



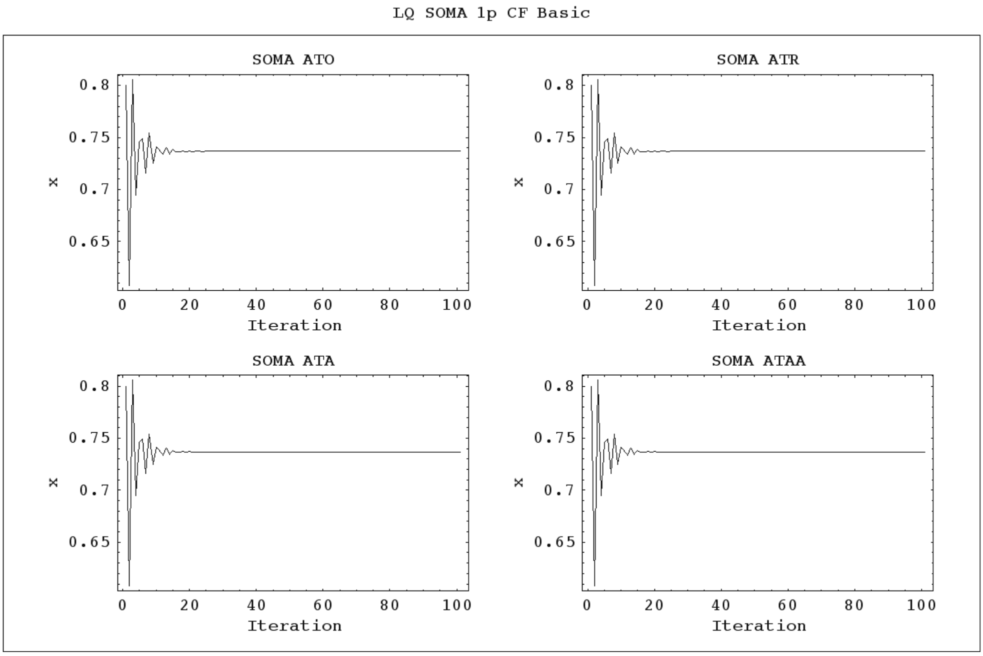
CF Surface HENON 4p CF Targ1 Advanced

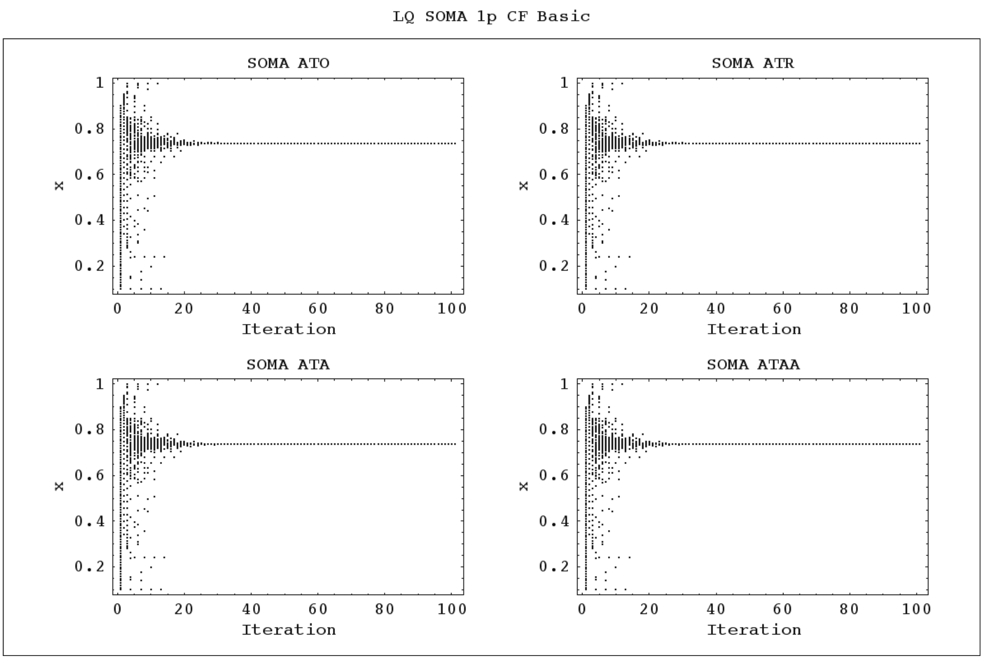


CF Surface HENON 4p CF Targ2 Advanced

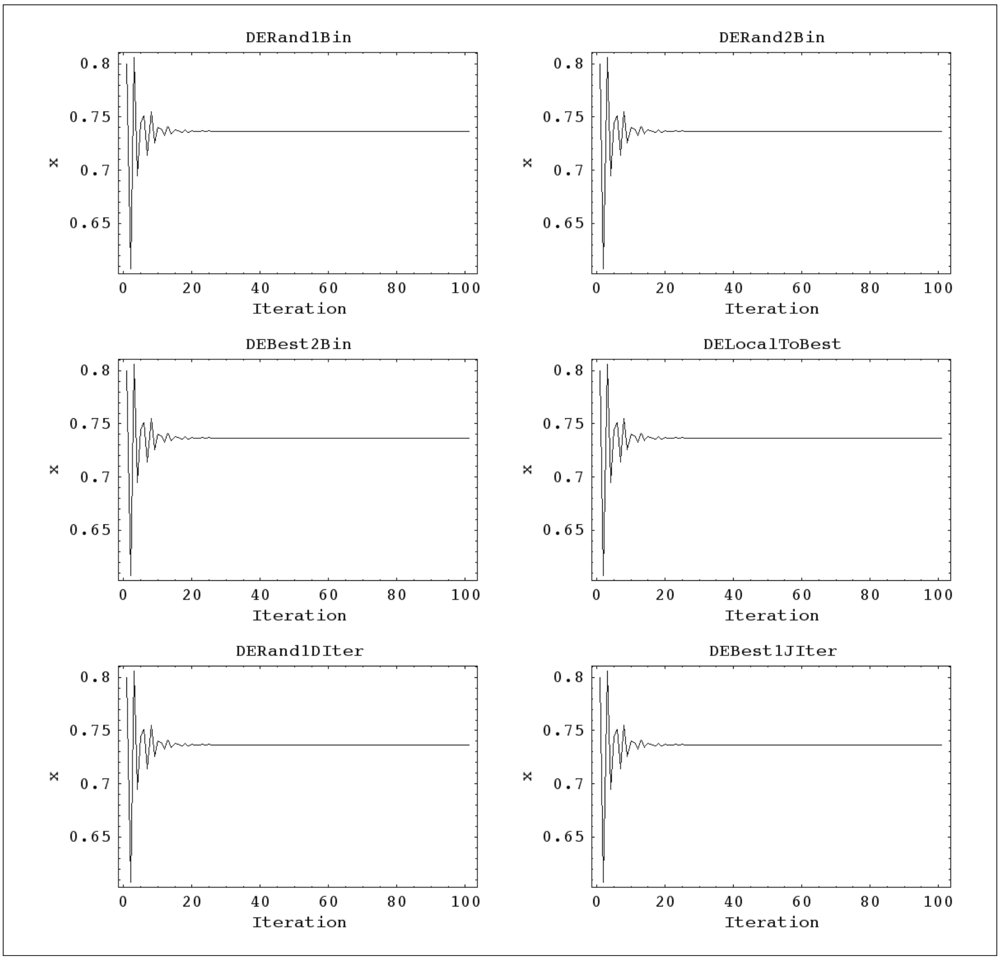


8.7 Summary of results, Case study 1, CF Basic, LQ

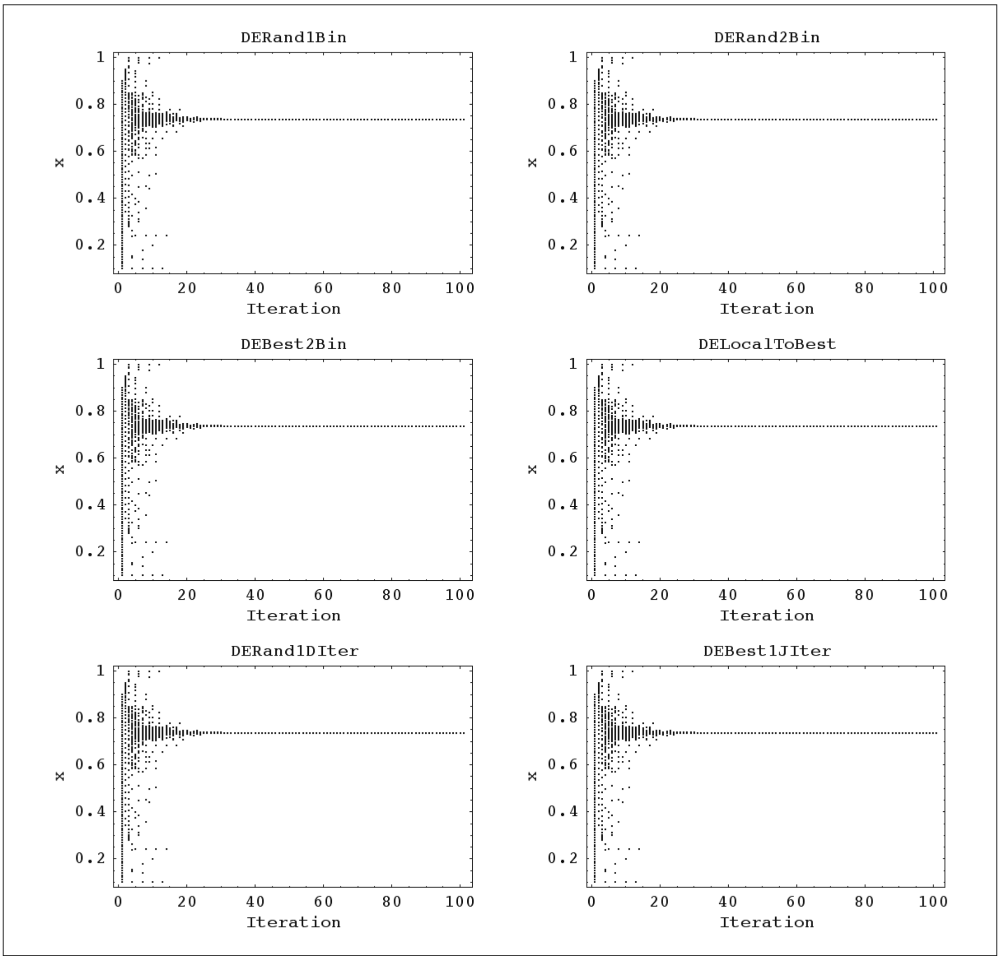




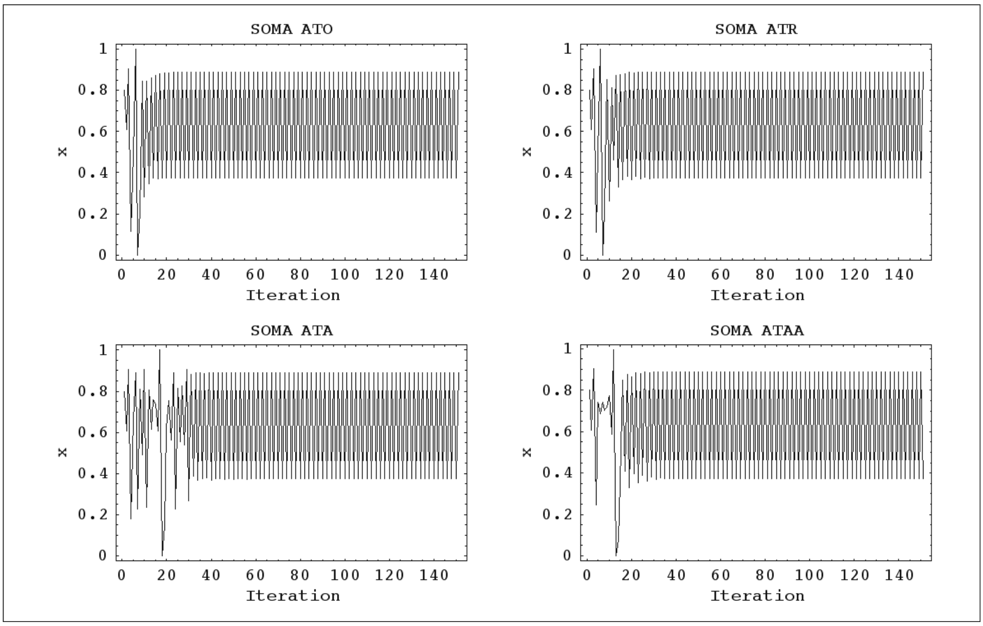
LQ SOMA 1p CF Basic



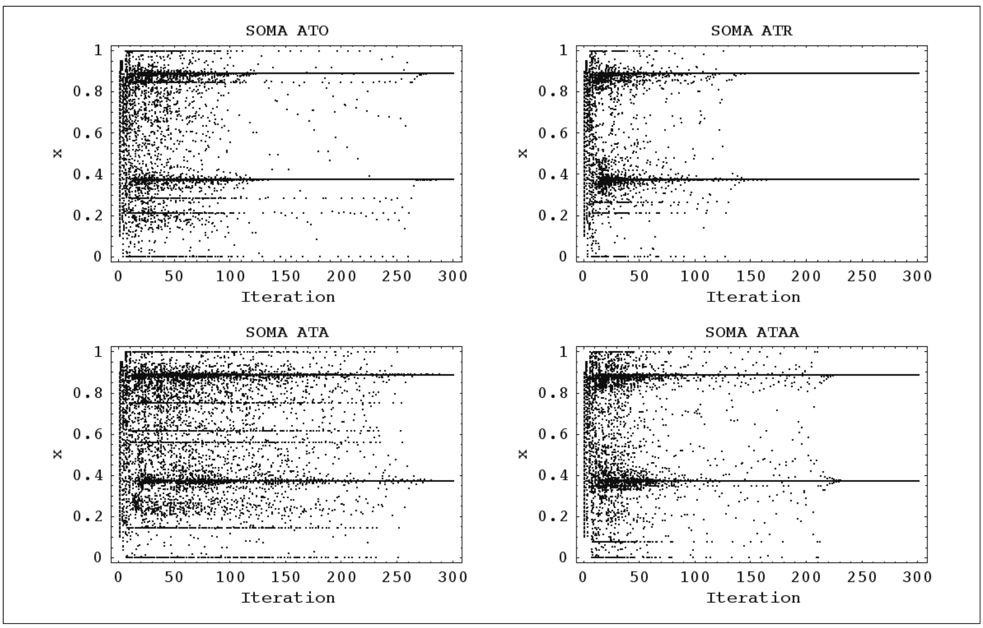
LQ DE 1p CF Basic



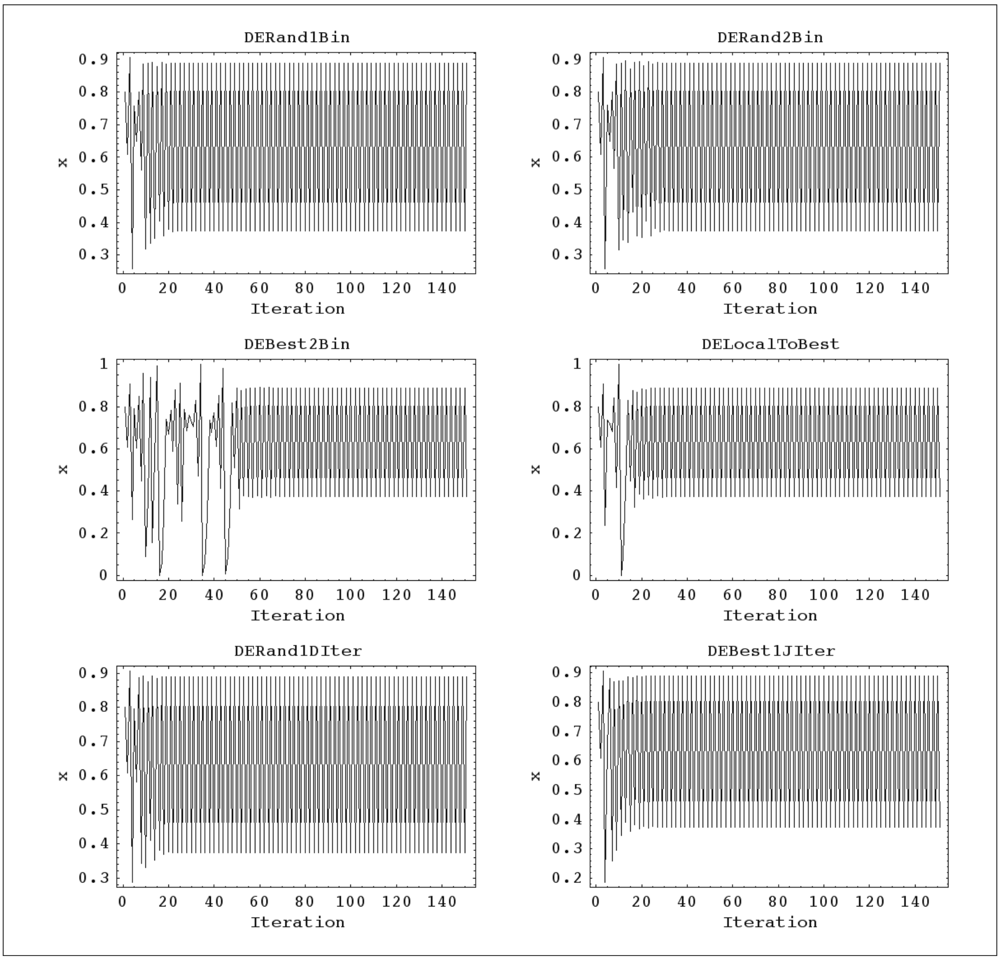
LQ SOMA 2p CF Basic



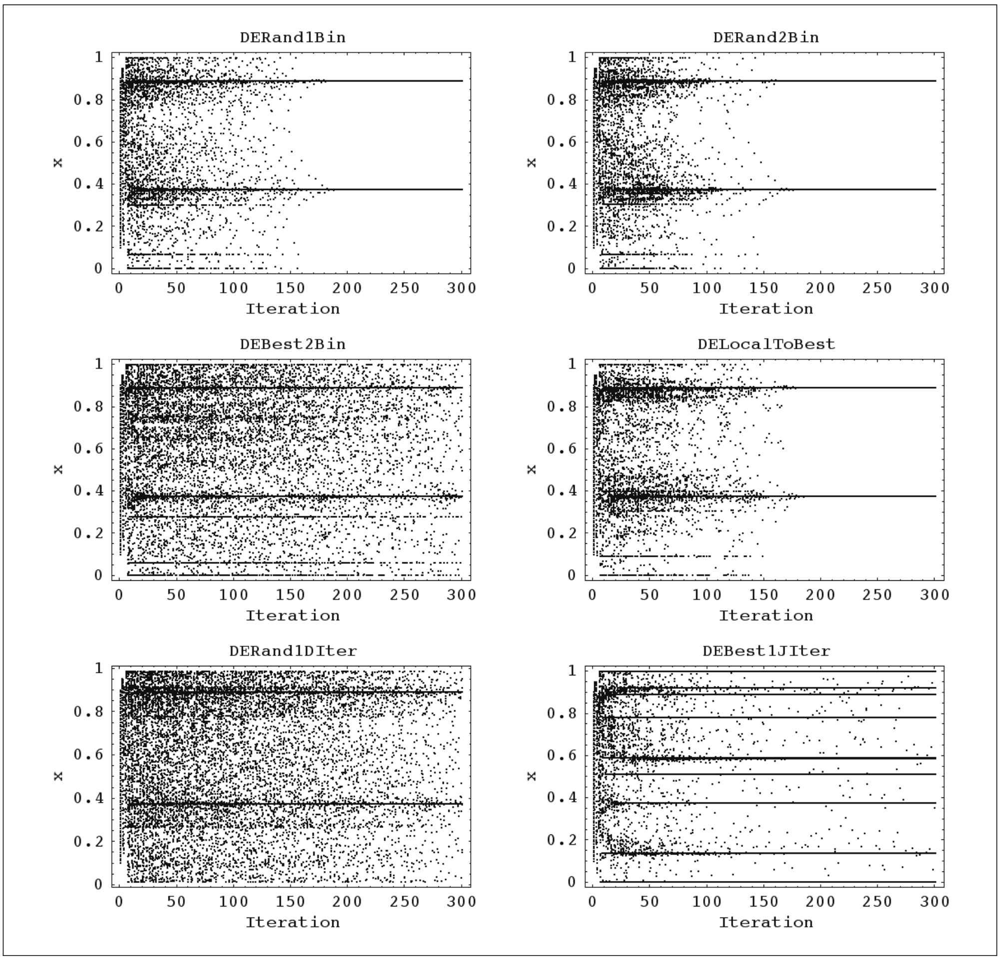
LQ SOMA 2p CF Basic



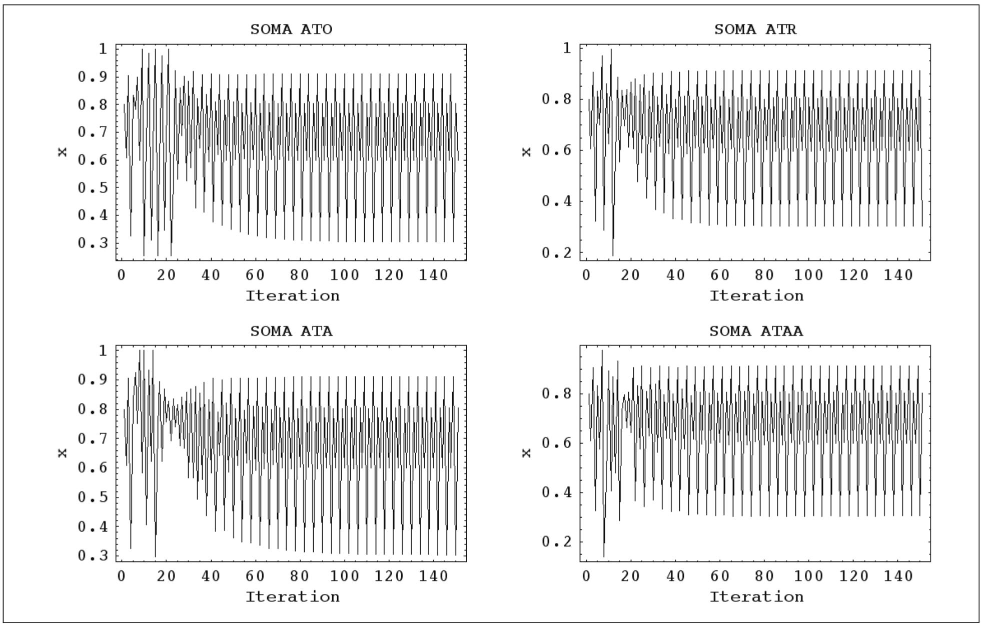
LQ DE 2p CF Basic



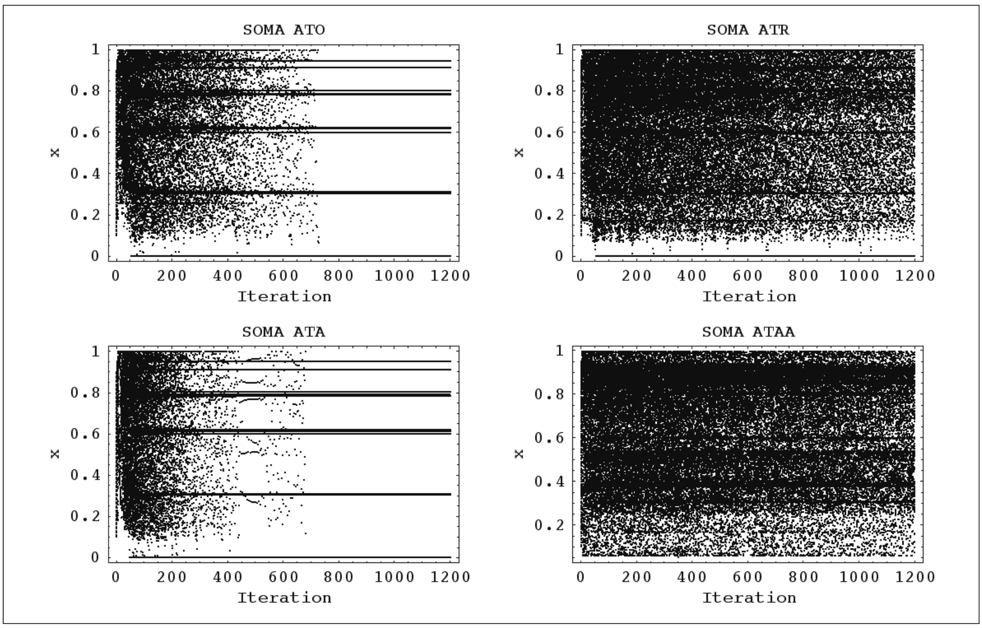
LQ DE 2p CF Basic



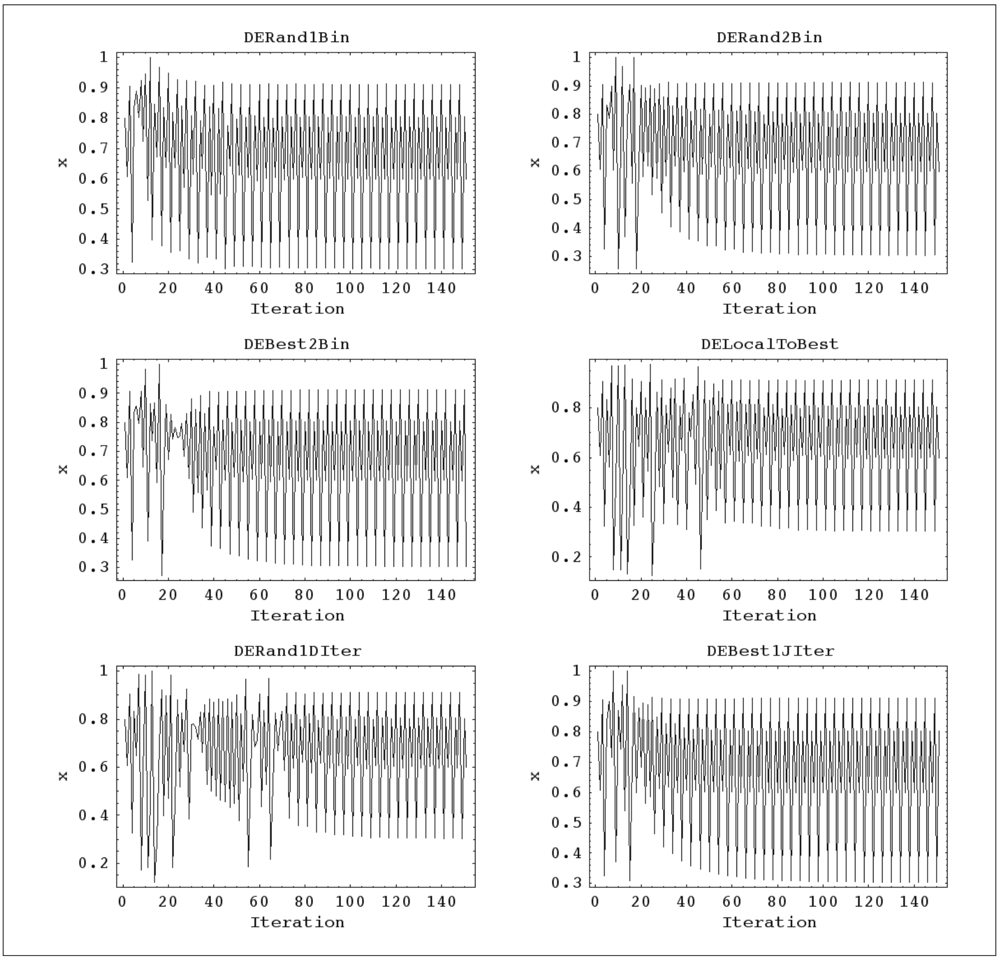
LQ SOMA 4p CF Basic

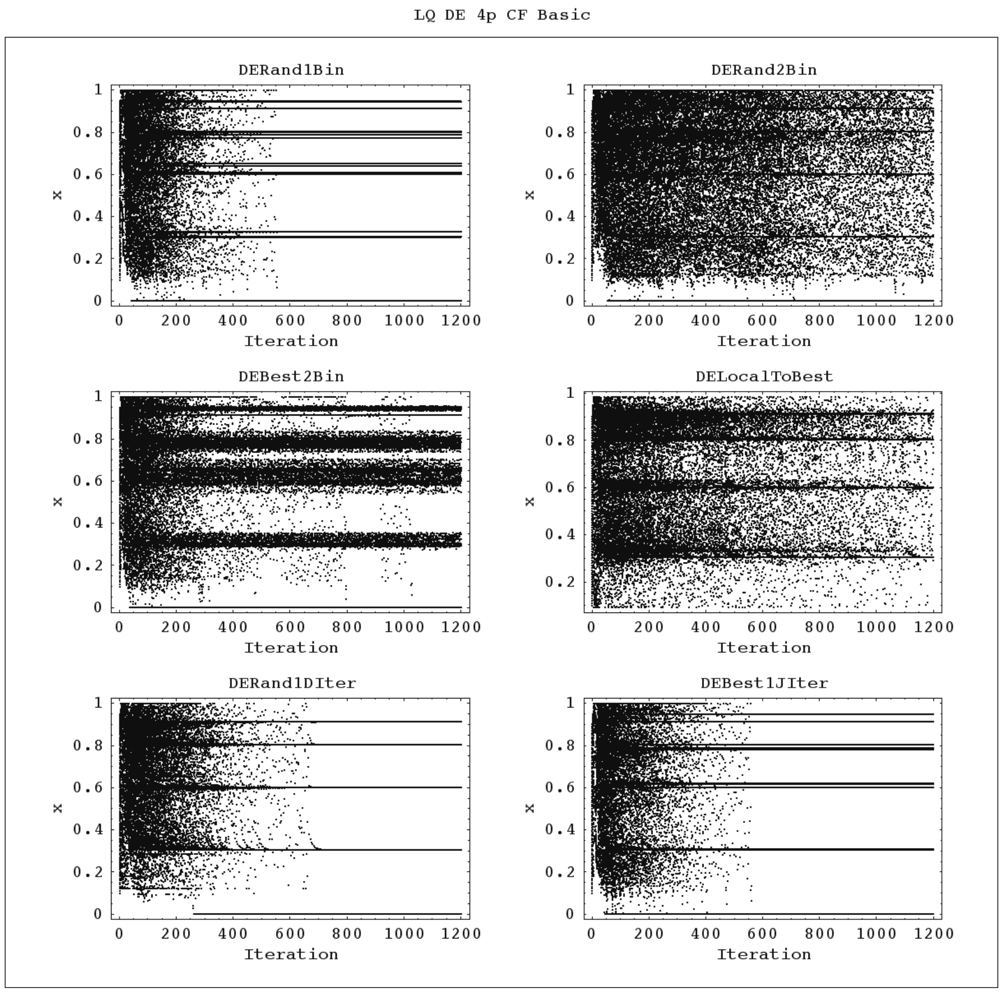


LQ SOMA 4p CF Basic

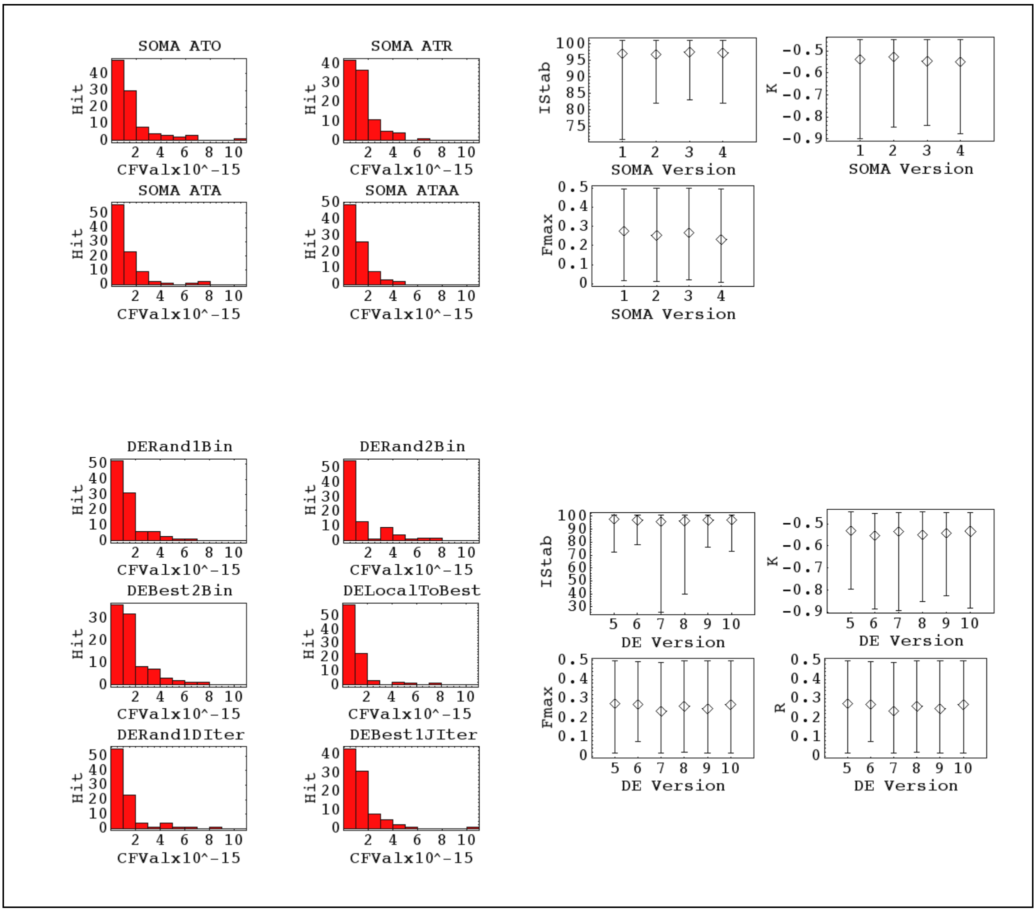


LQ DE 4p CF Basic

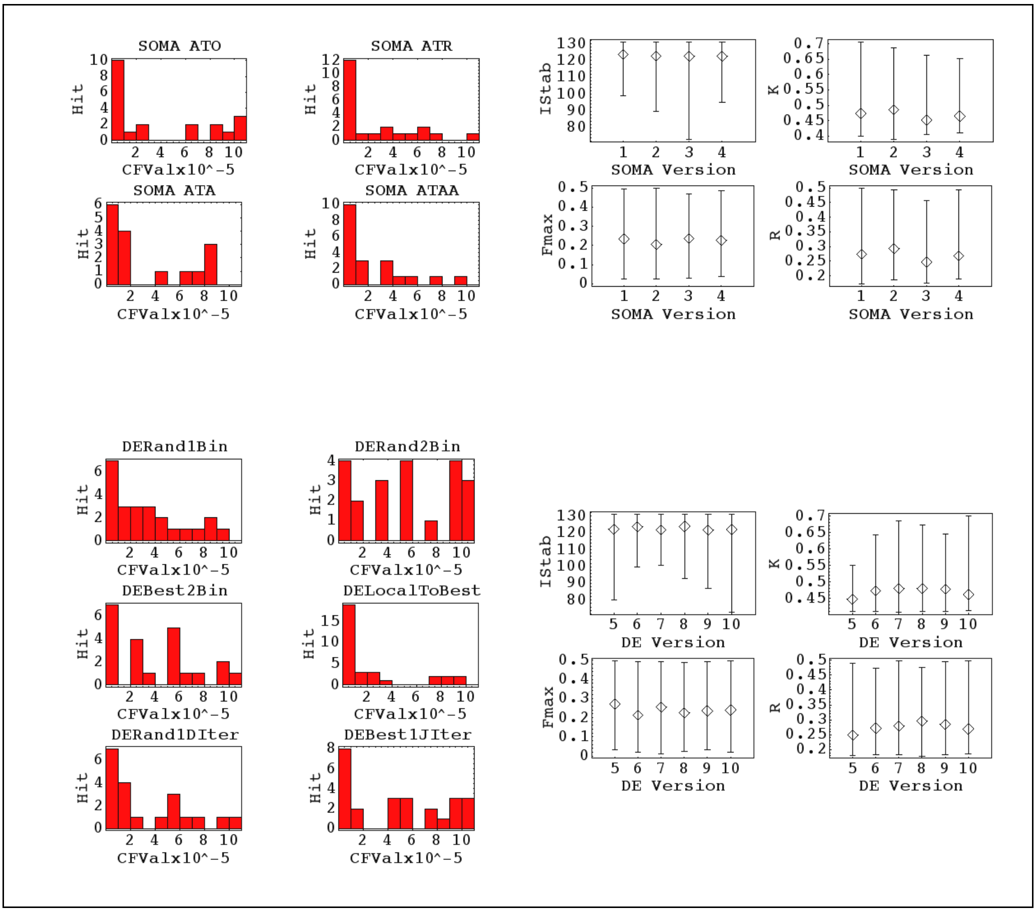



LQ DE 4p CF Basic
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration
0
0.2
0.4
0.6
0.8
1
200
400
600
800
1000
1200

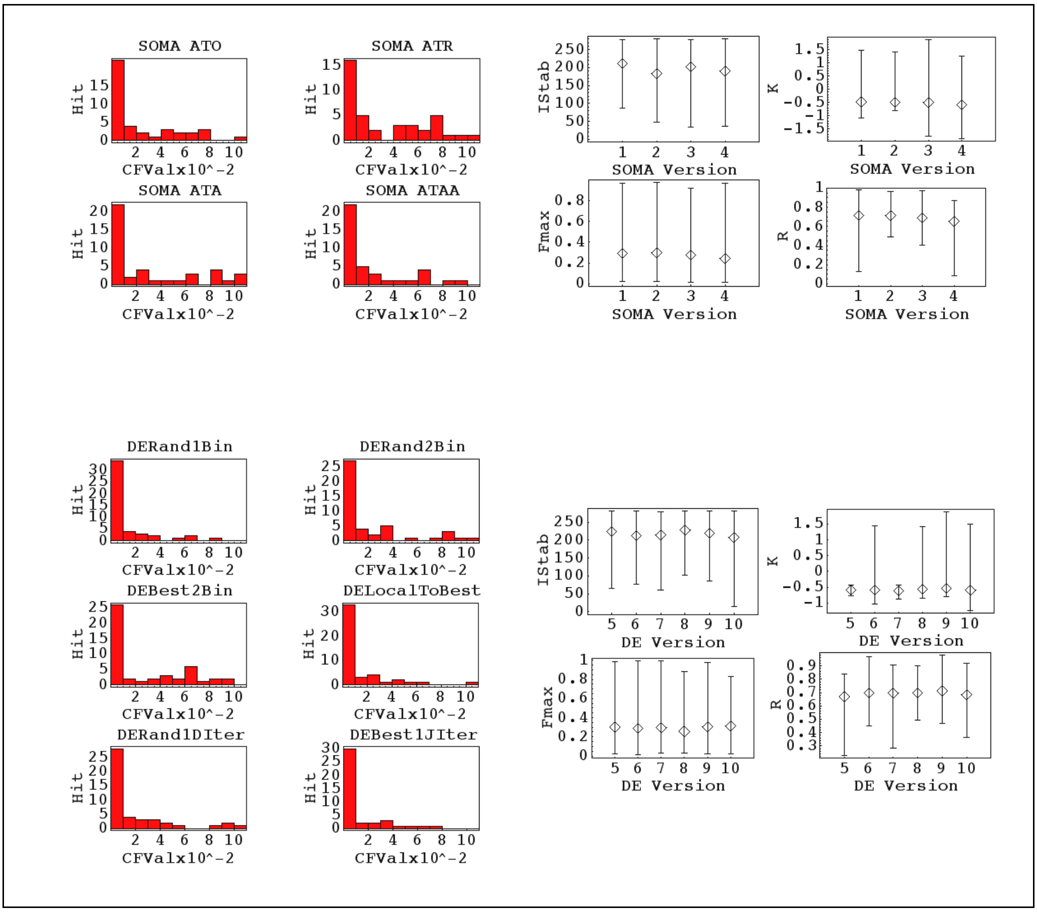
LQ SOMA & DE 1p CF Basic



LQ SOMA & DE 2p CF Basic

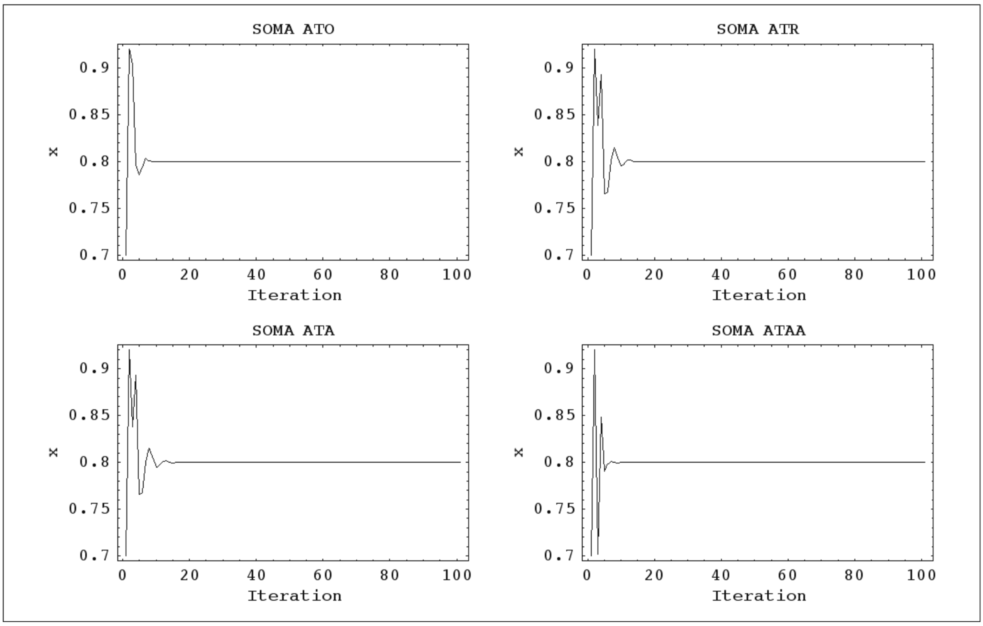


LQ SOMA & DE 4p CF Basic

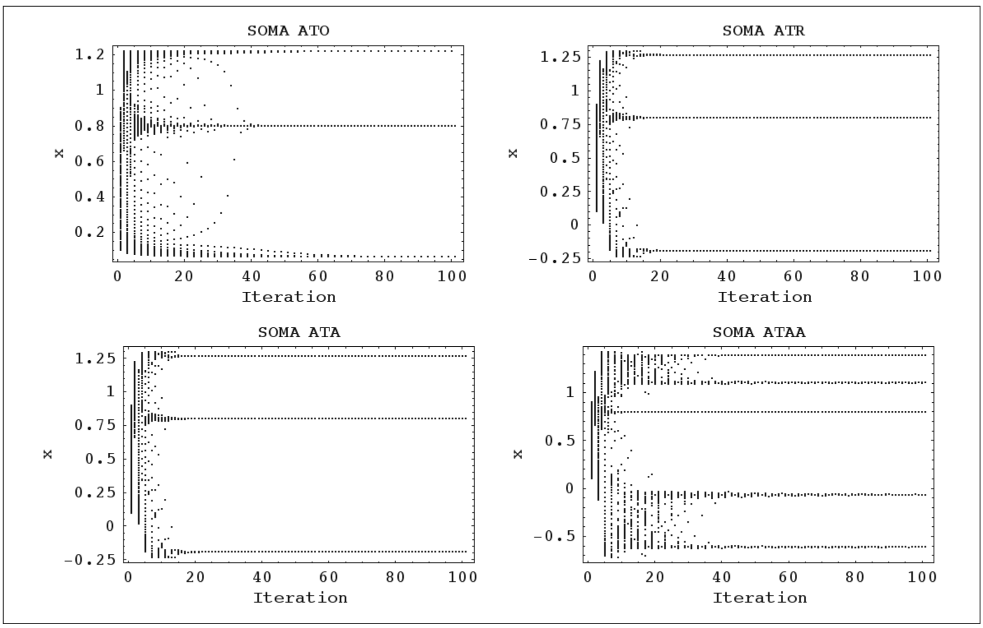


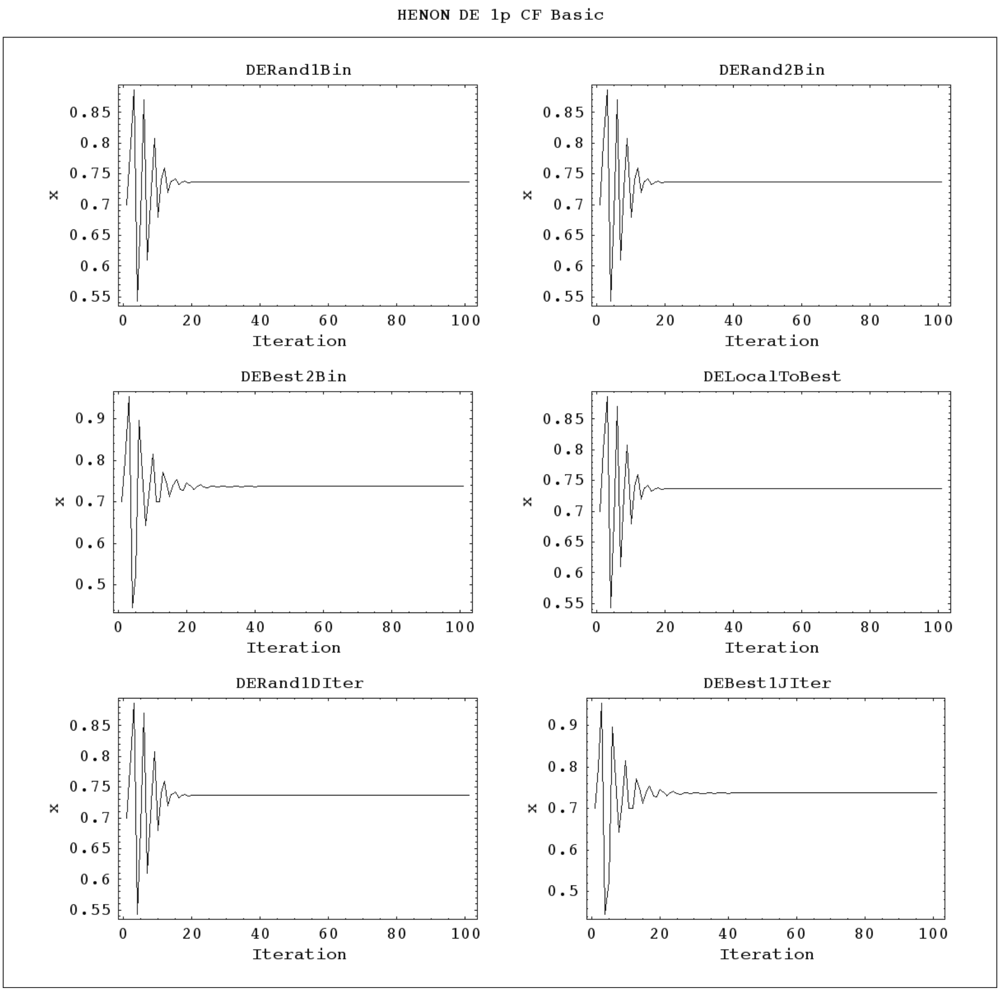
8.8 Summary of results, Case study 1, CF Basic, HENON

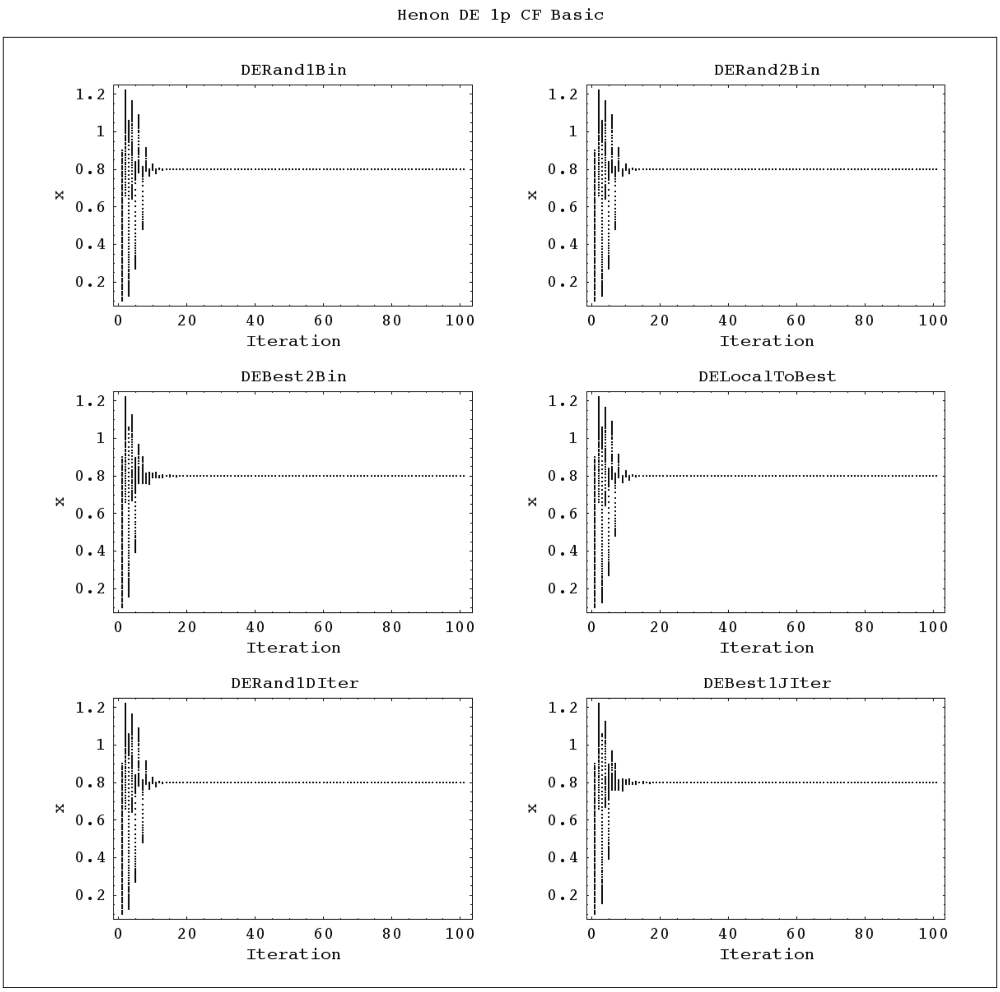
HENON SOMA 1p CF Basic



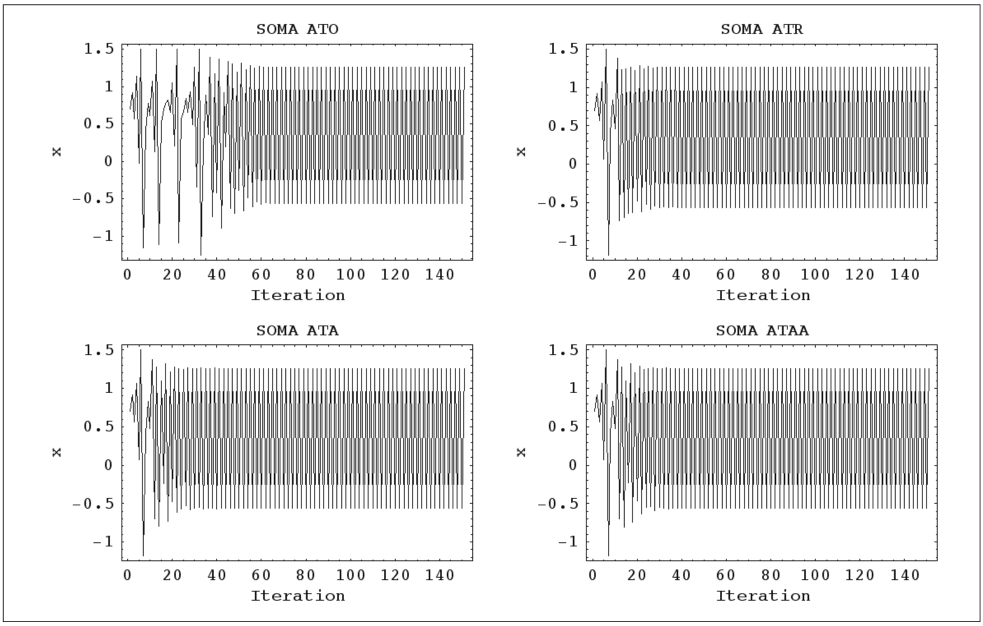
Henon SOMA 1p CF Basic



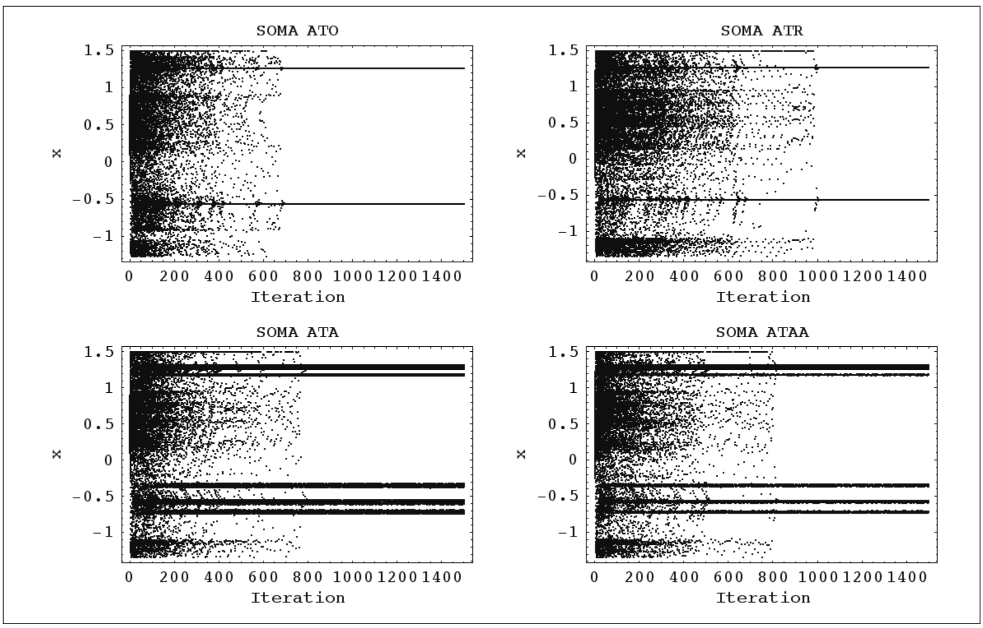

HENON DE 1p CF Basic
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.9
0.5
0
20
40
60
80
100

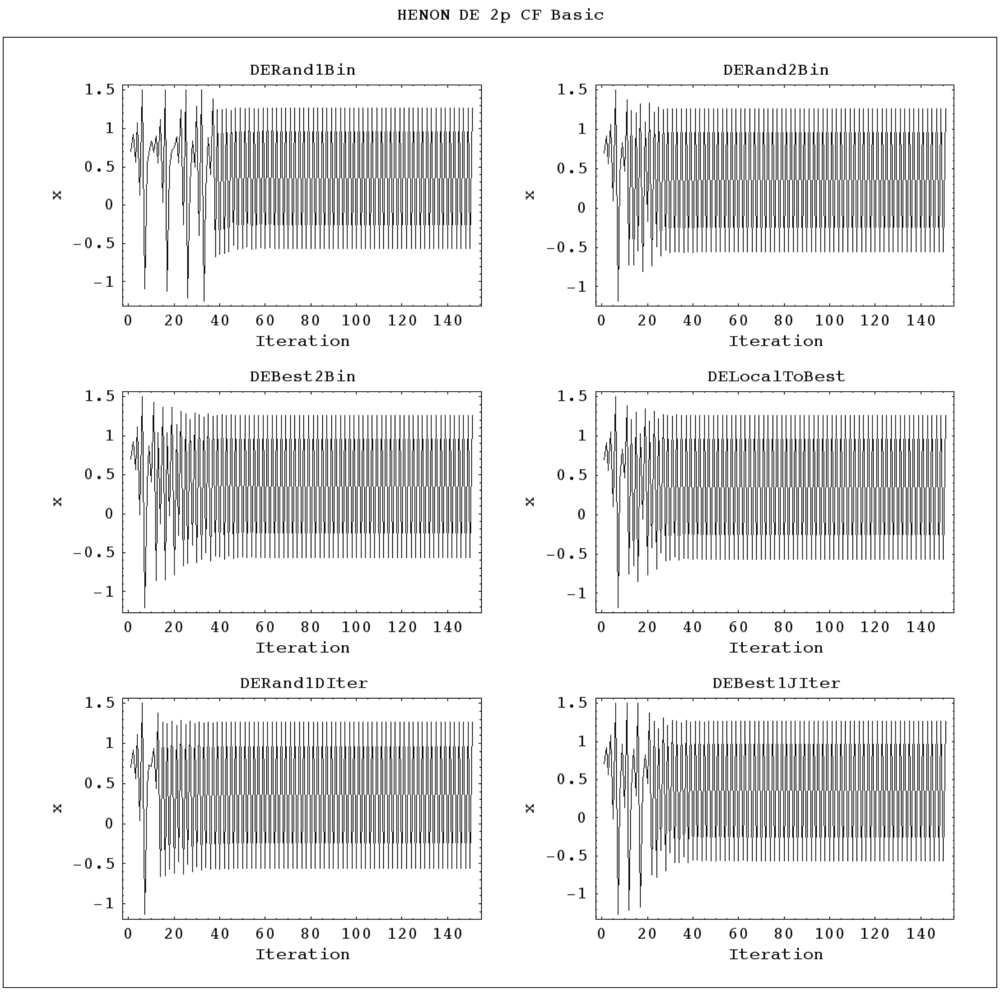

Henon DE 1p CF Basic
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration
1.2
1
0.8
0.6
0.4
0.2
0
20
40
60
80
100

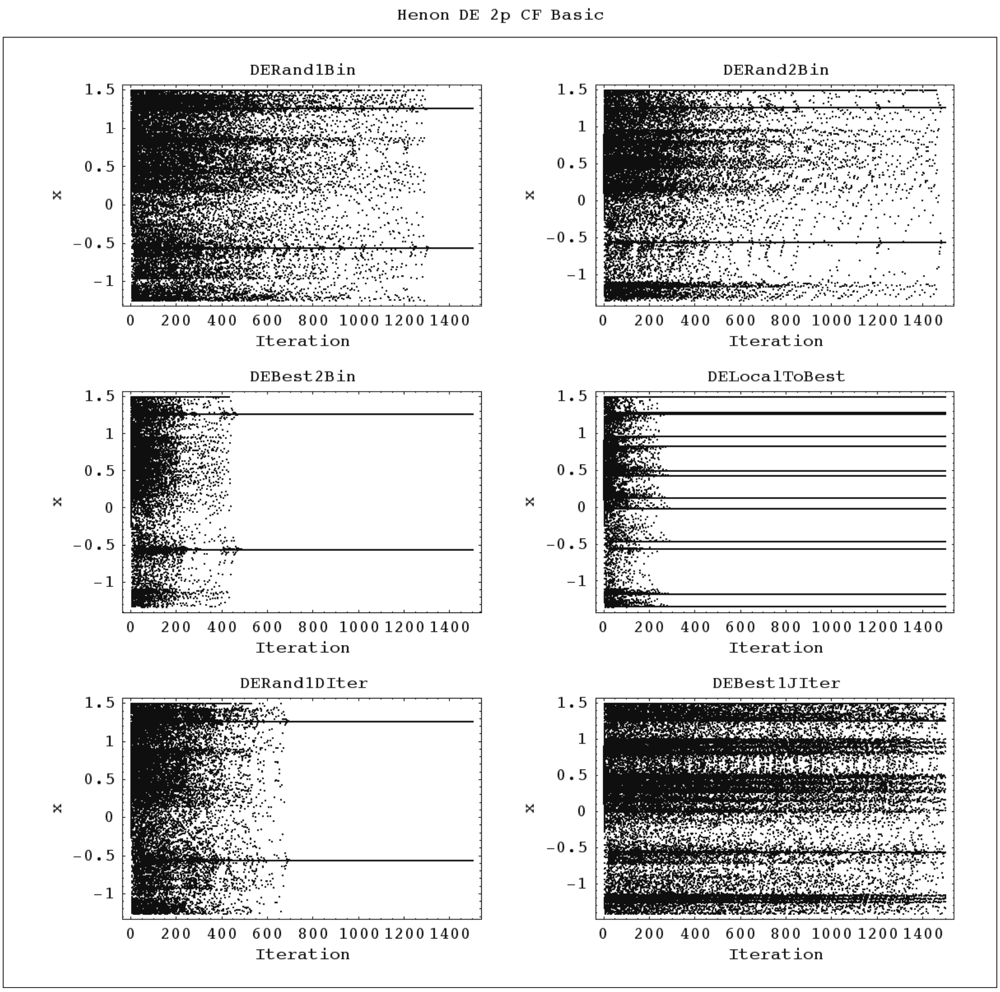
HENON SOMA 2p CF Basic



Henon SOMA 2p CF Basic




HENON DE 2p CF Basic
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration
1.5
1
0.5
0
-0.5
-1
0 20 40 60 80 100 120 140


Henon DE 2p CF Basic
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
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0.5
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-0.5
-1
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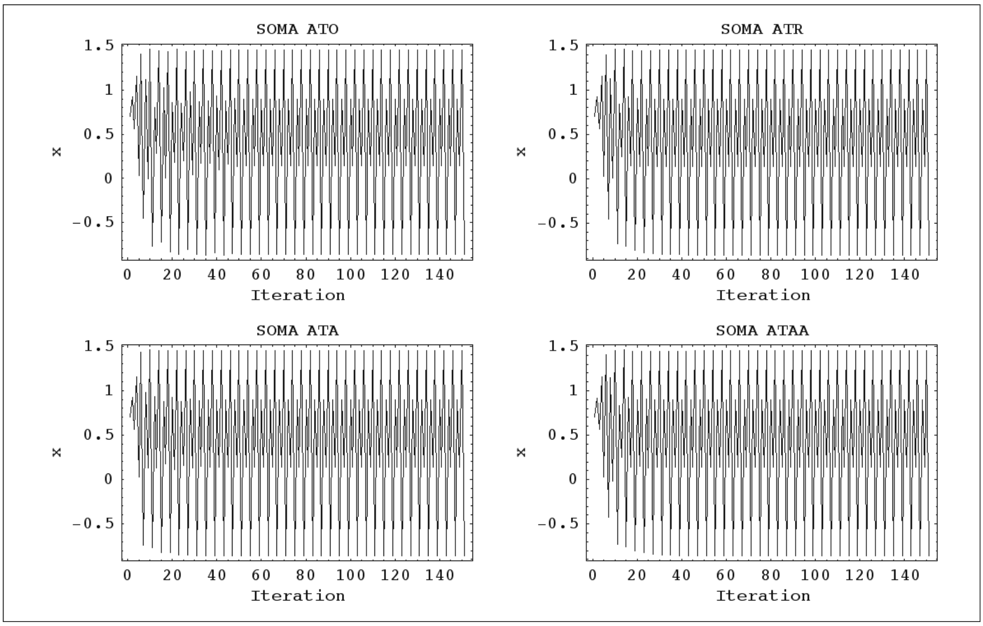
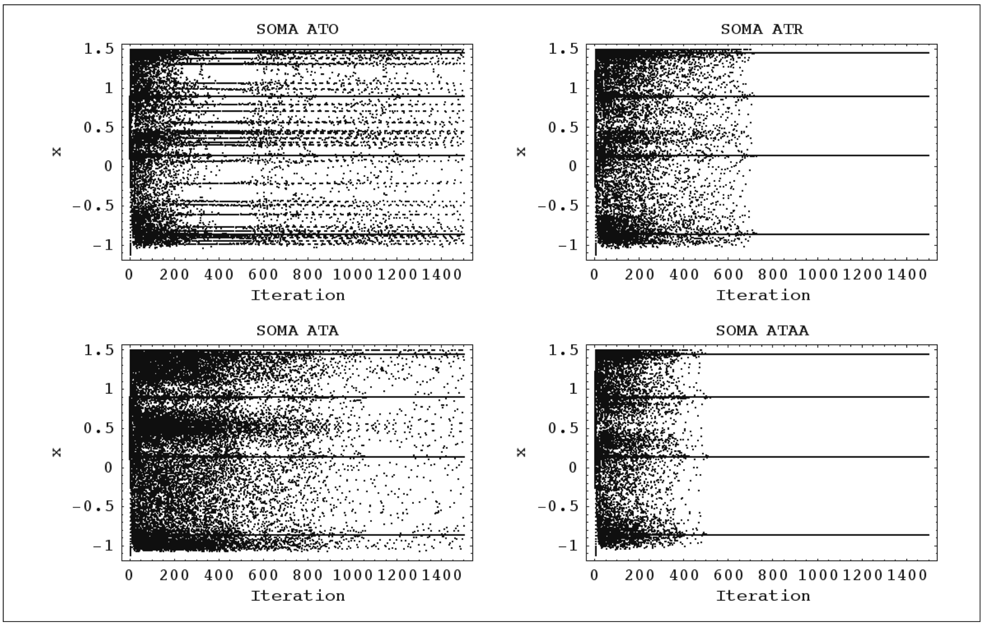
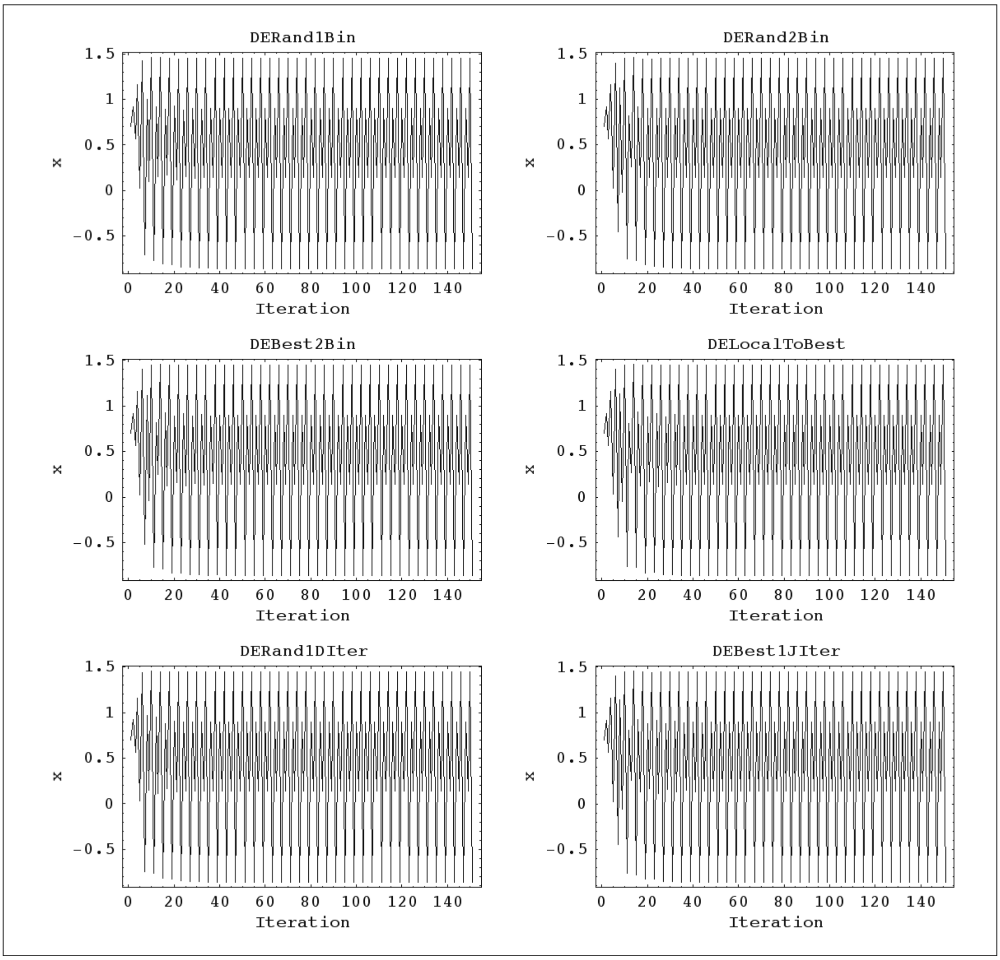
HENON SOMA 4p CF Basic



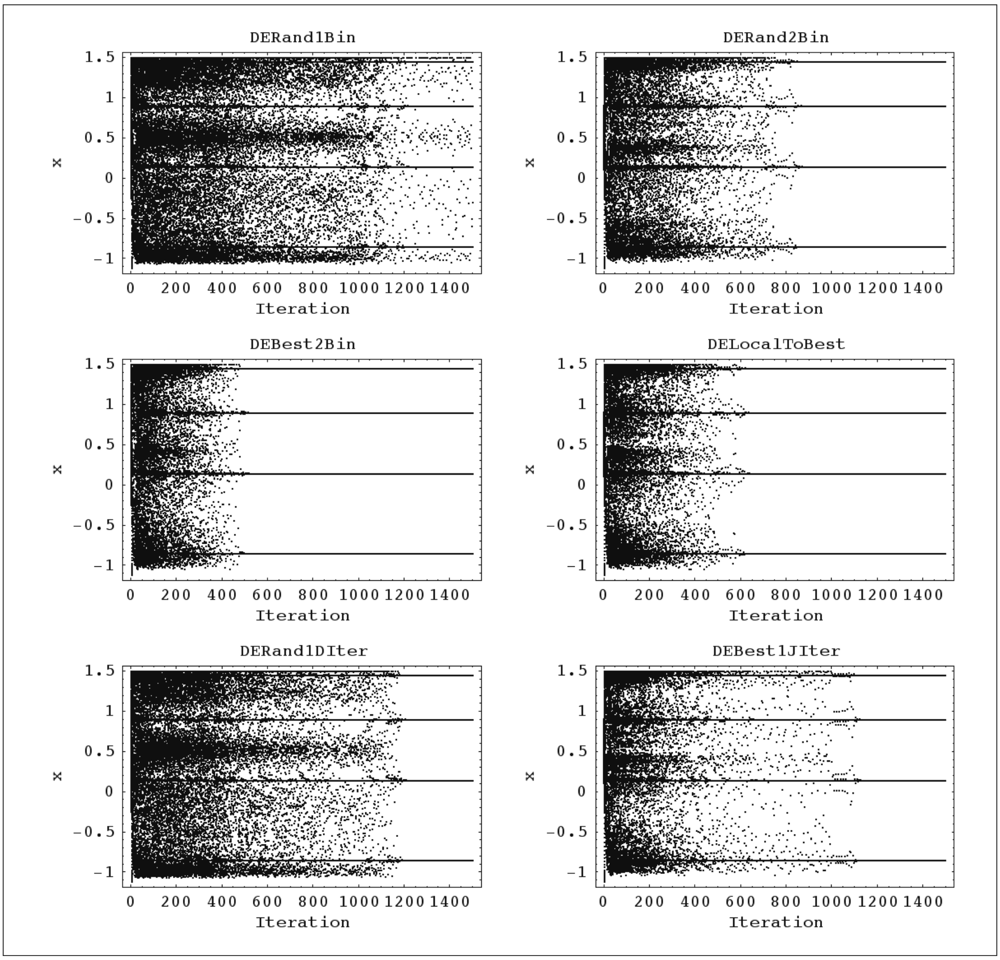
Henon SOMA 4p CF Basic



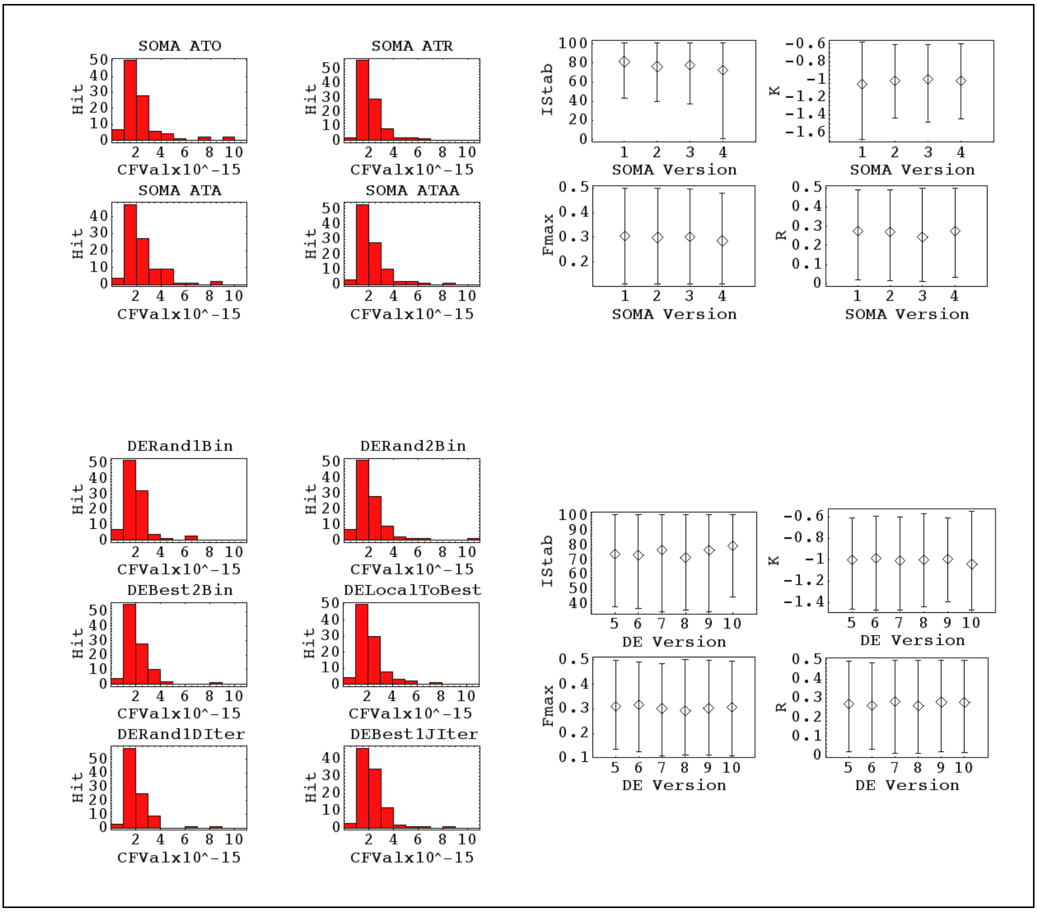
HENON DE 4p CF Basic



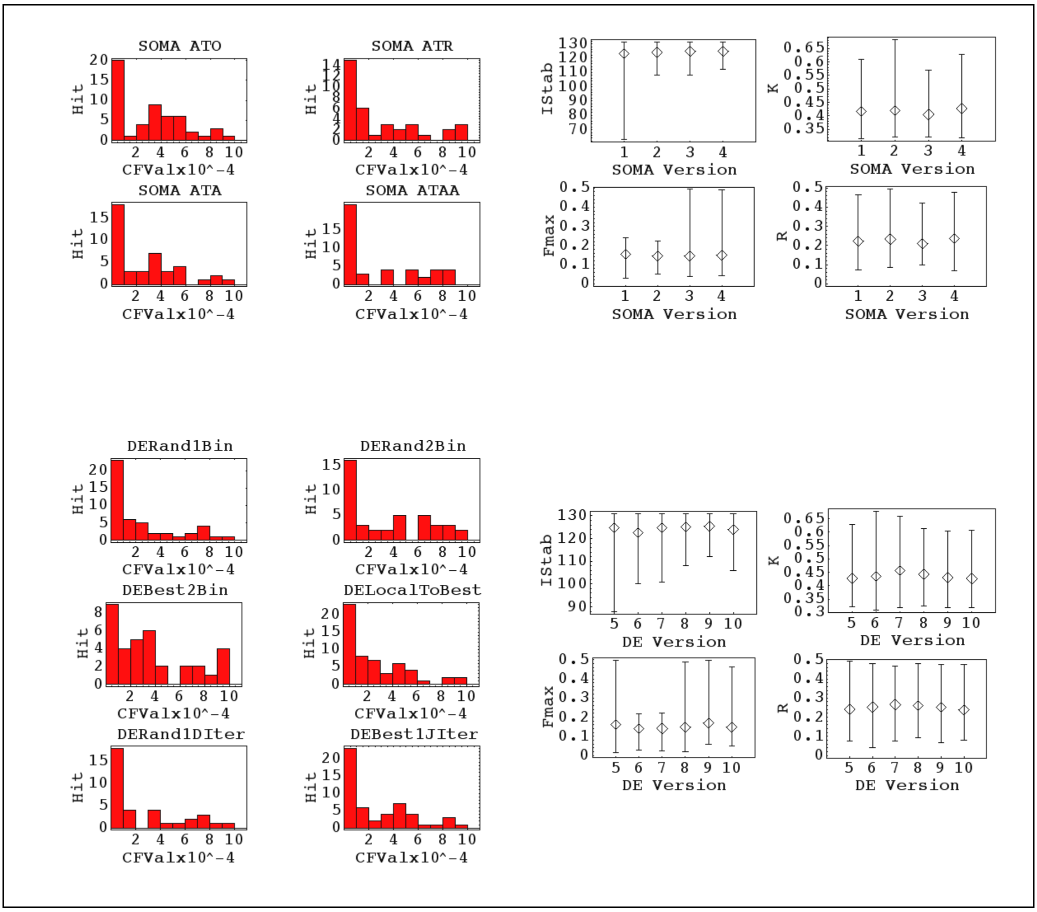
Henon DE 4p CF Basic



HENON SOMA & DE 1p CF Basic



HENON SOMA & DE 2p CF Basic



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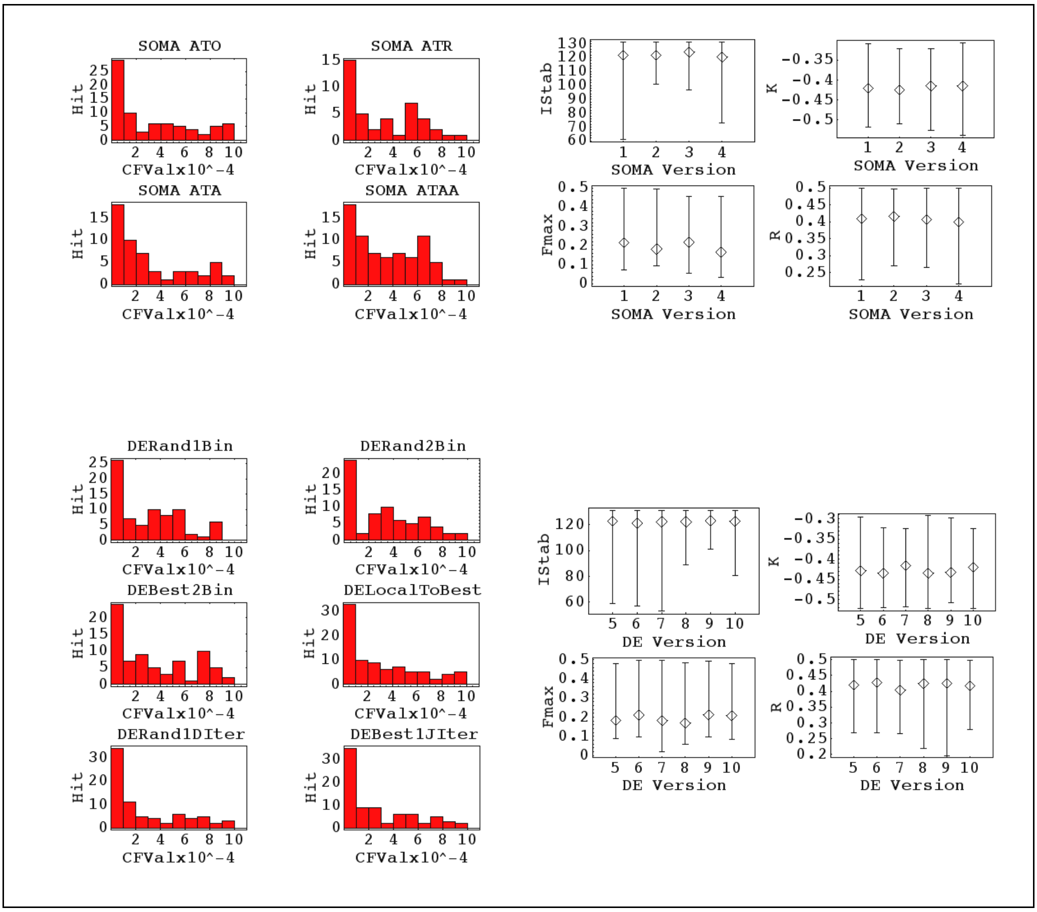
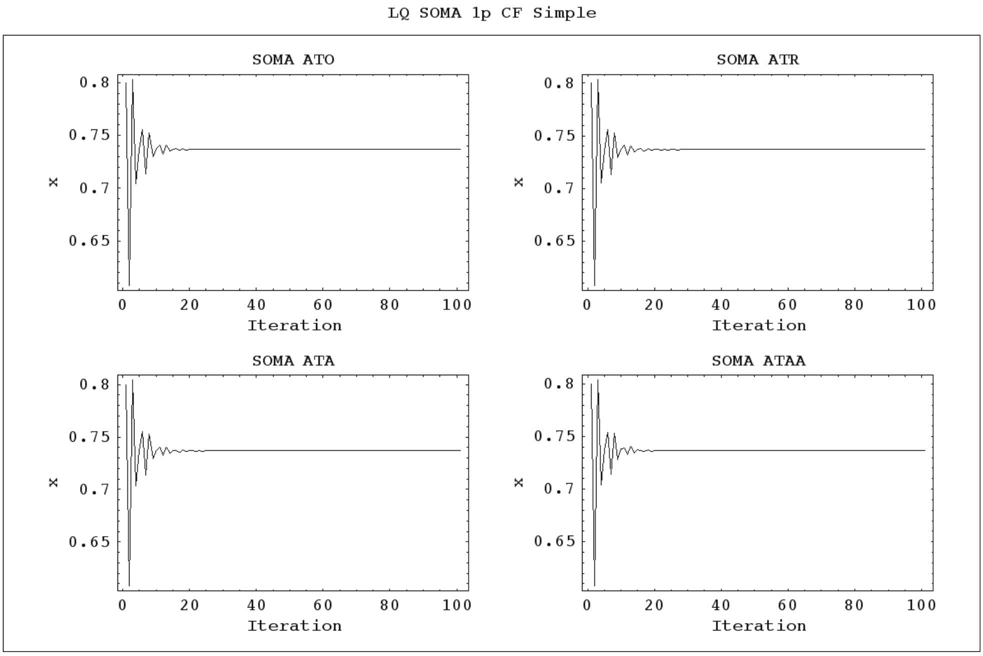
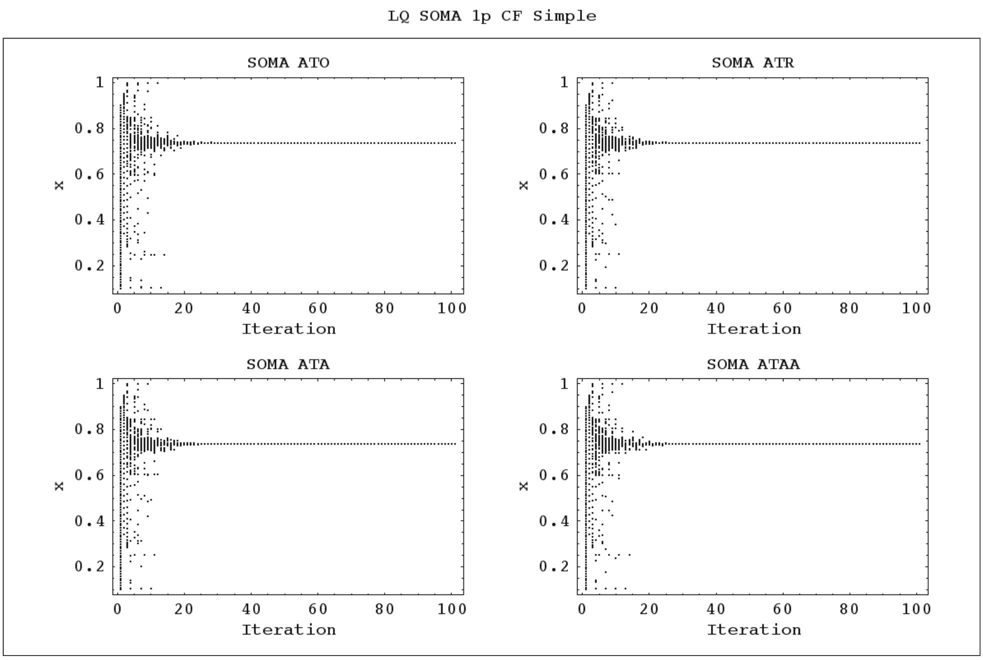
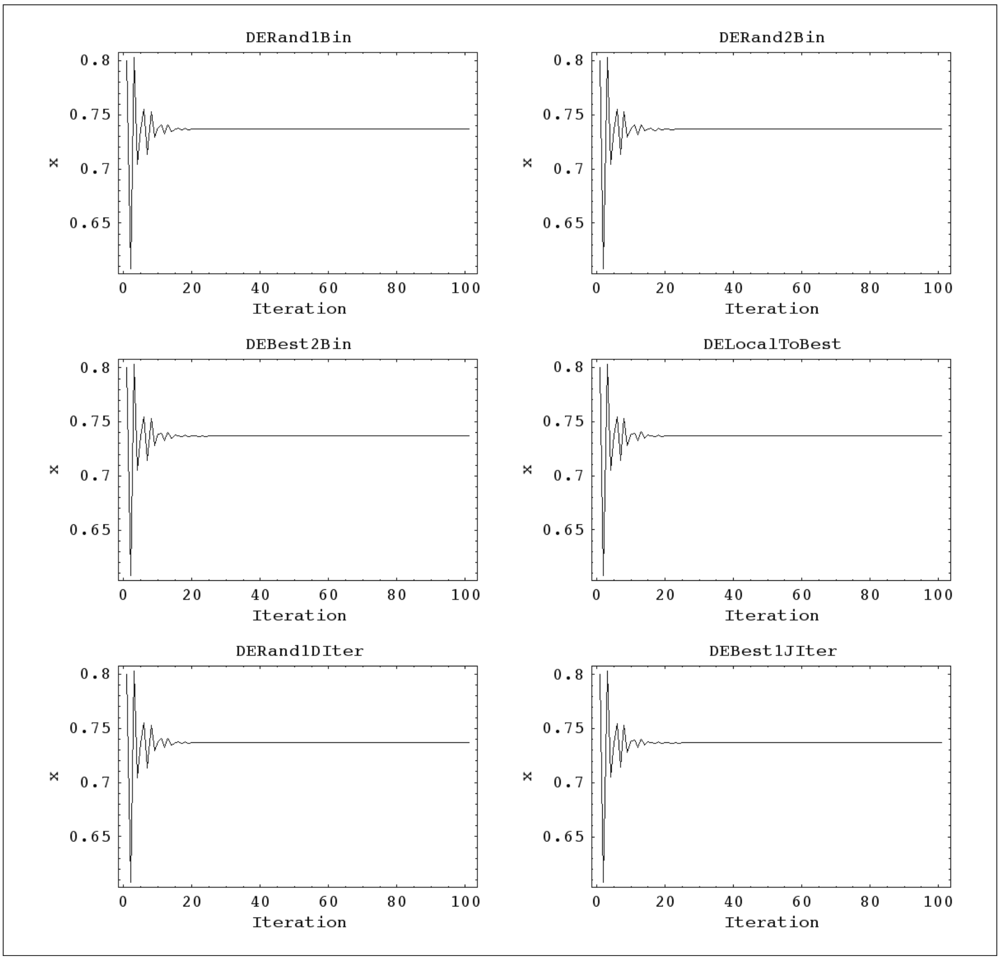


8.9 Summary of results, Case study 2, CF Simple, LQ

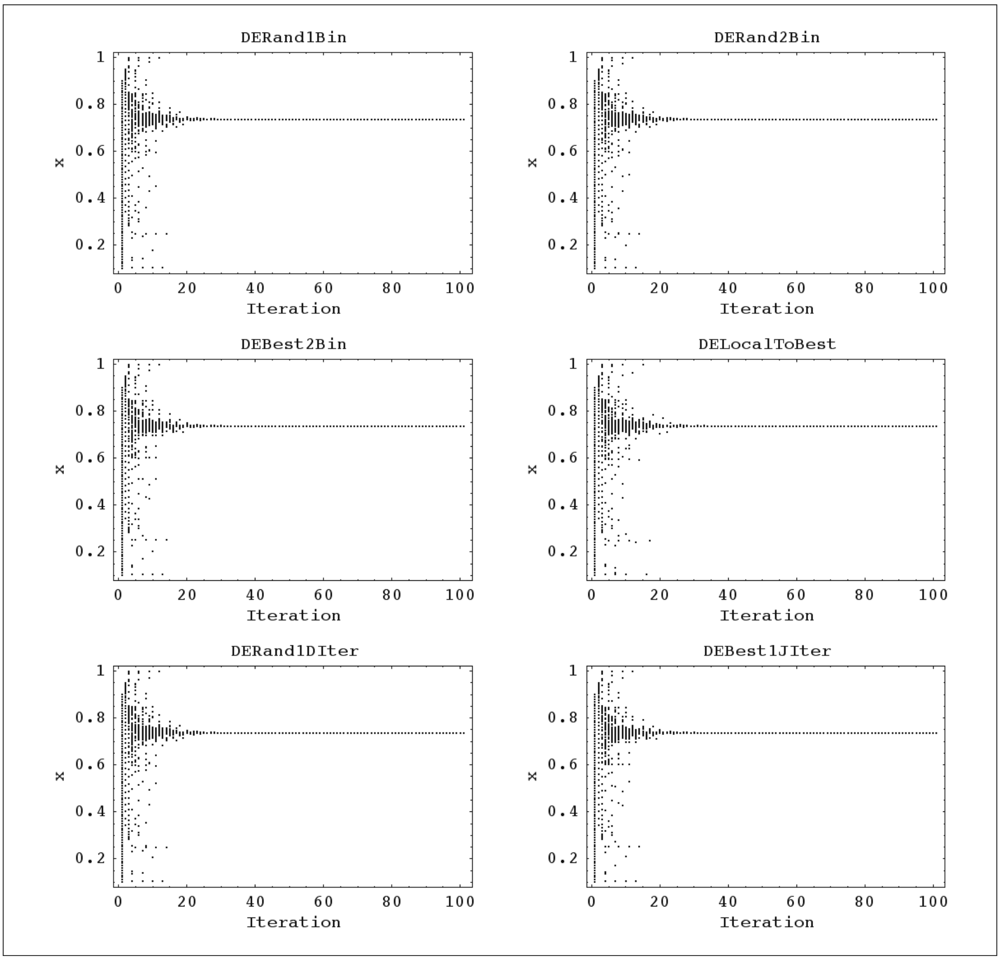




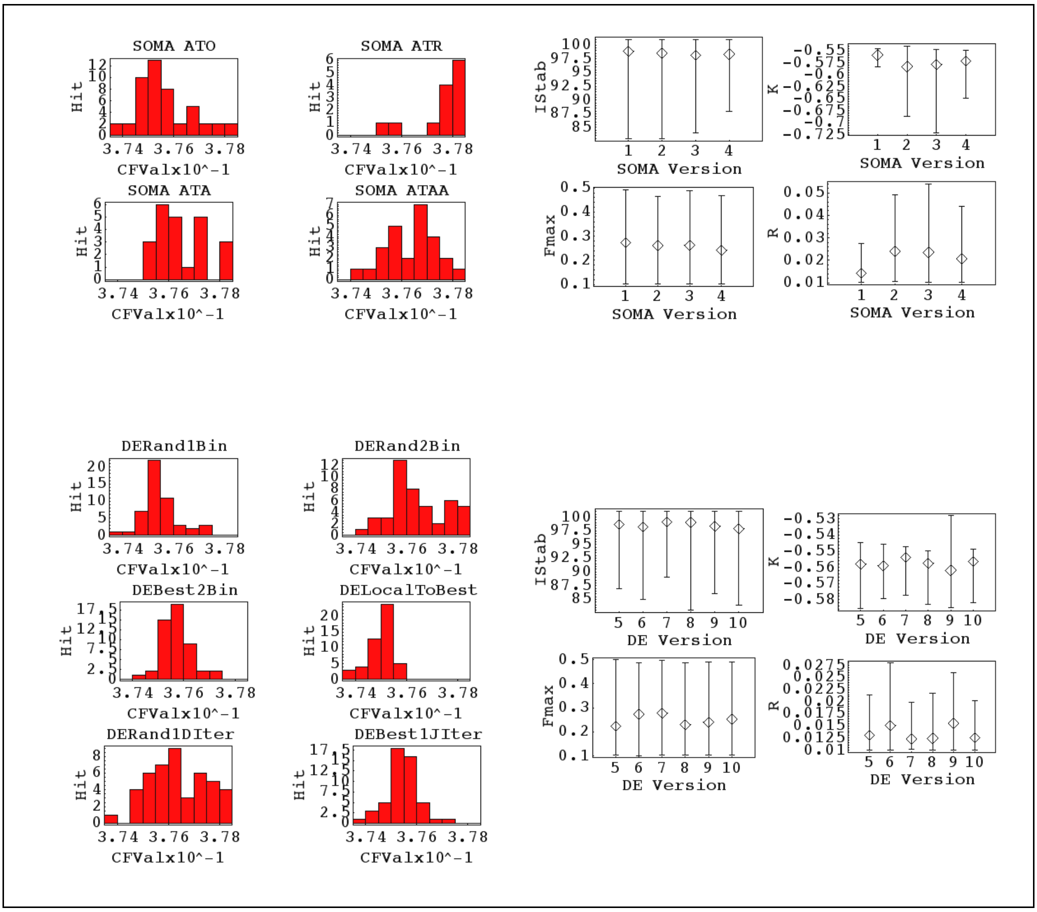
LQ DE 1p CF Simple



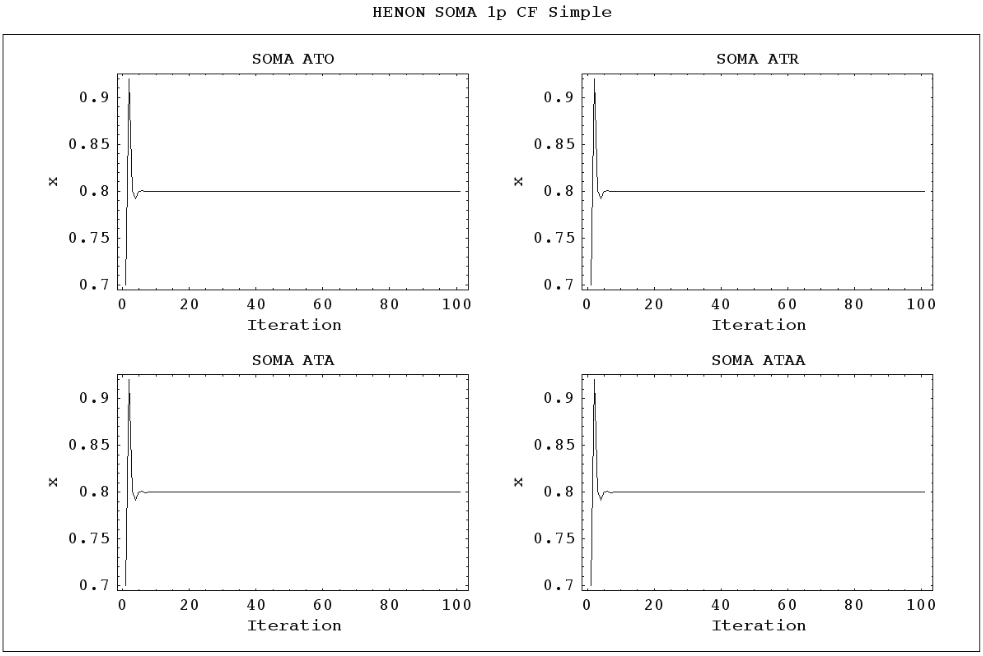
LQ DE 1p CF Simple

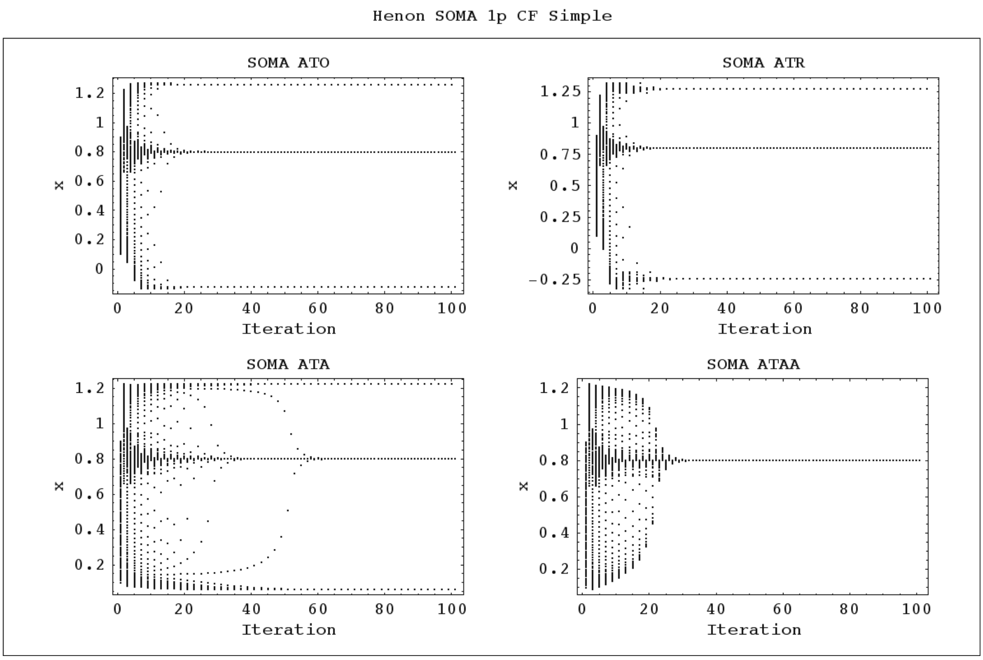


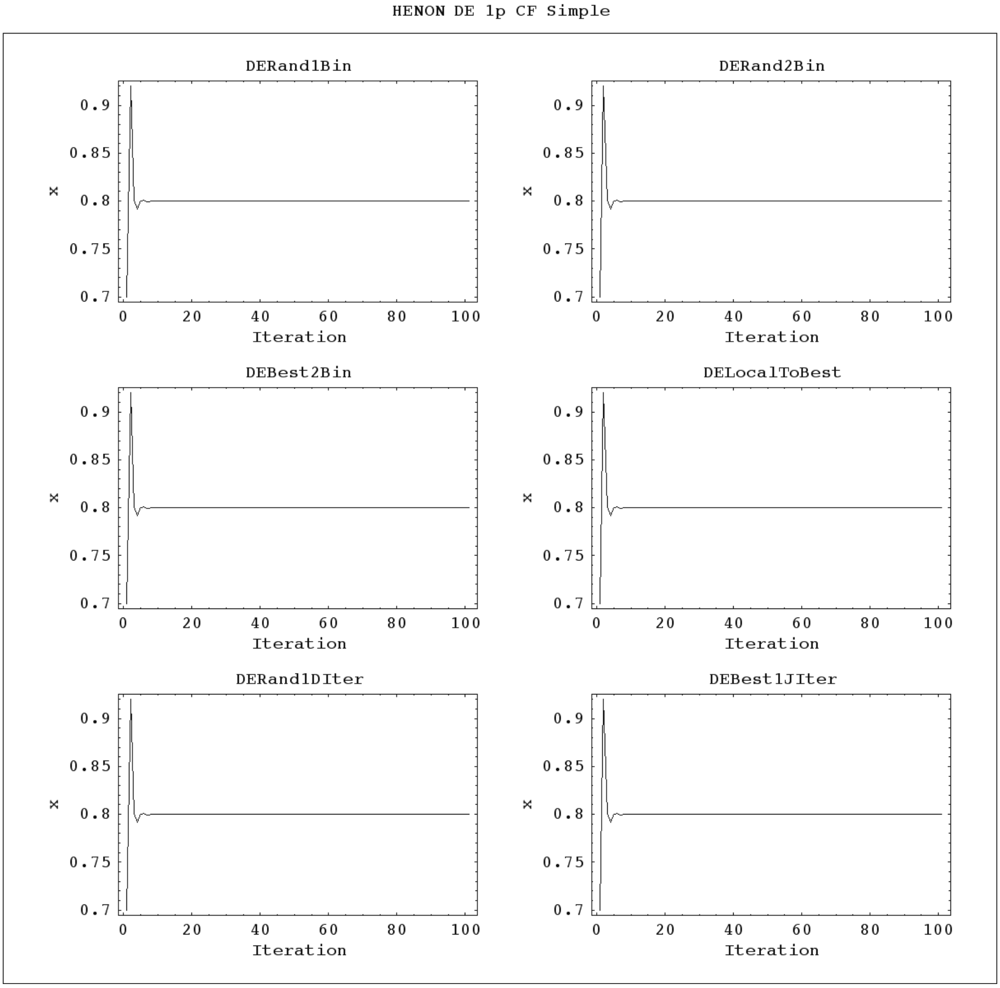
LQ SOMA & DE 1p CF Simple



8.10 Summary of results, Case study 2, CF Simple, HENON






HENON DE 1p CF Simple
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration
0.7
0.75
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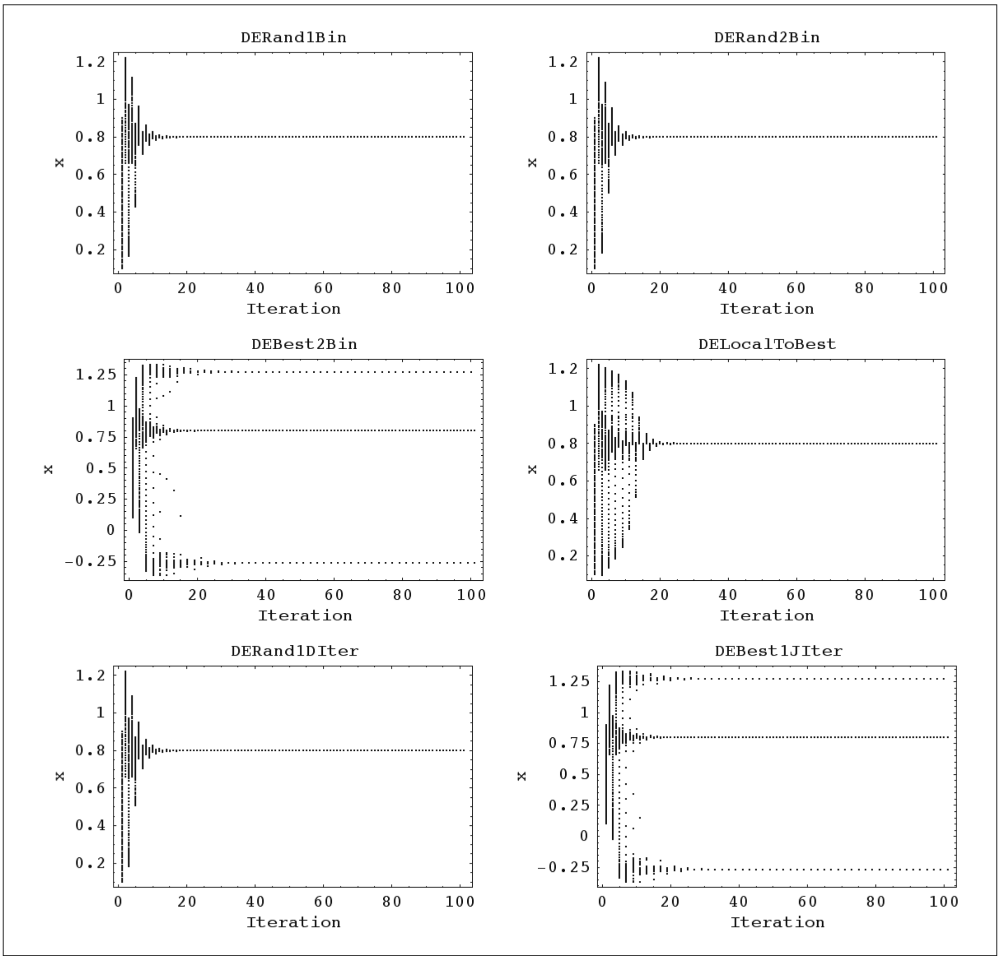
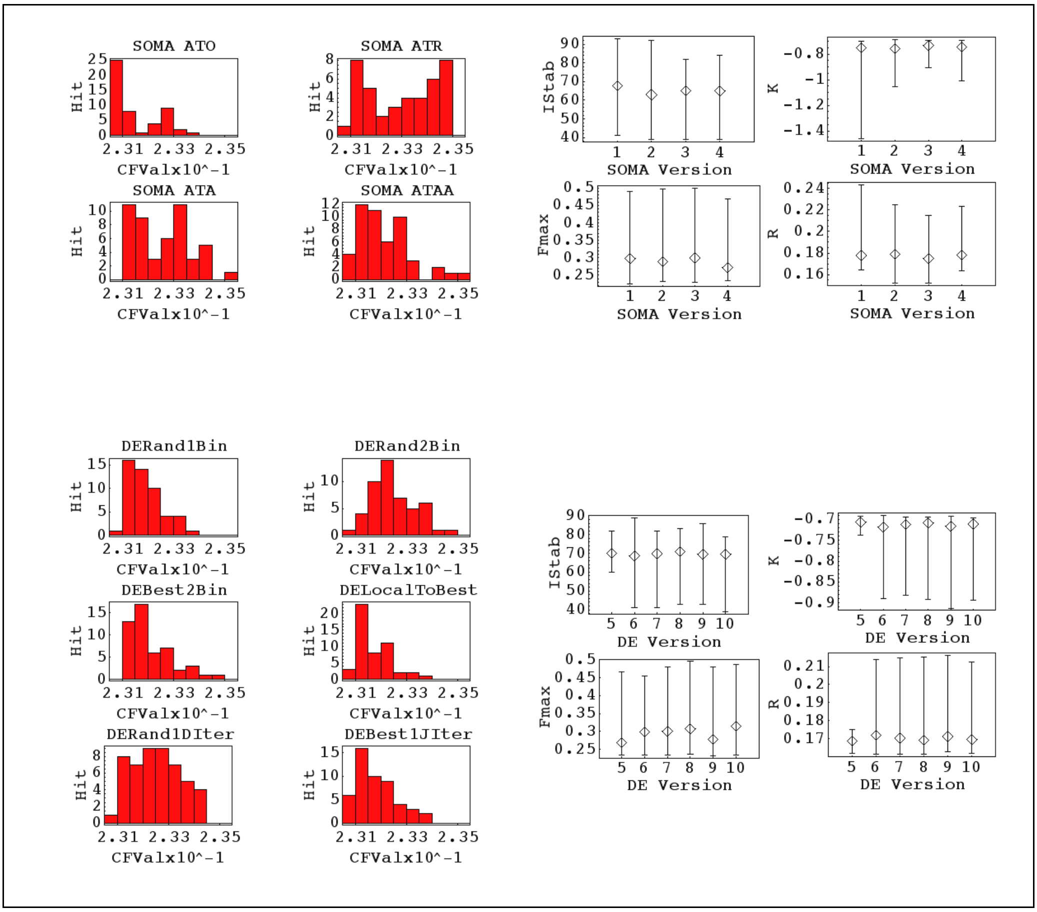
0.85
0.9
0
20
40
60
80
100

Henon DE 1p CF Simple

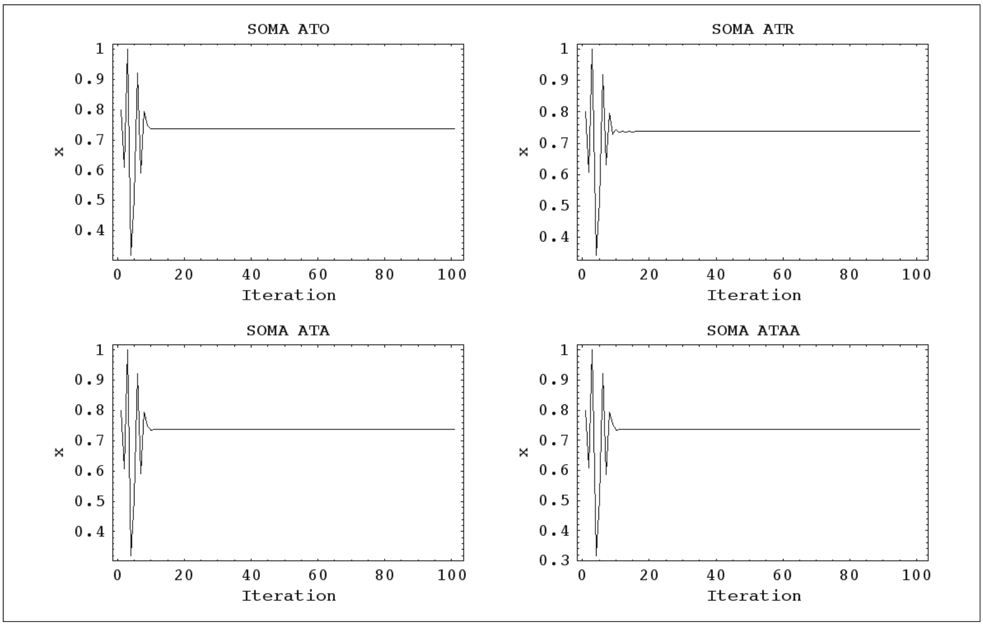


HENON SOMA & DE 1p CF Simple

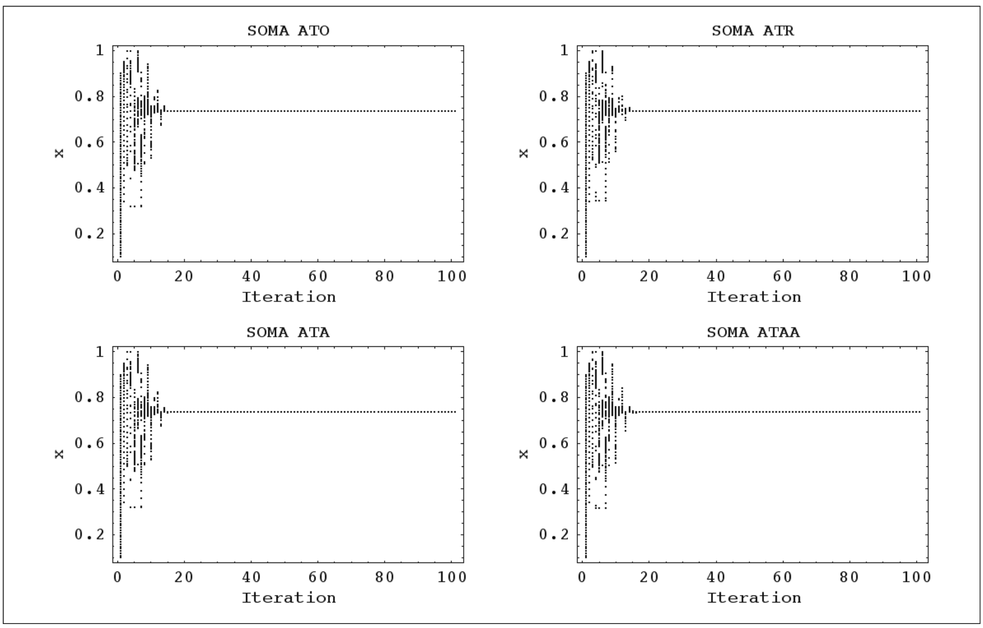


8.11 Summary of results, Case study 3, CF NA, LQ

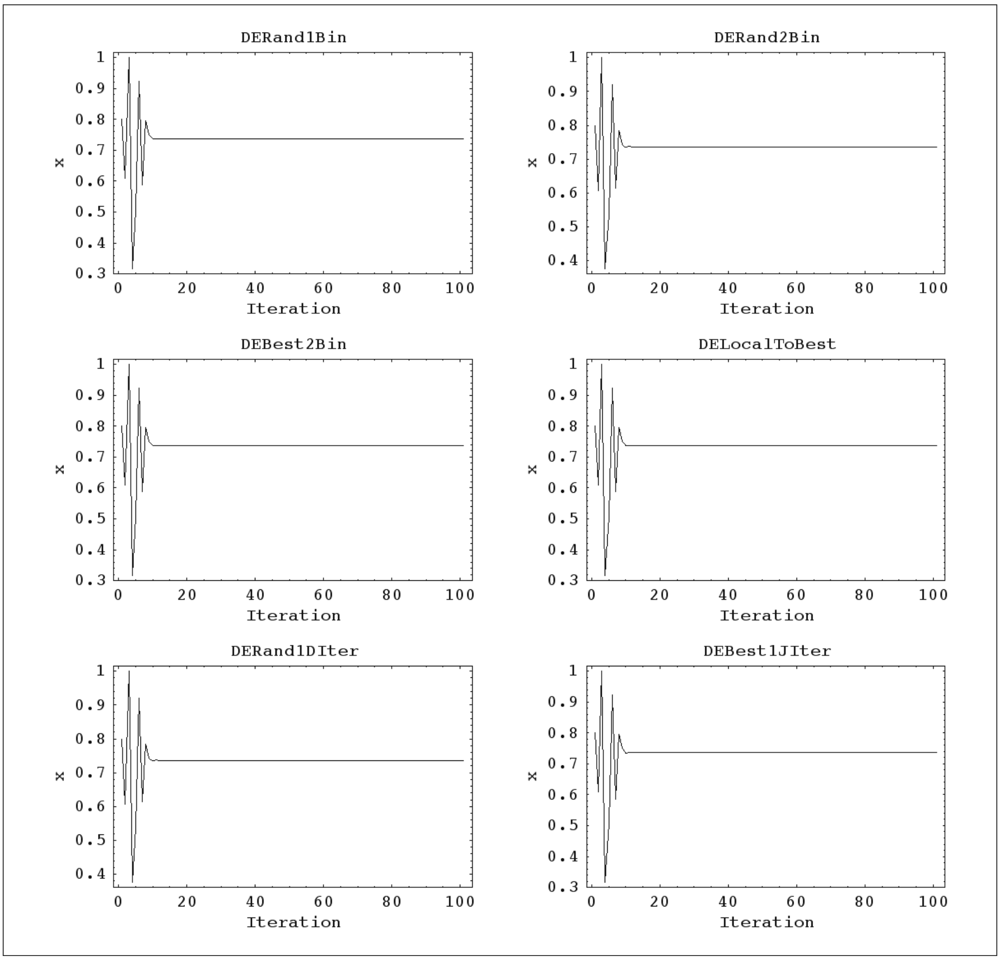
LQ SOMA 1p CF Non-Auto



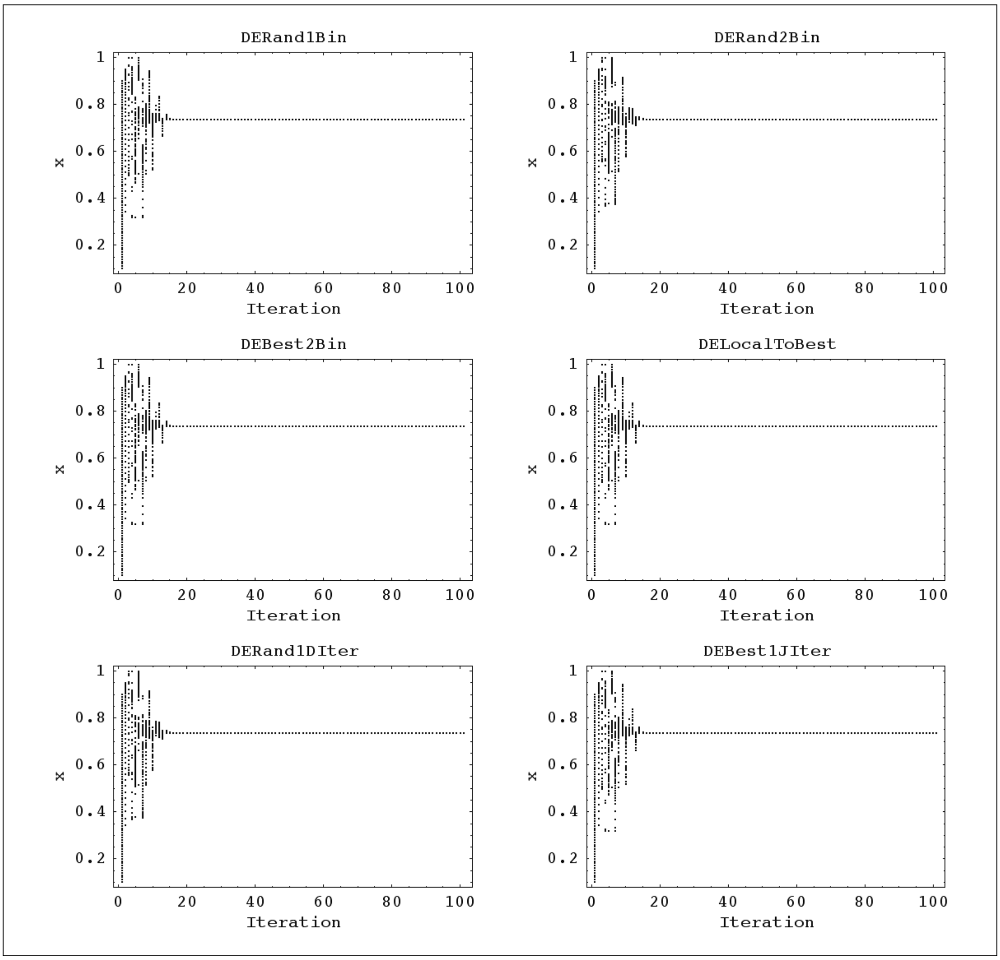
LQ SOMA 1p CF Non-Auto



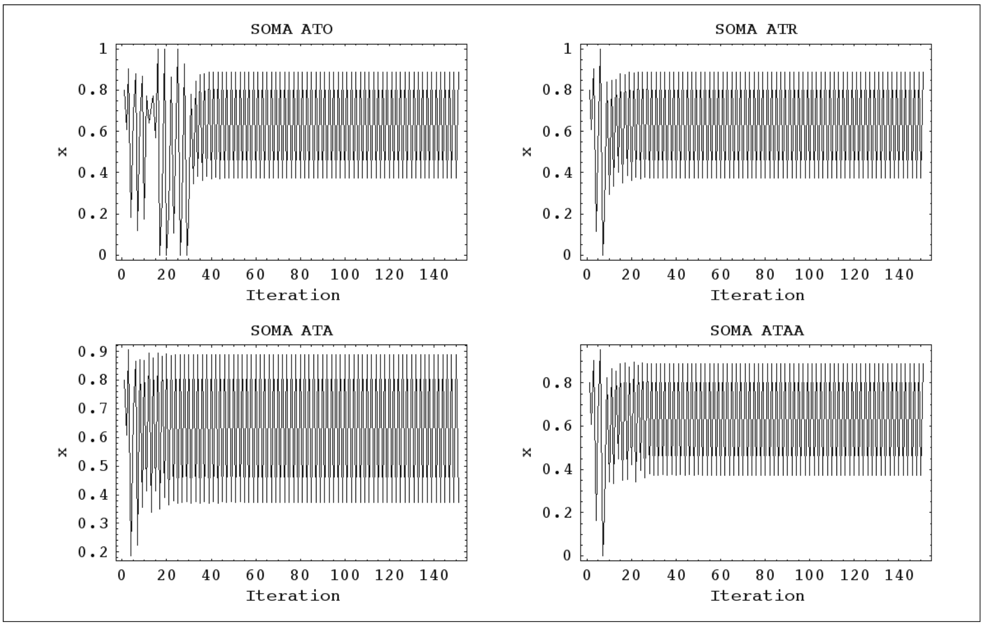
LQ DE 1p CF Non-Auto



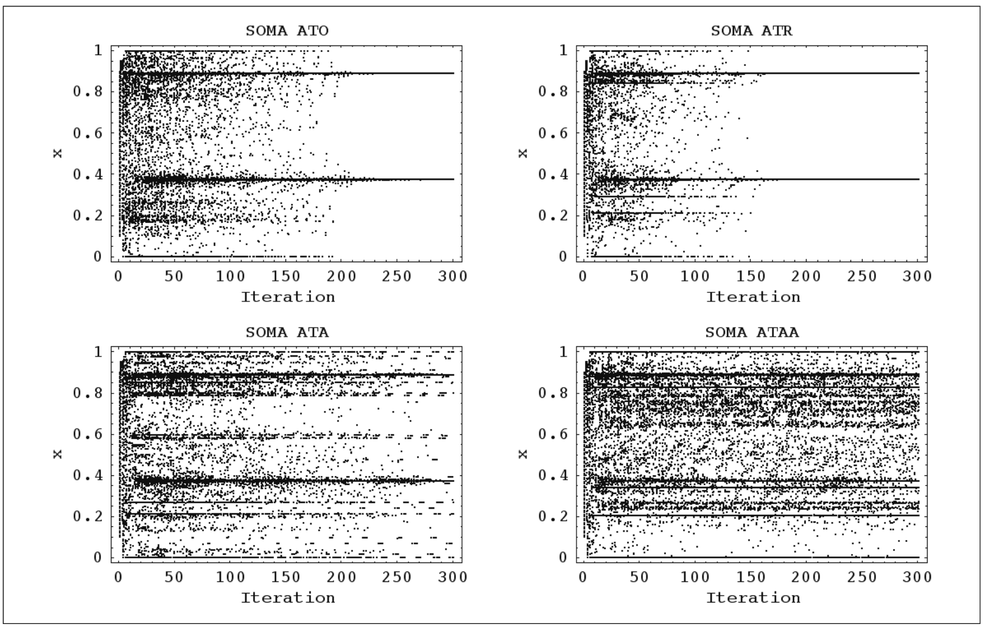
LQ DE 1p CF Non-Auto



LQ SOMA 2p CF Non-Auto



LQ SOMA 2p CF Non-Auto



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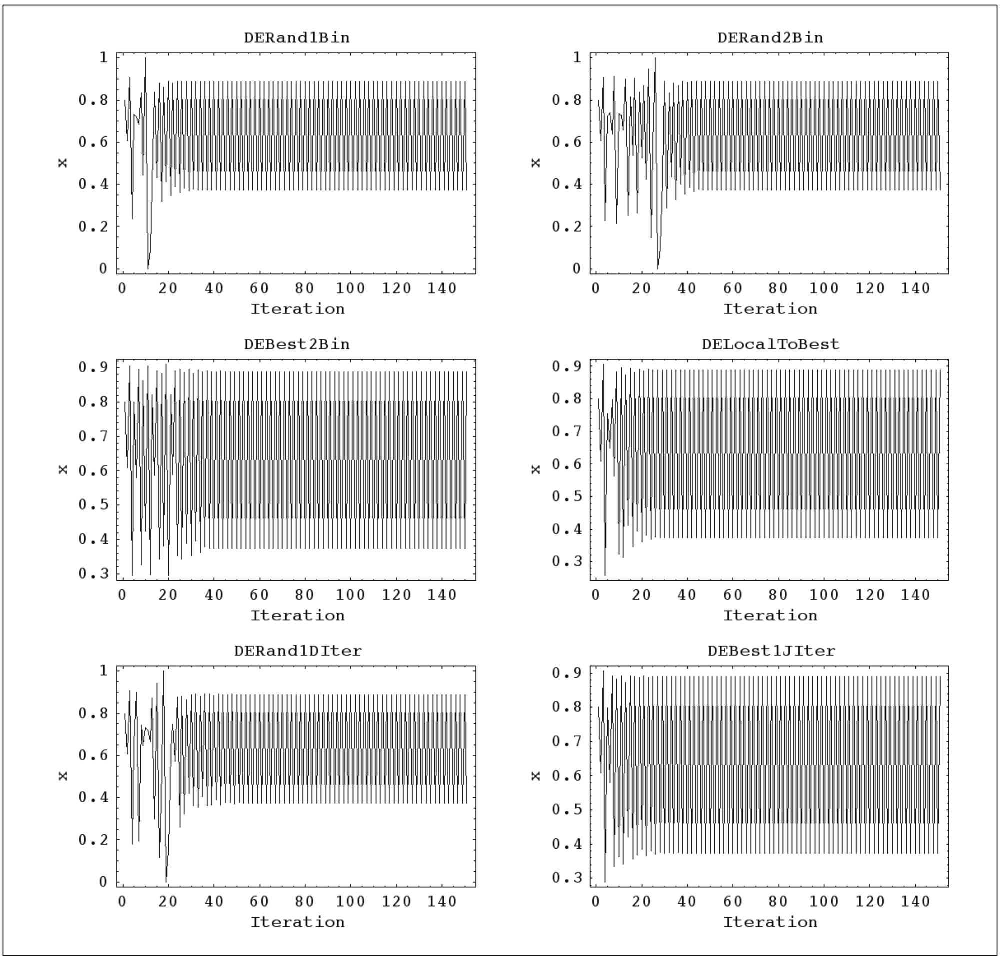

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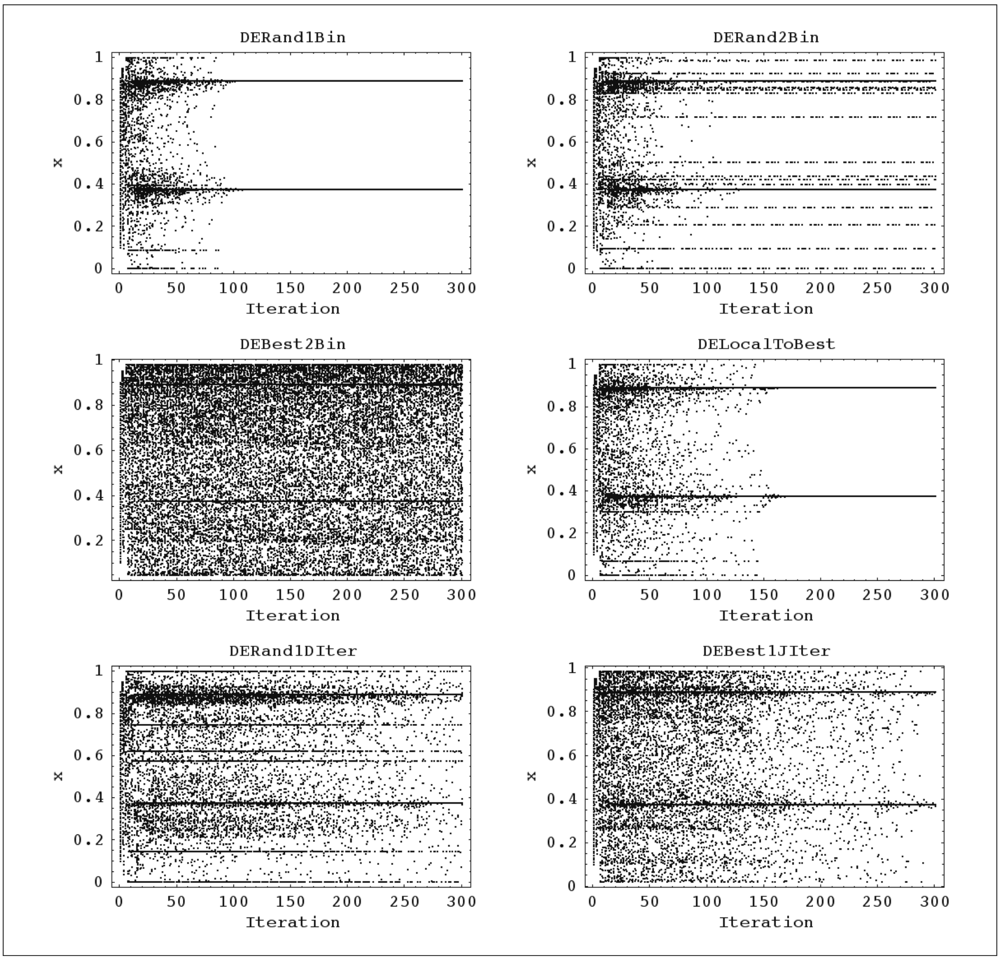
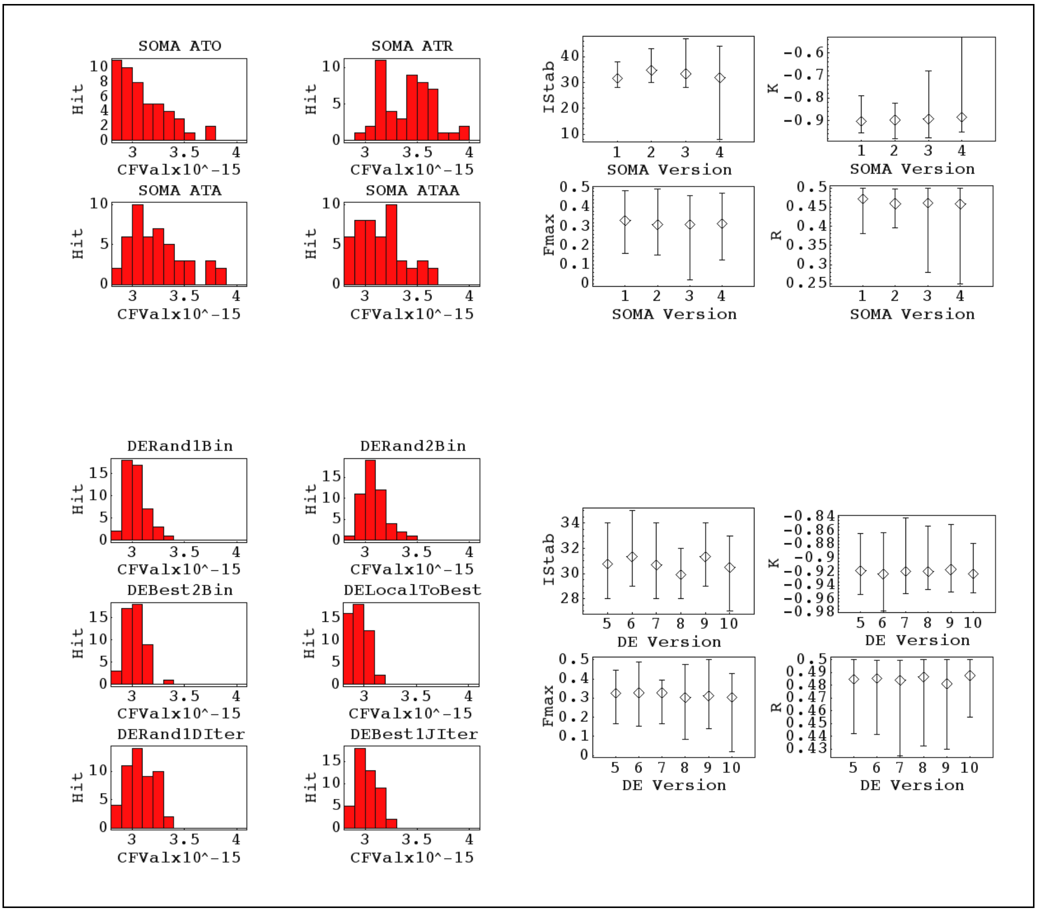
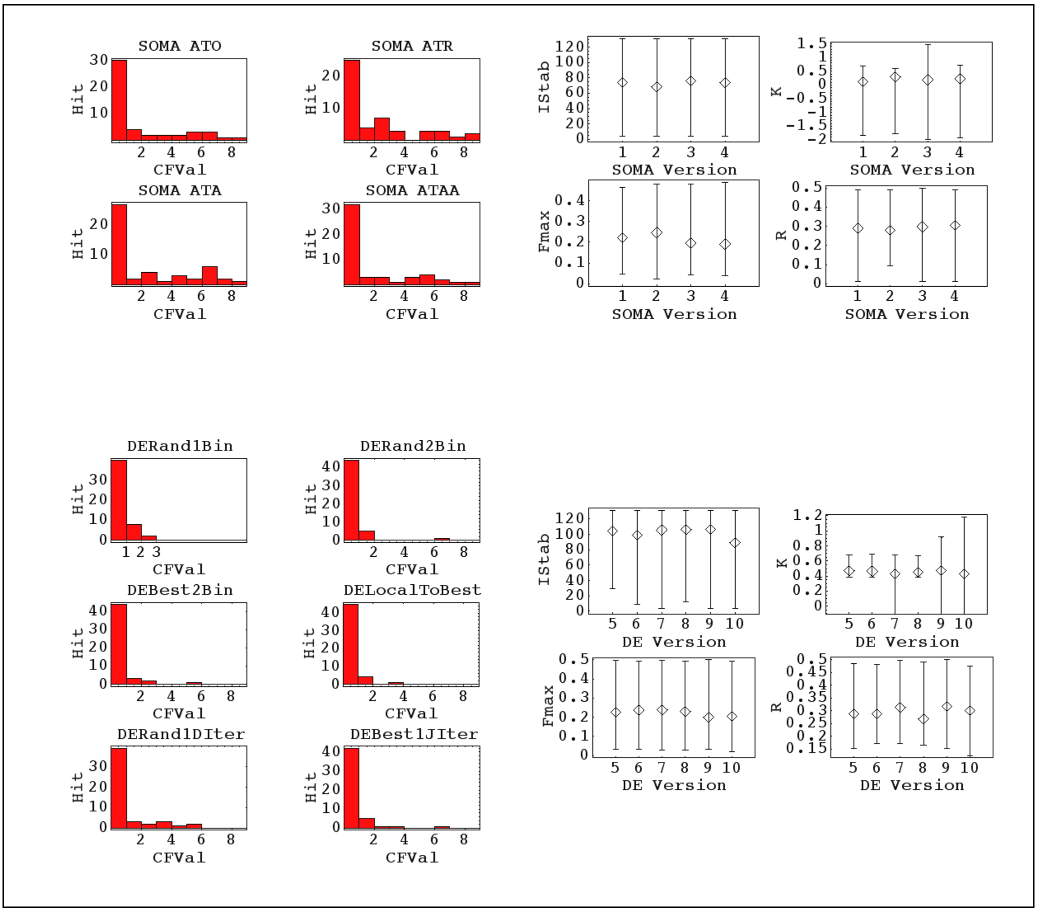


LQ SOMA & DE 1p CF NA

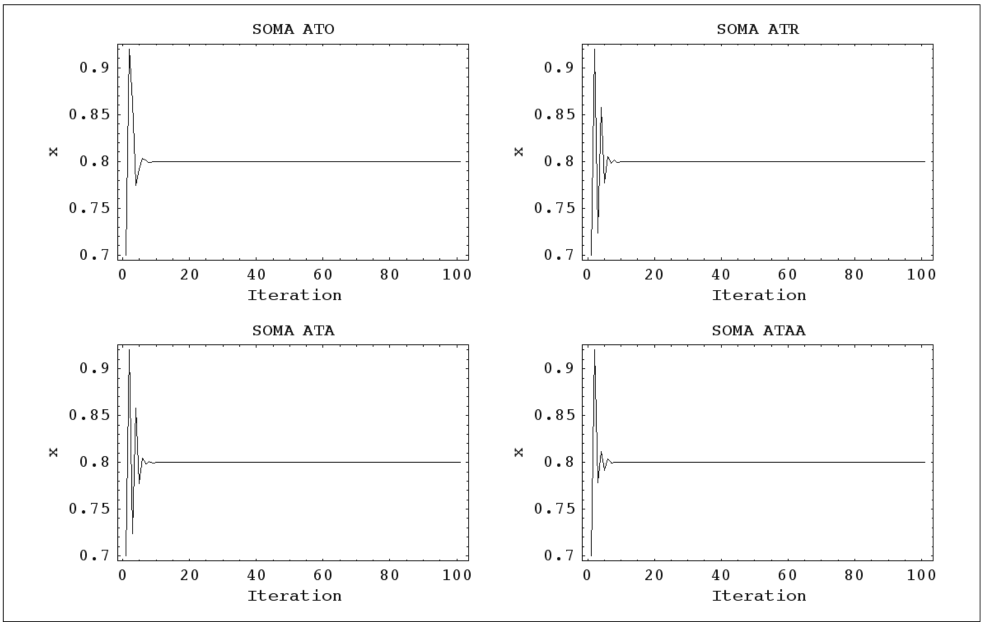


LQ SOMA & DE 2p CF NA

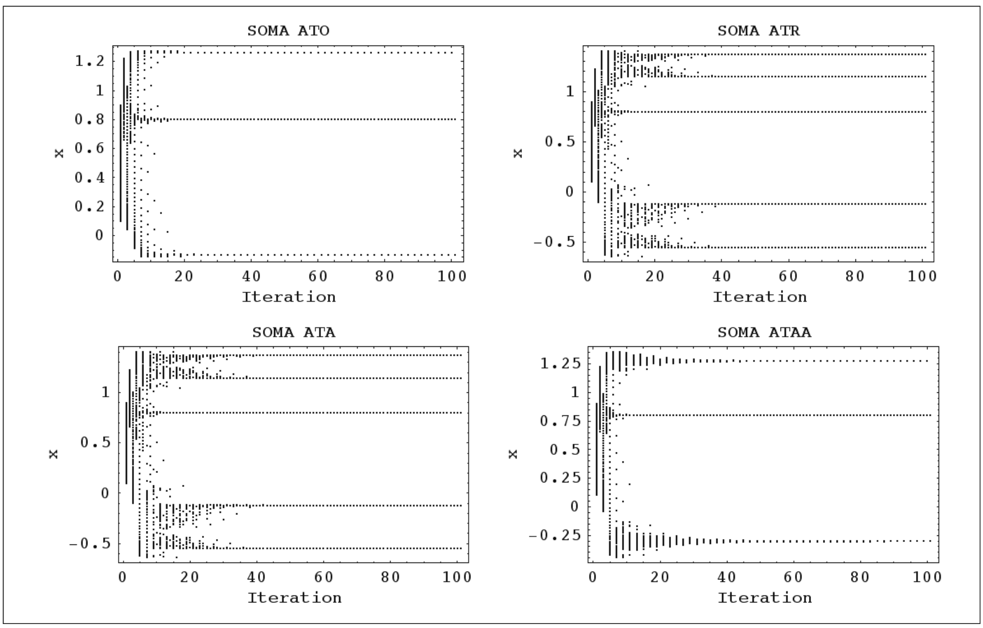


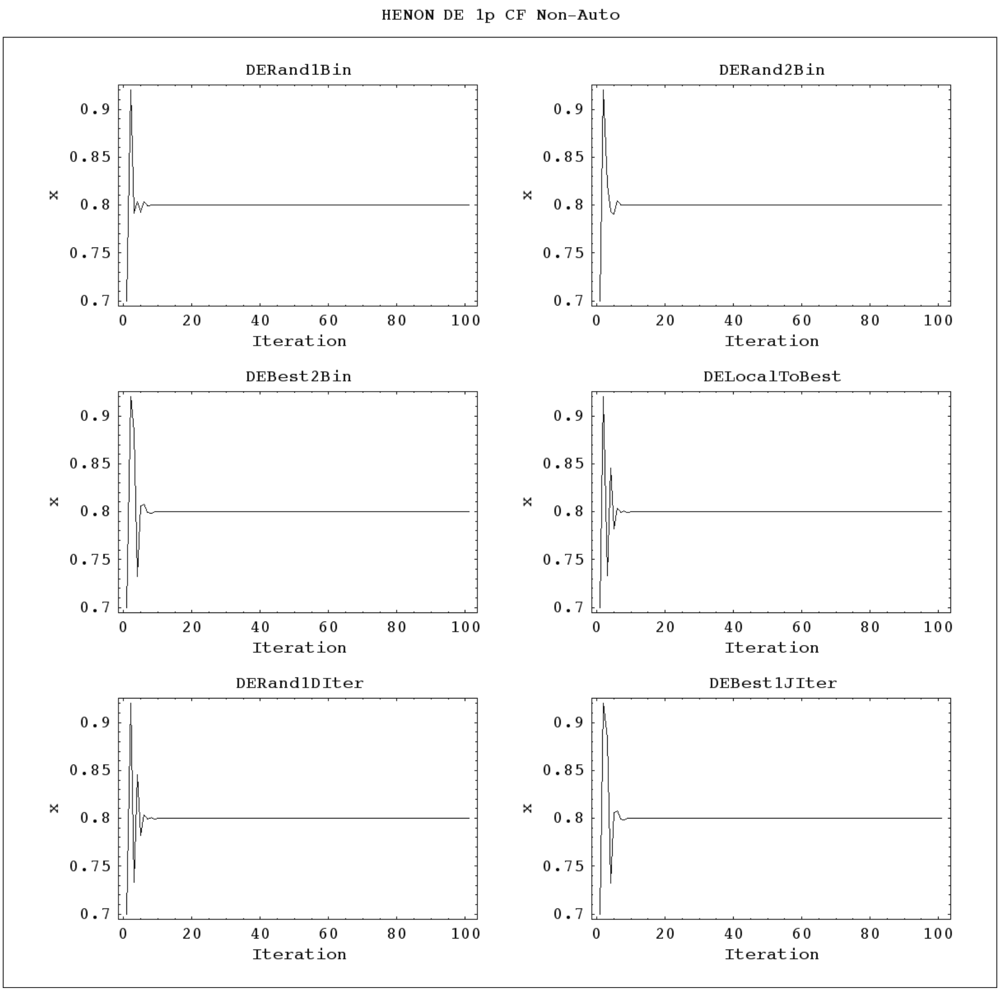
8.12 Summary of results, Case study 3, CF NA, HENON

HENON SOMA 1p CF Non-Auto



Henon SOMA 1p CF Non-Auto




HENON DE 1p CF Non-Auto
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration
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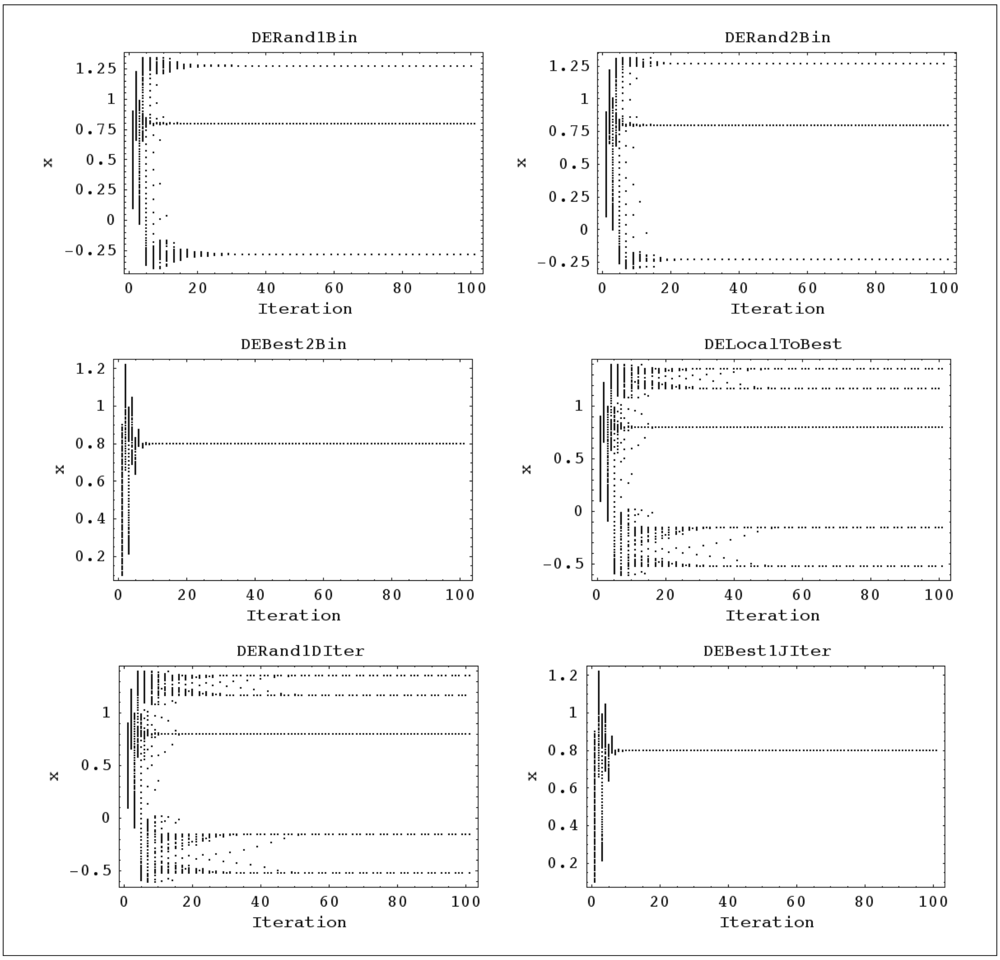
0.75
0.8
0.85
0.9
0
20
40
60
80
100

Henon DE 1p CF Non-Auto



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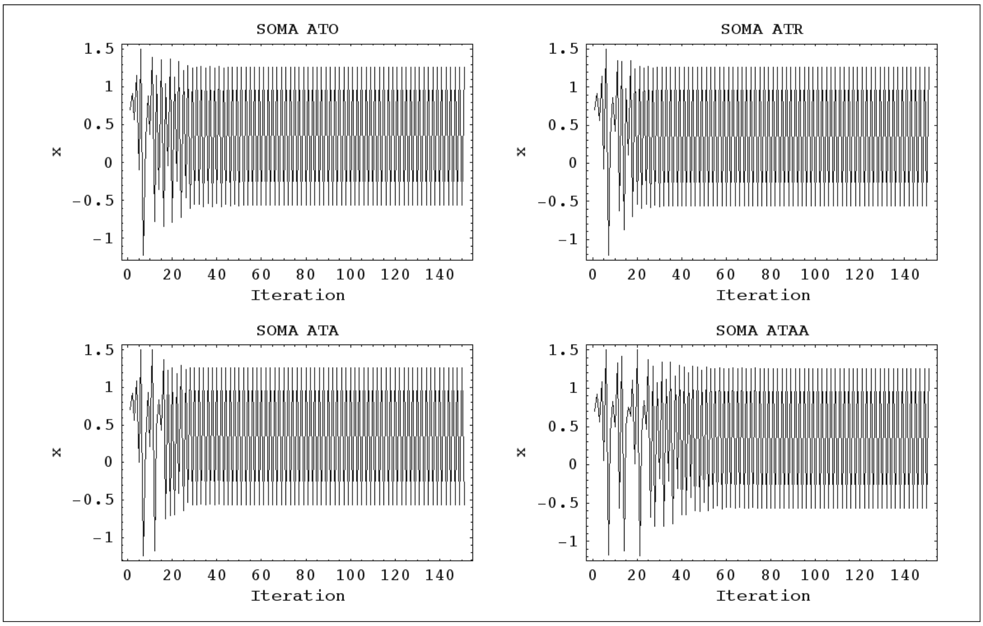

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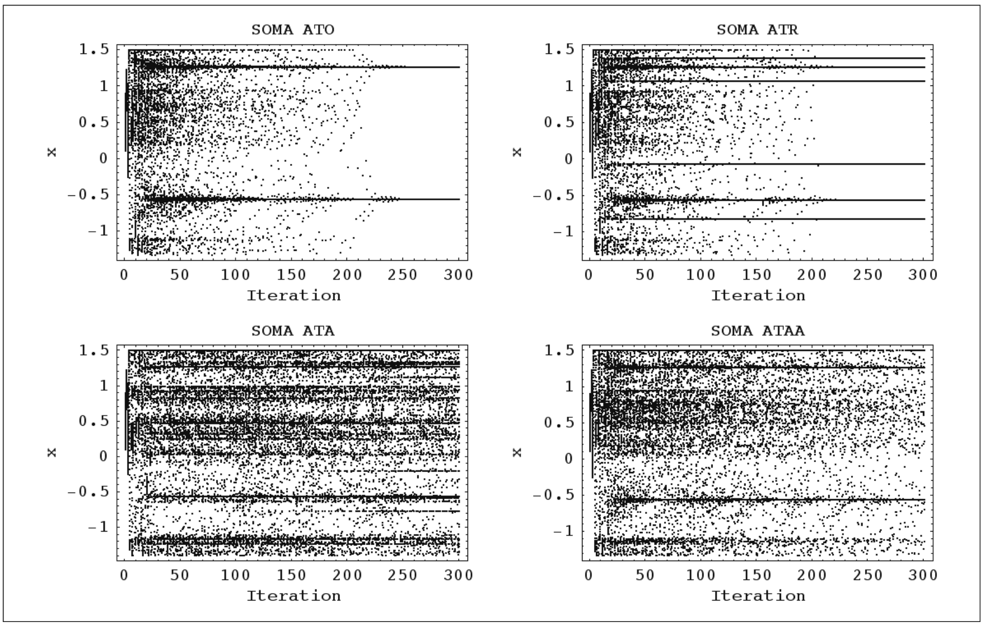

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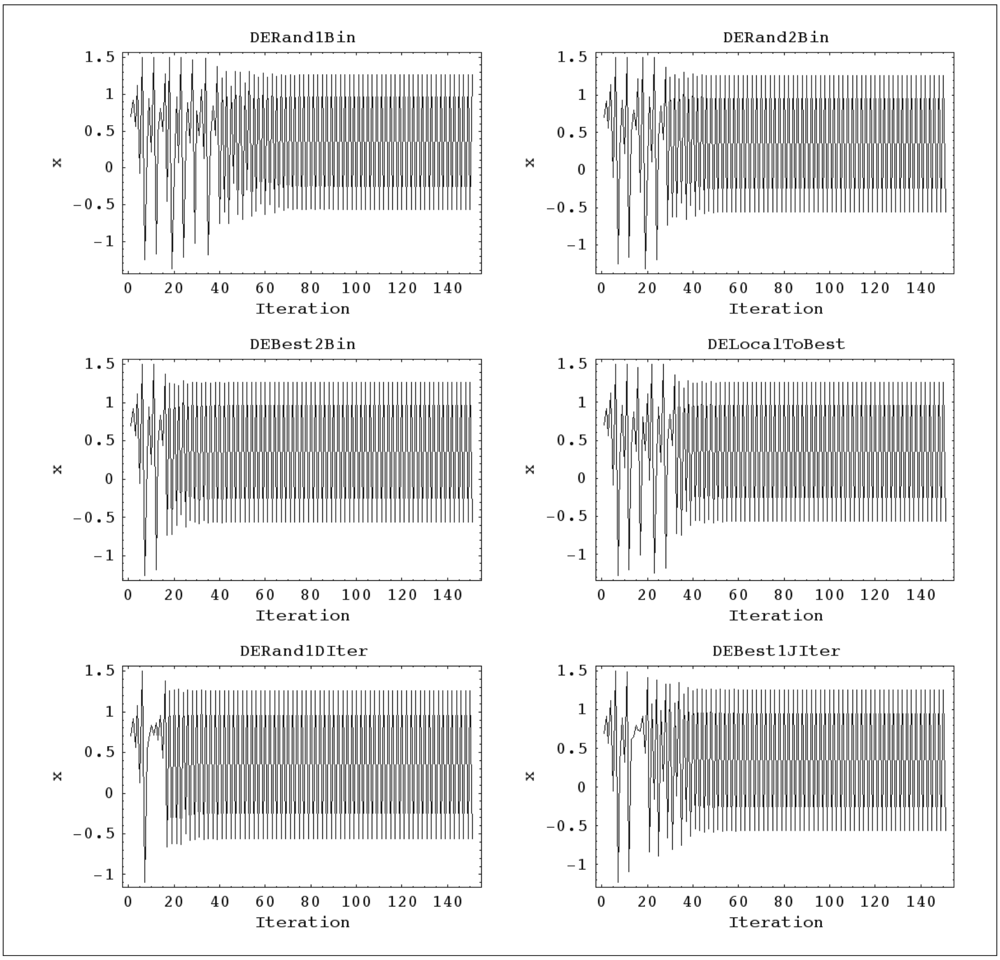

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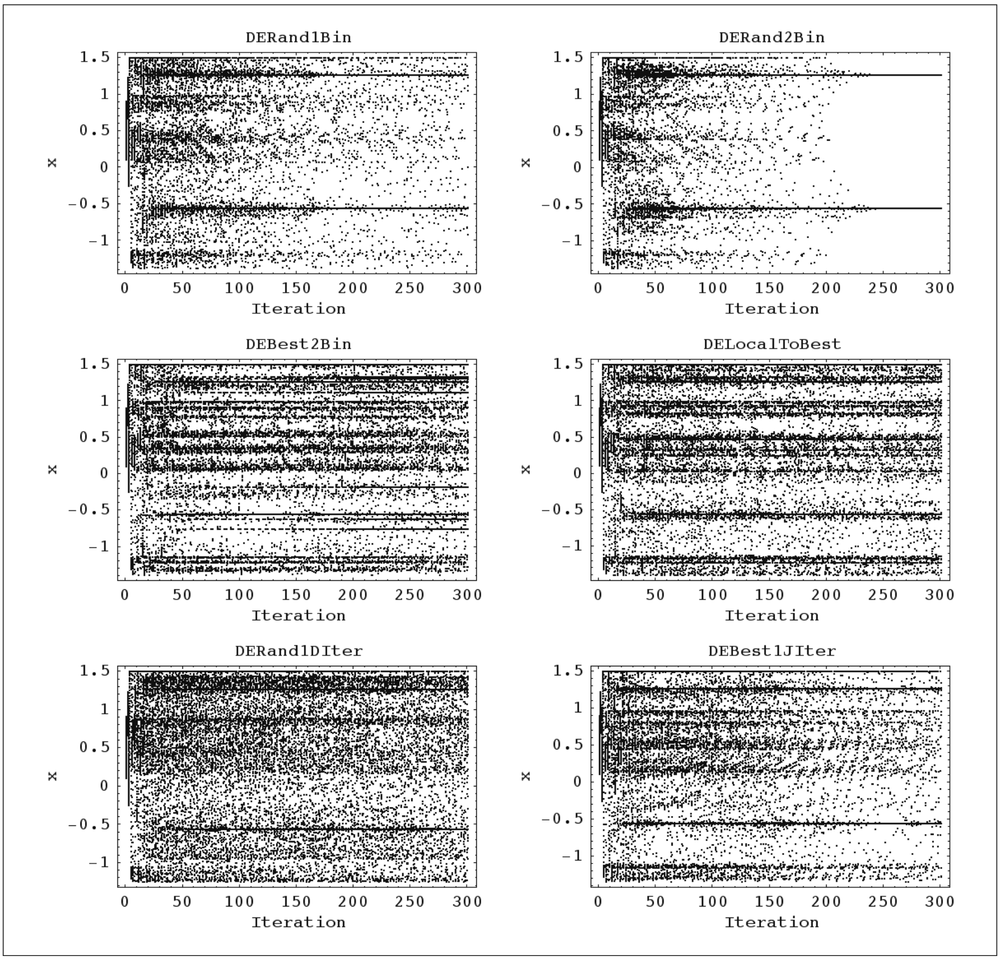
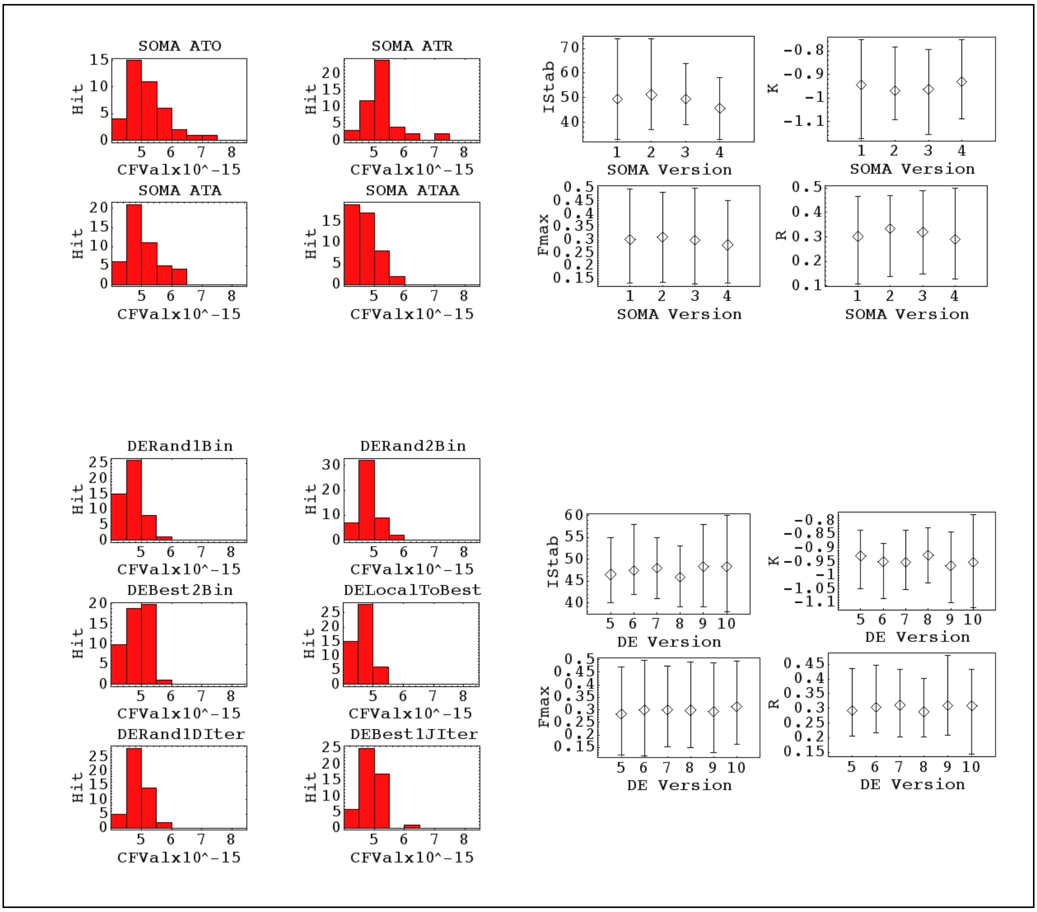


HENON SOMA & DE 1p CF NA



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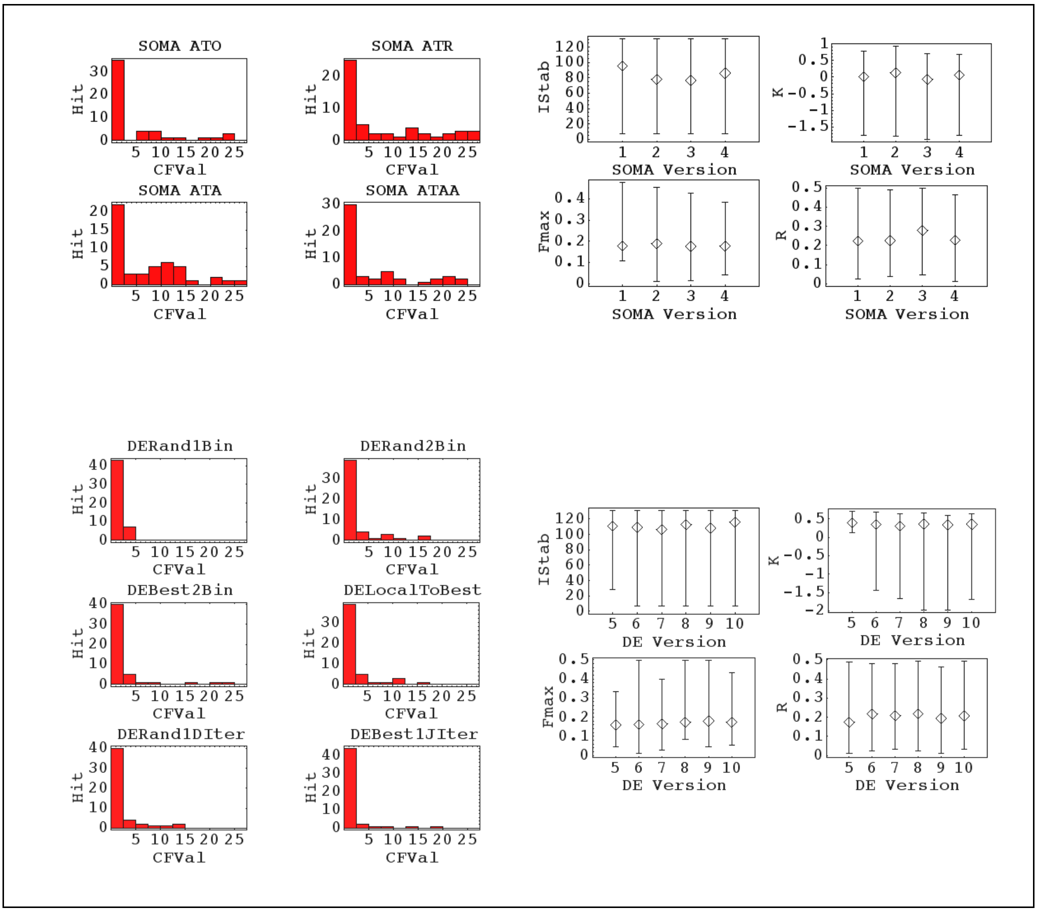
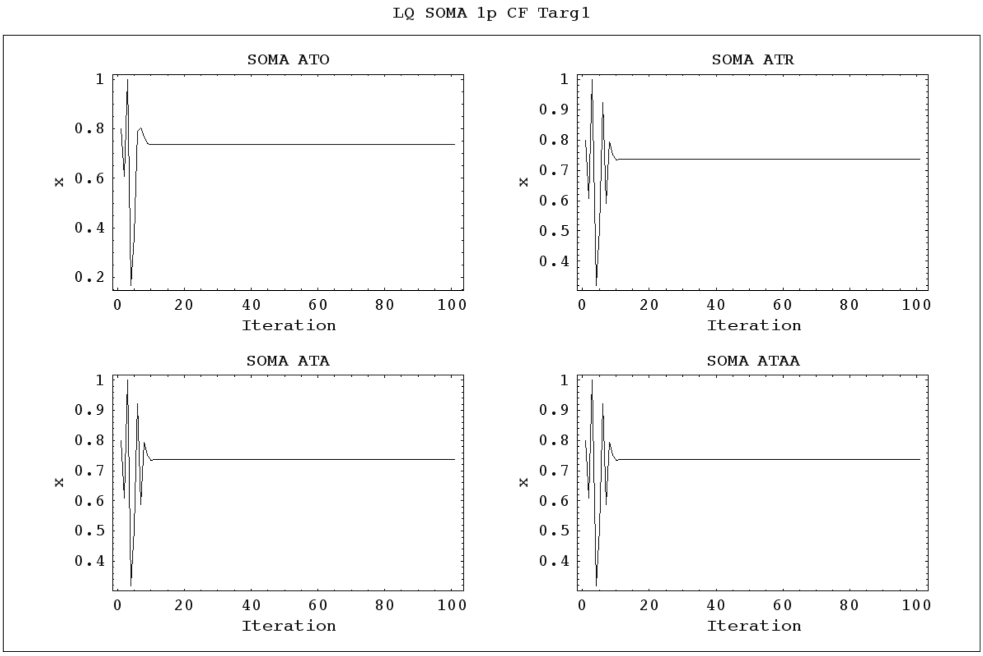
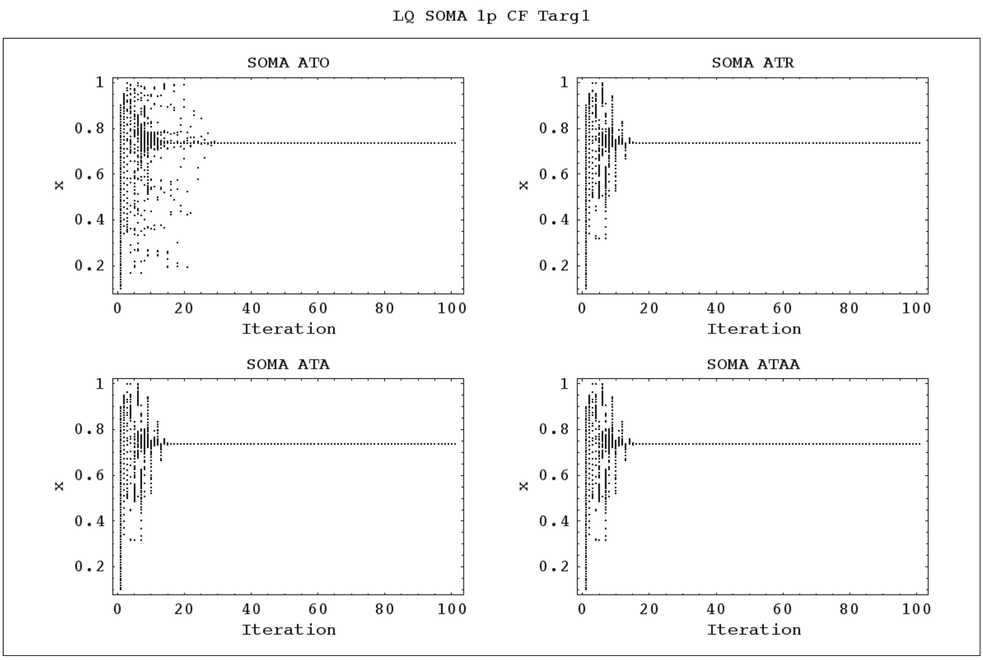
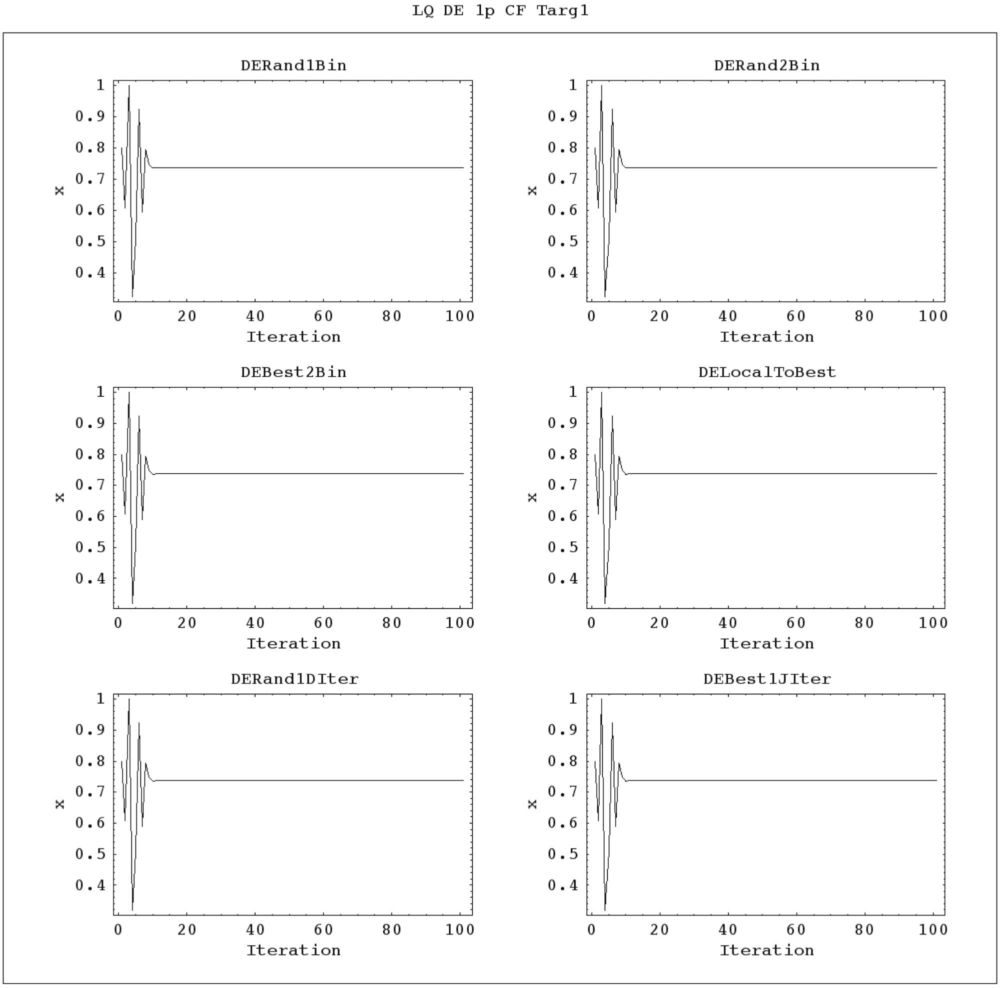


8.13 Summary of results, Case study 4, CF Targ1, LQ






LQ DE 1p CF Targ1
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x
Iteration
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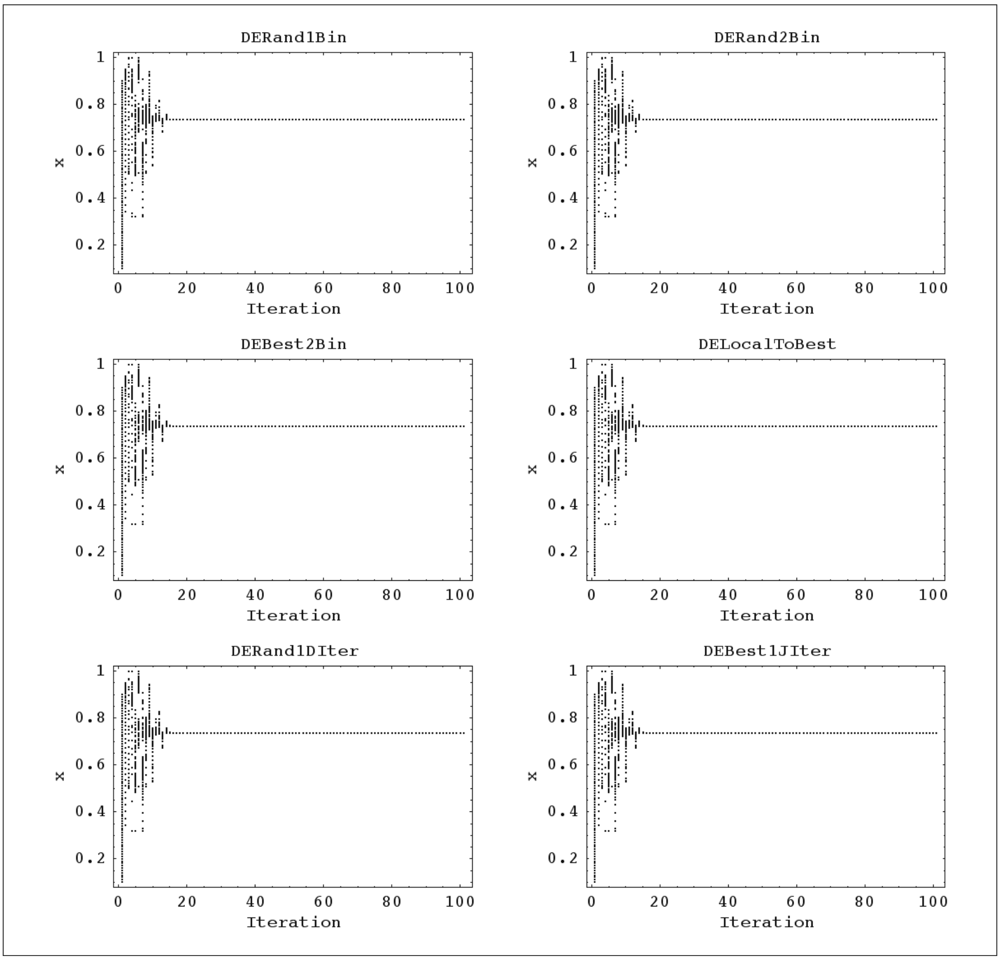
LQ DE 1p CF Targ1



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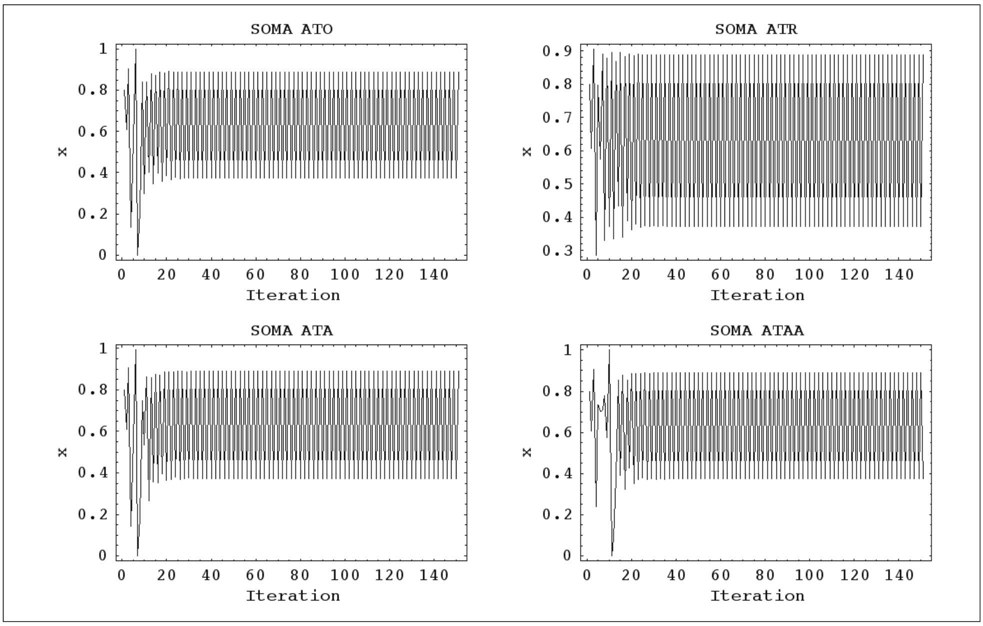

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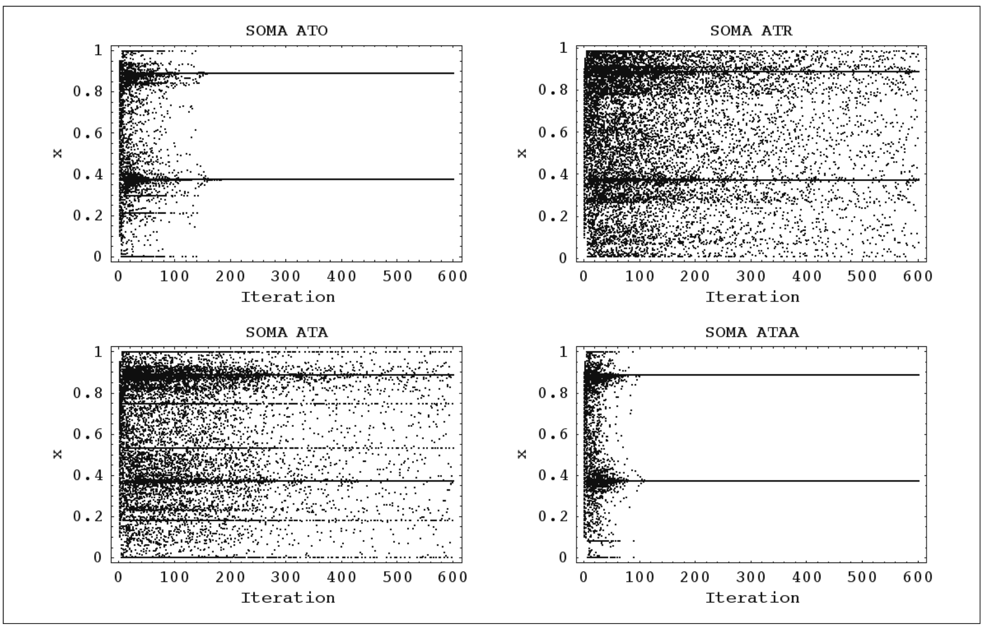

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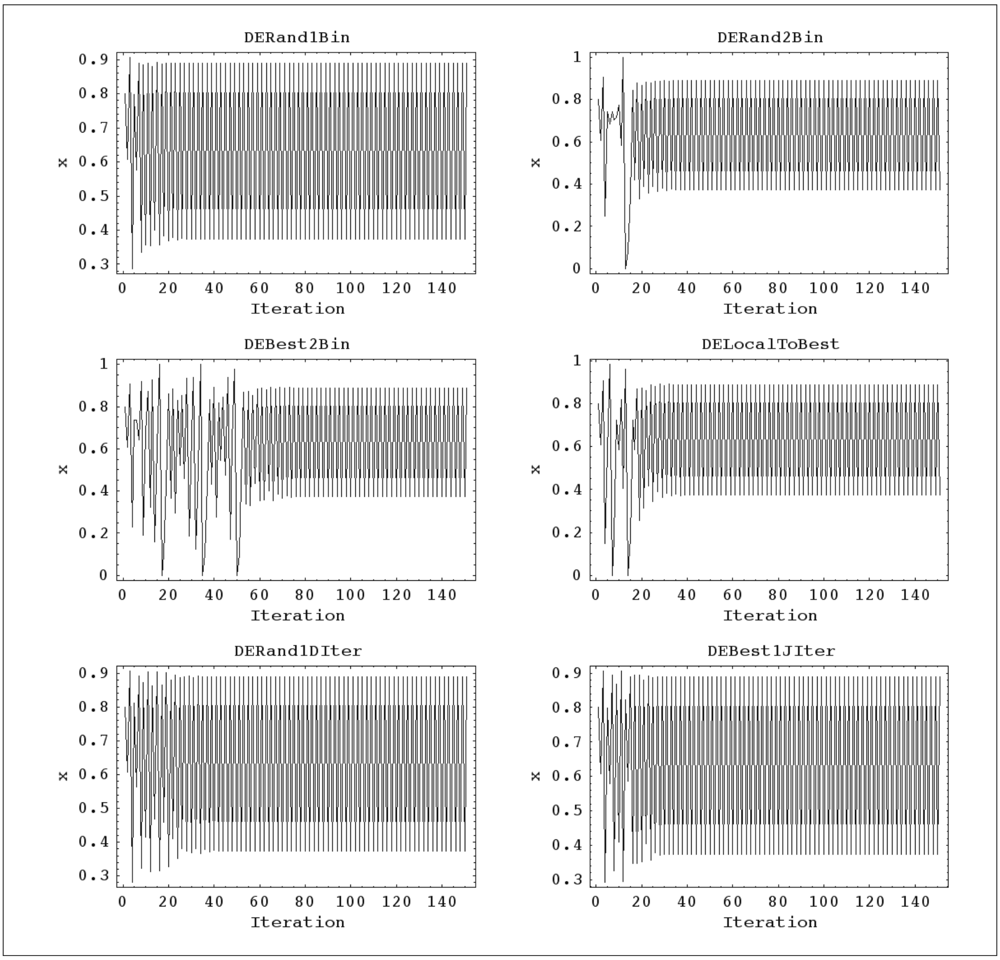
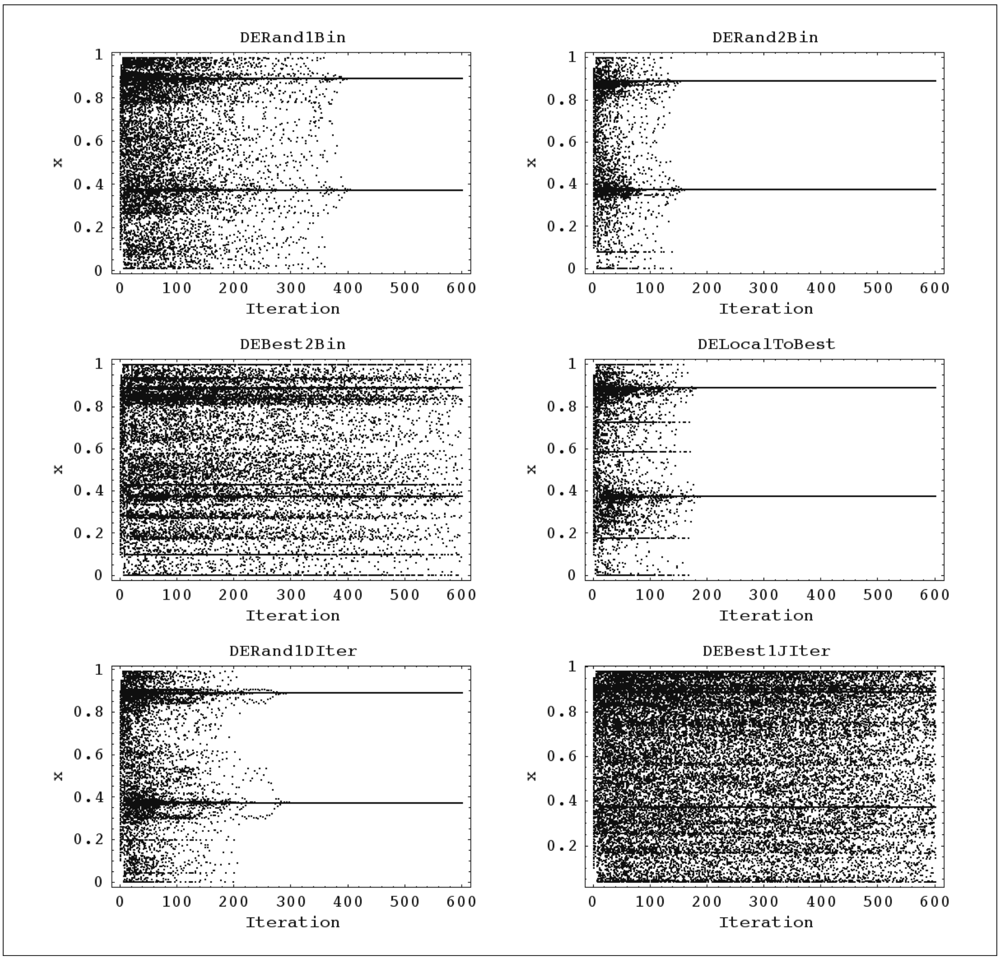
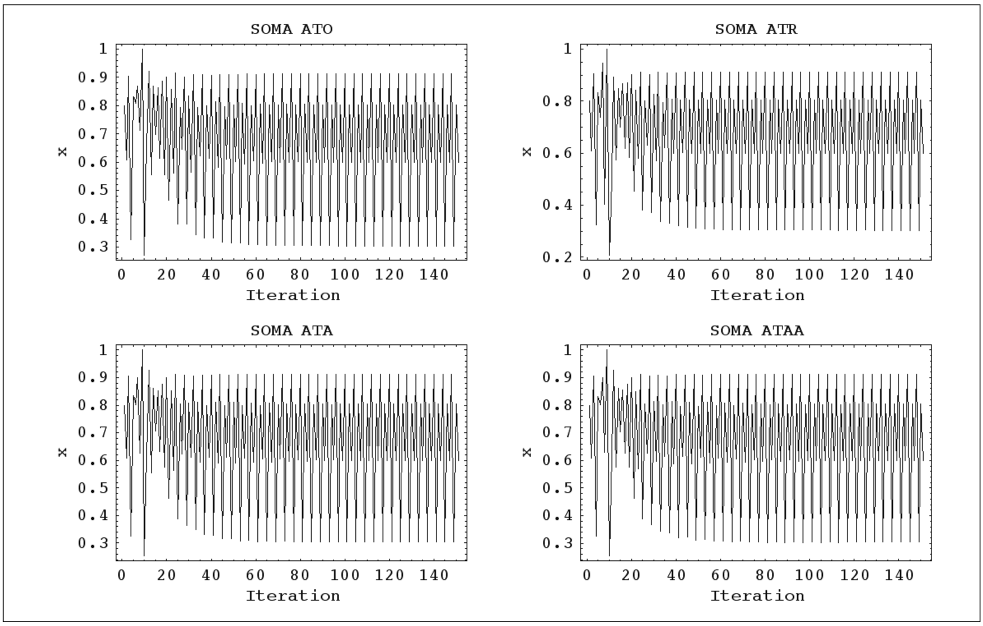


LQ DE 2p CF Targ1



LQ SOMA 4p CF Targ1



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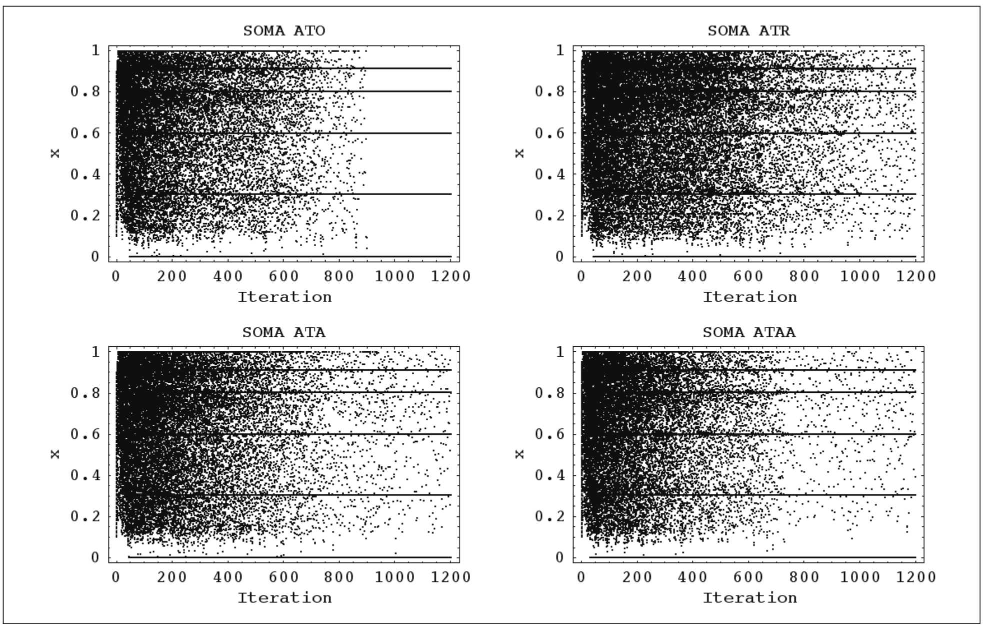

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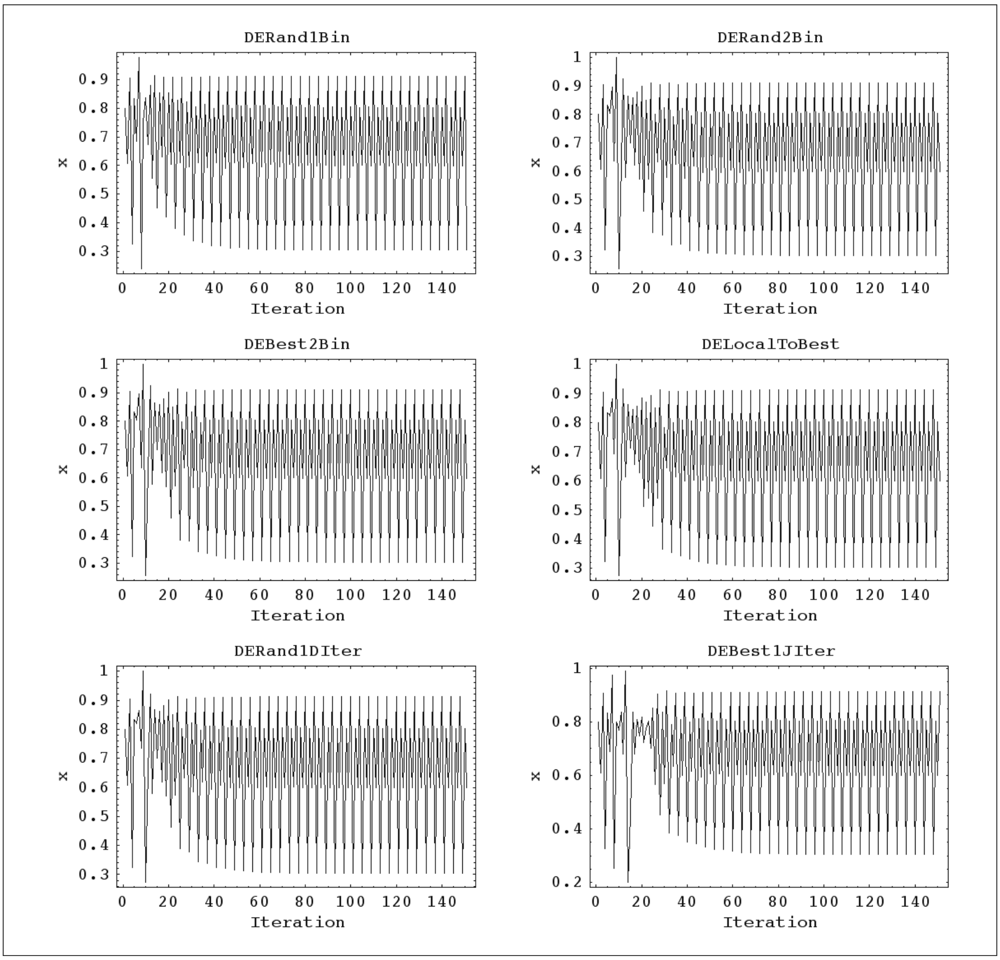

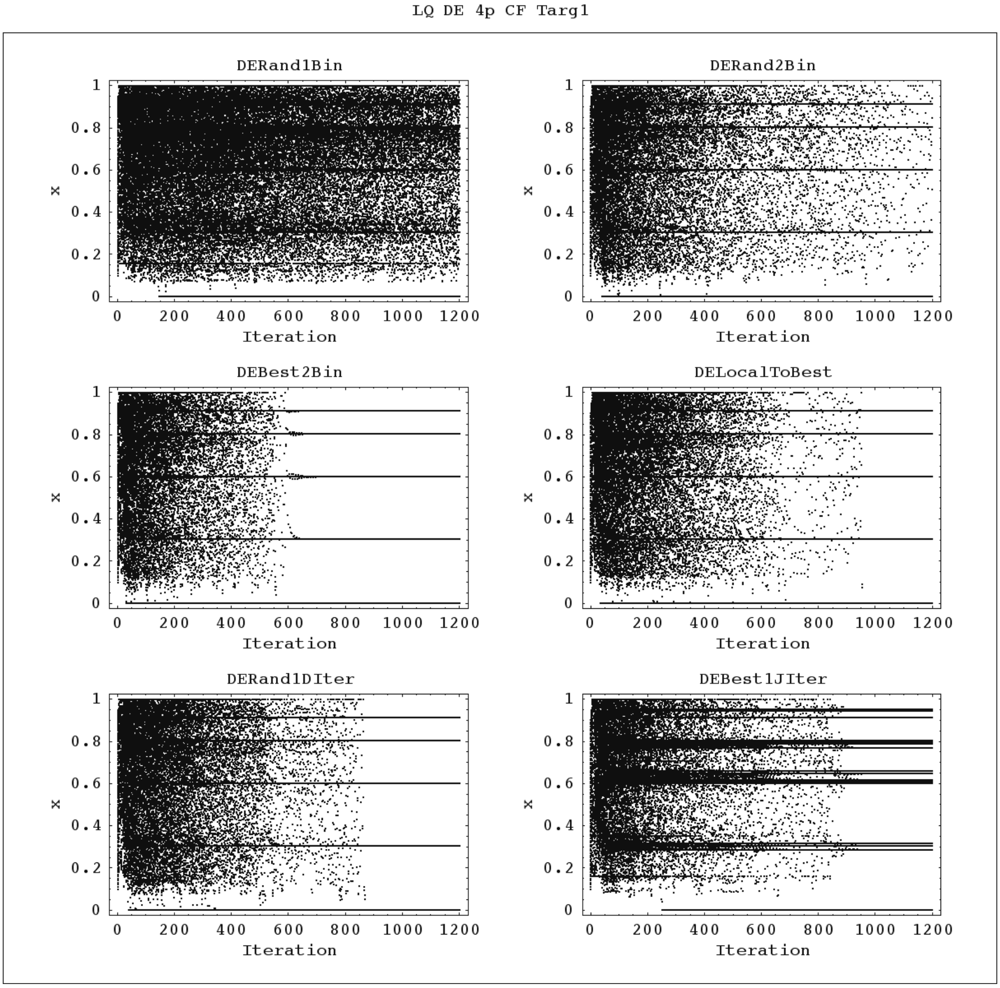
LQ DE 4p CF Targ1
DERand1Bin
DERand2Bin
DEBest2Bin
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DERand1DIter
DEBest1JIter
x
Iteration
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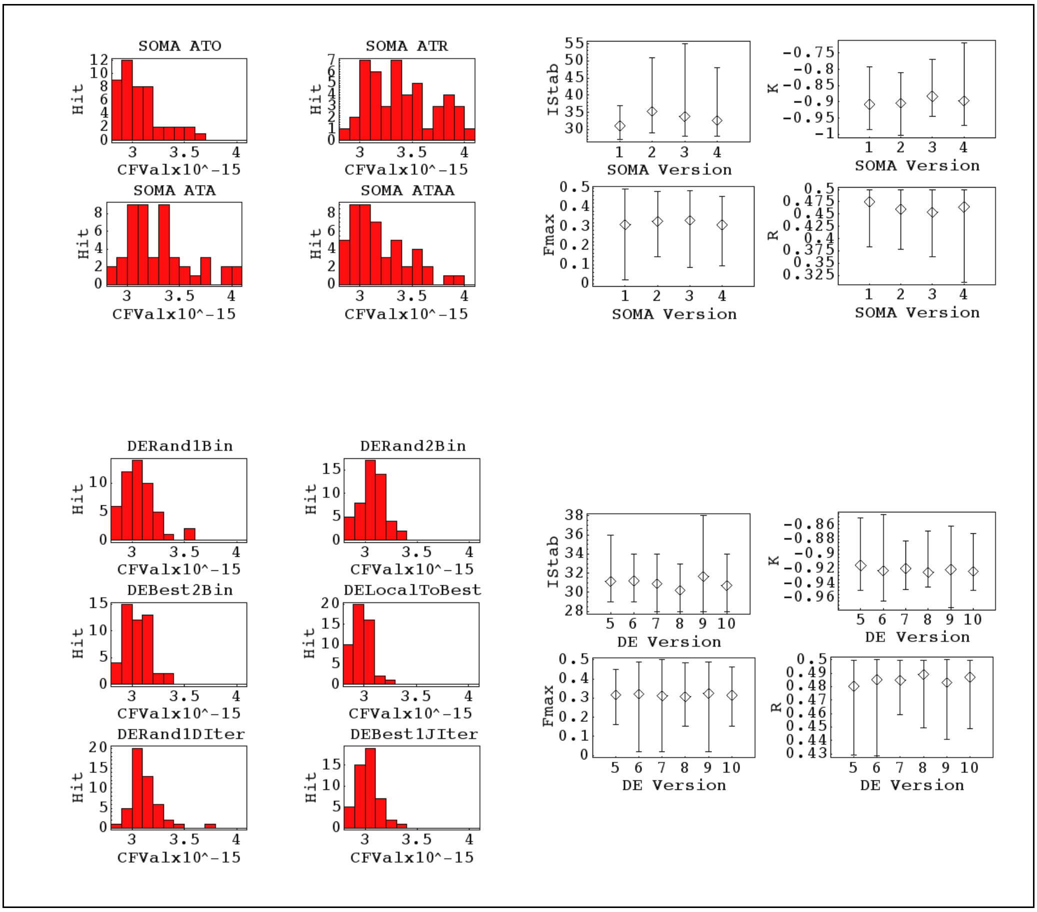
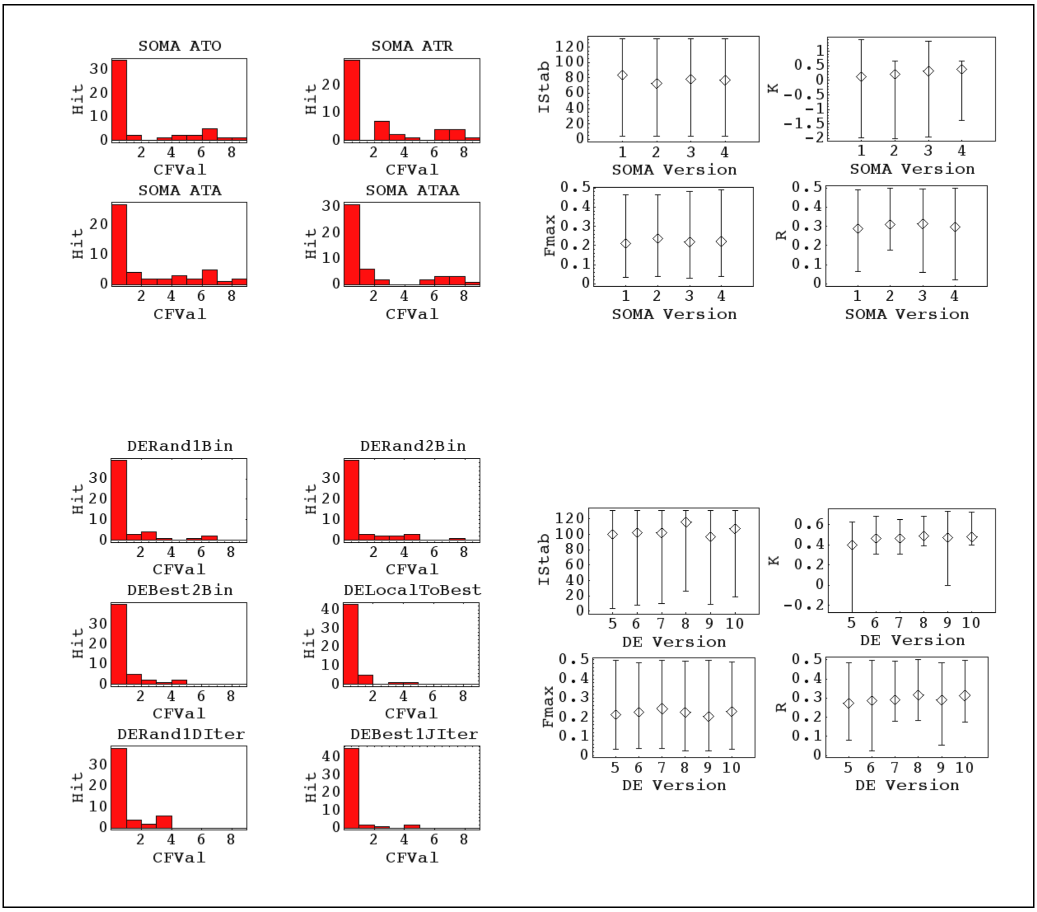
1200

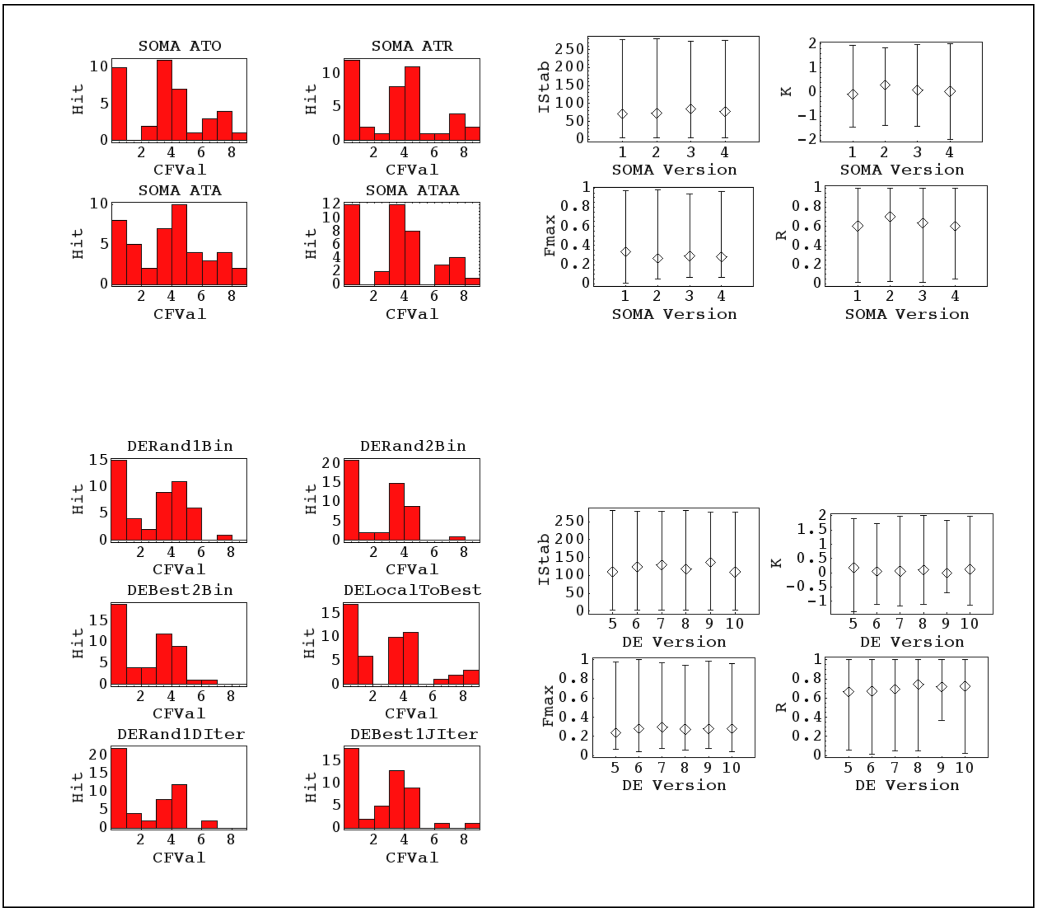
LQ SOMA & DE 1p CF Targ1



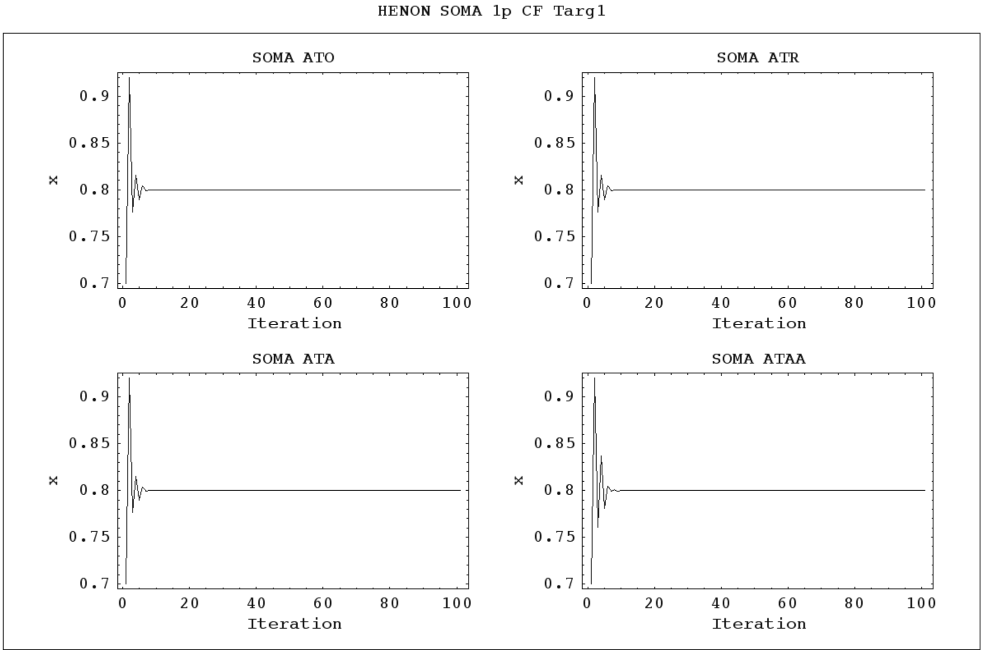
LQ SOMA & DE 2p CF Targ1

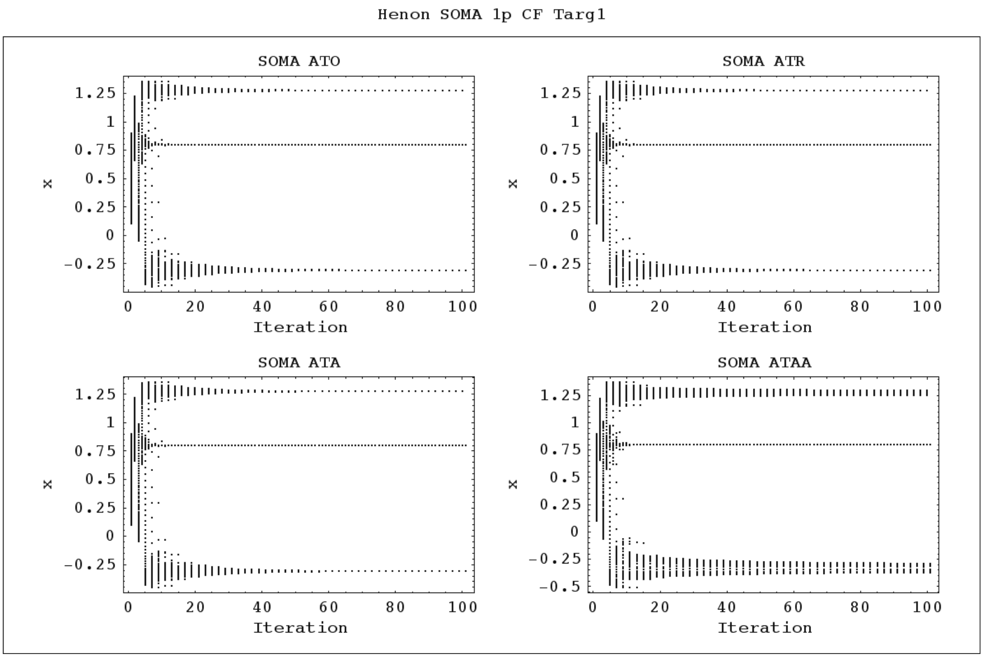


LQ SOMA & DE 4p CF Targ1

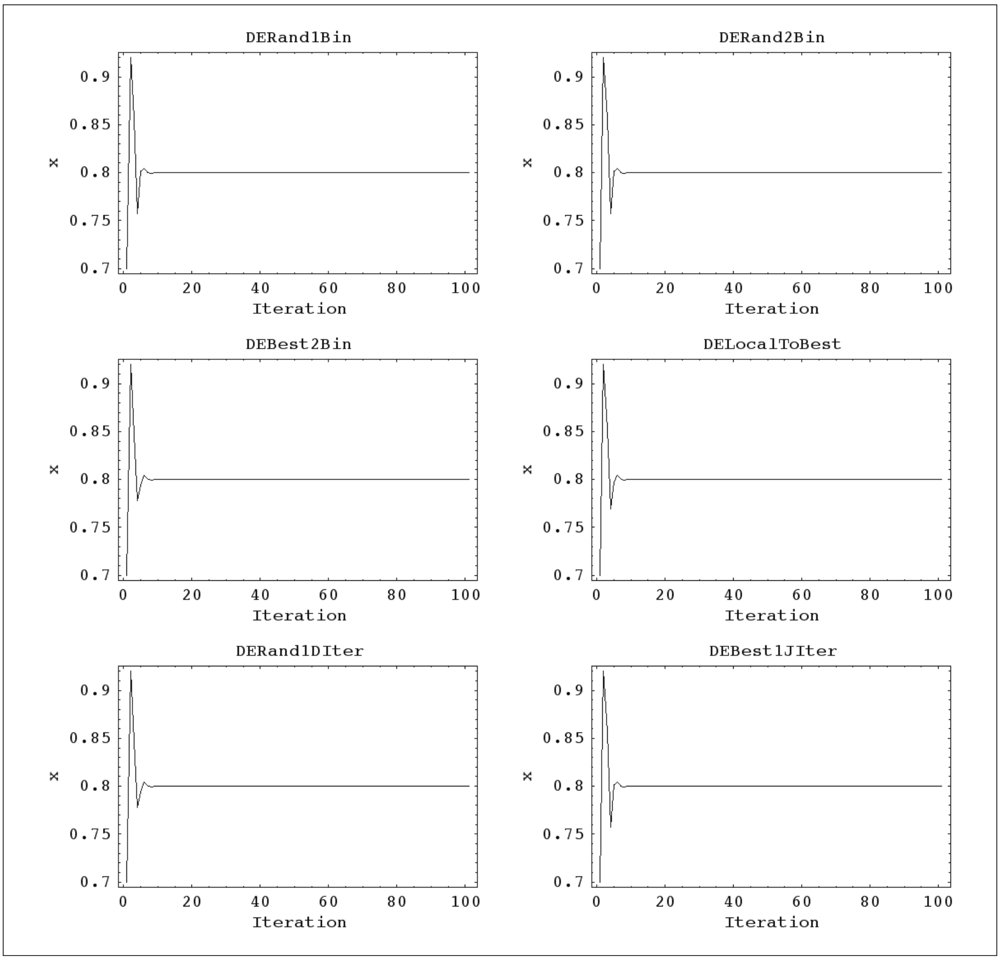


8.14 Summary of results, Case study 4, CF Targ1, HENON

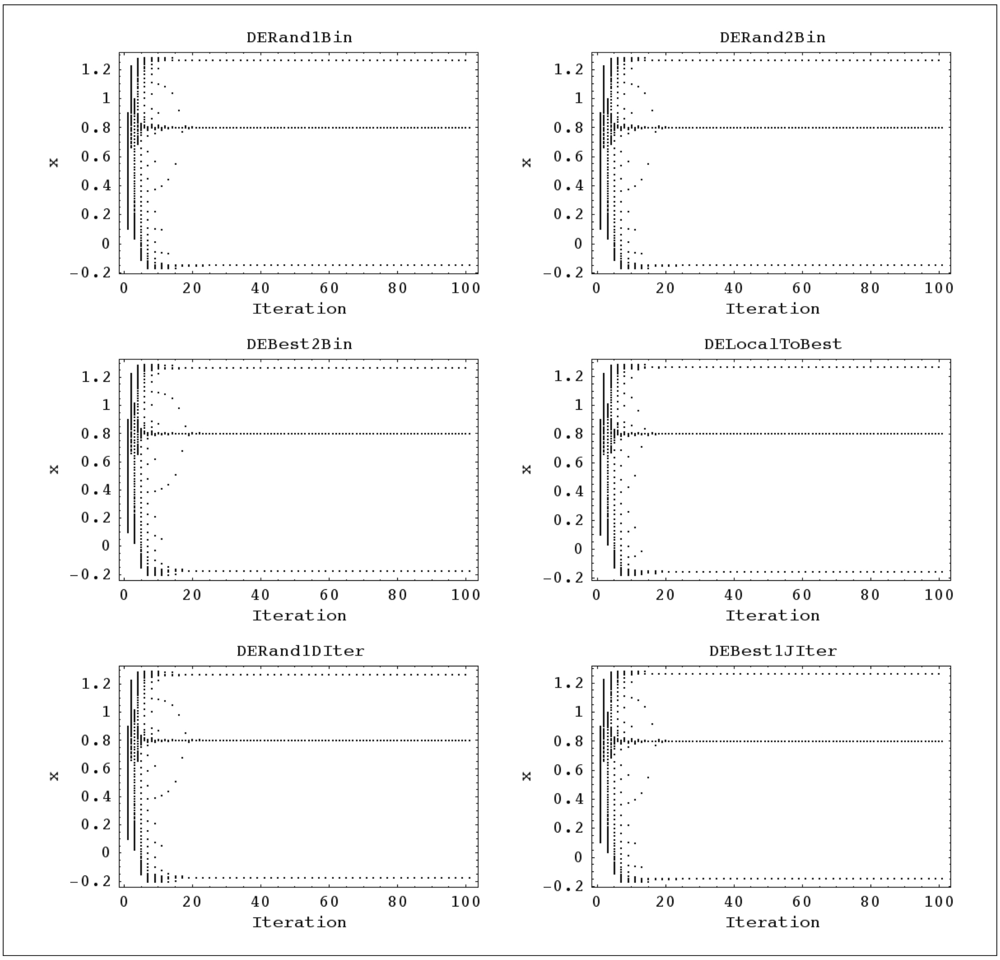




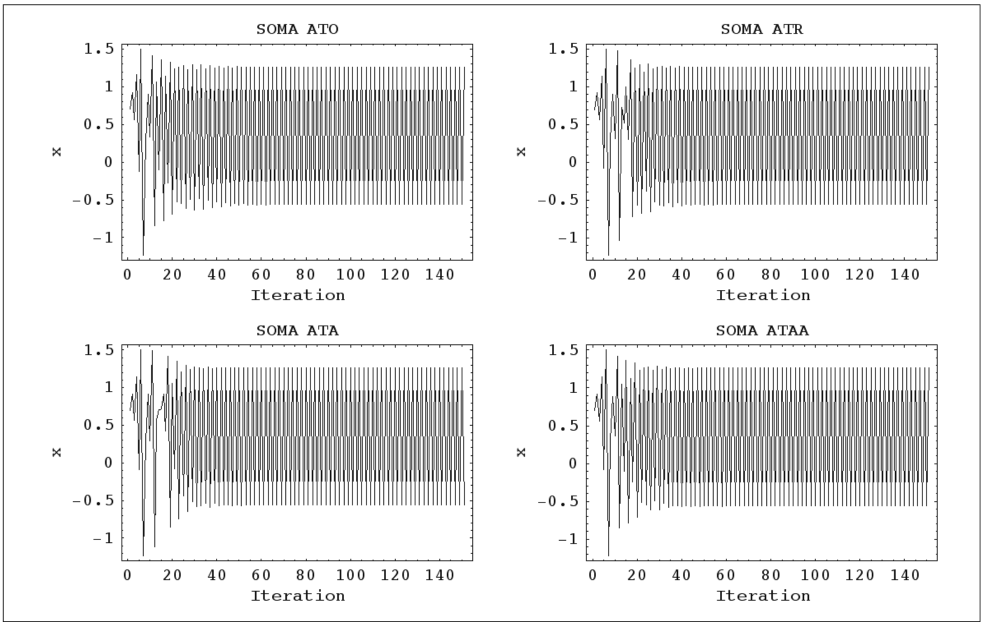
HENON DE 1p CF Targ1



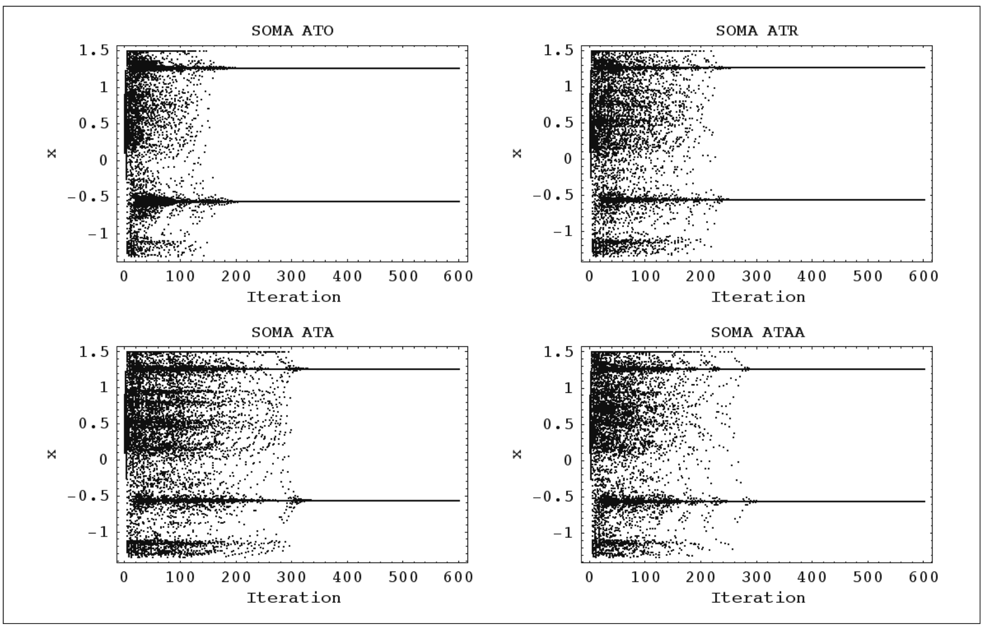
Henon DE 1p CF Targ1



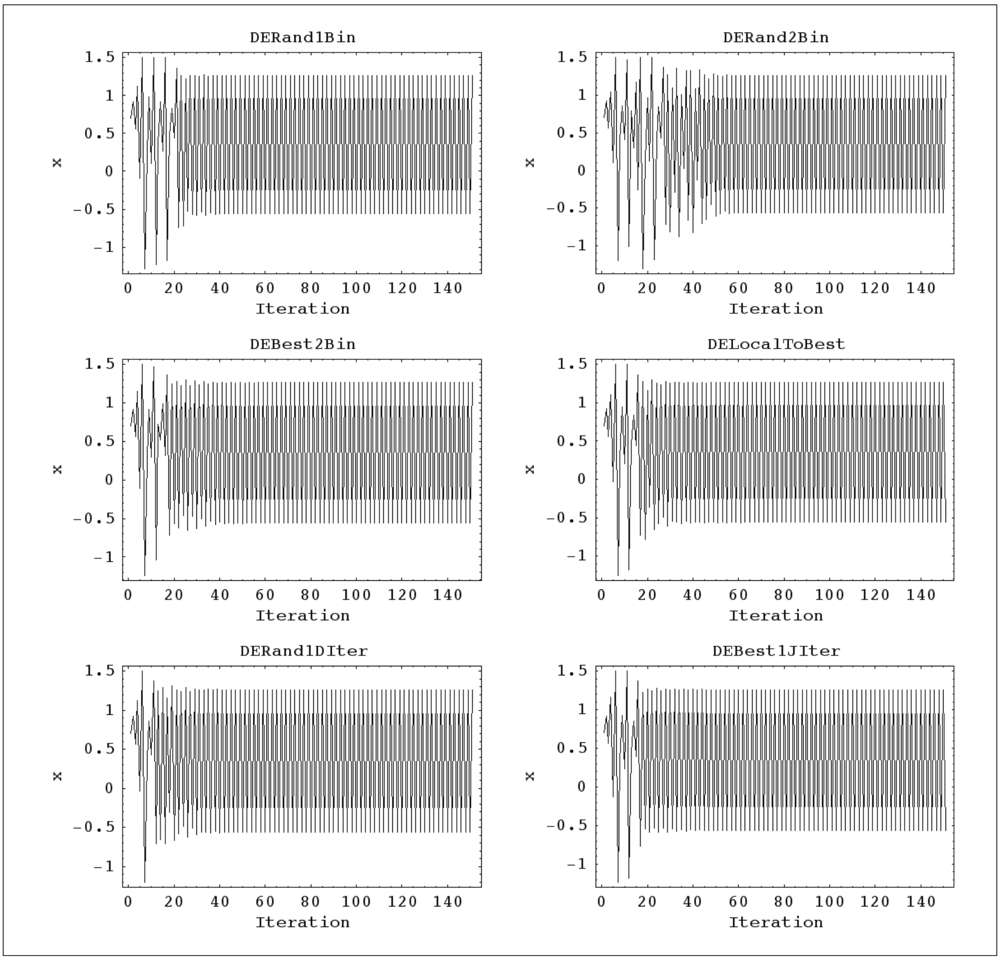
HENON SOMA 2p CF Targ1

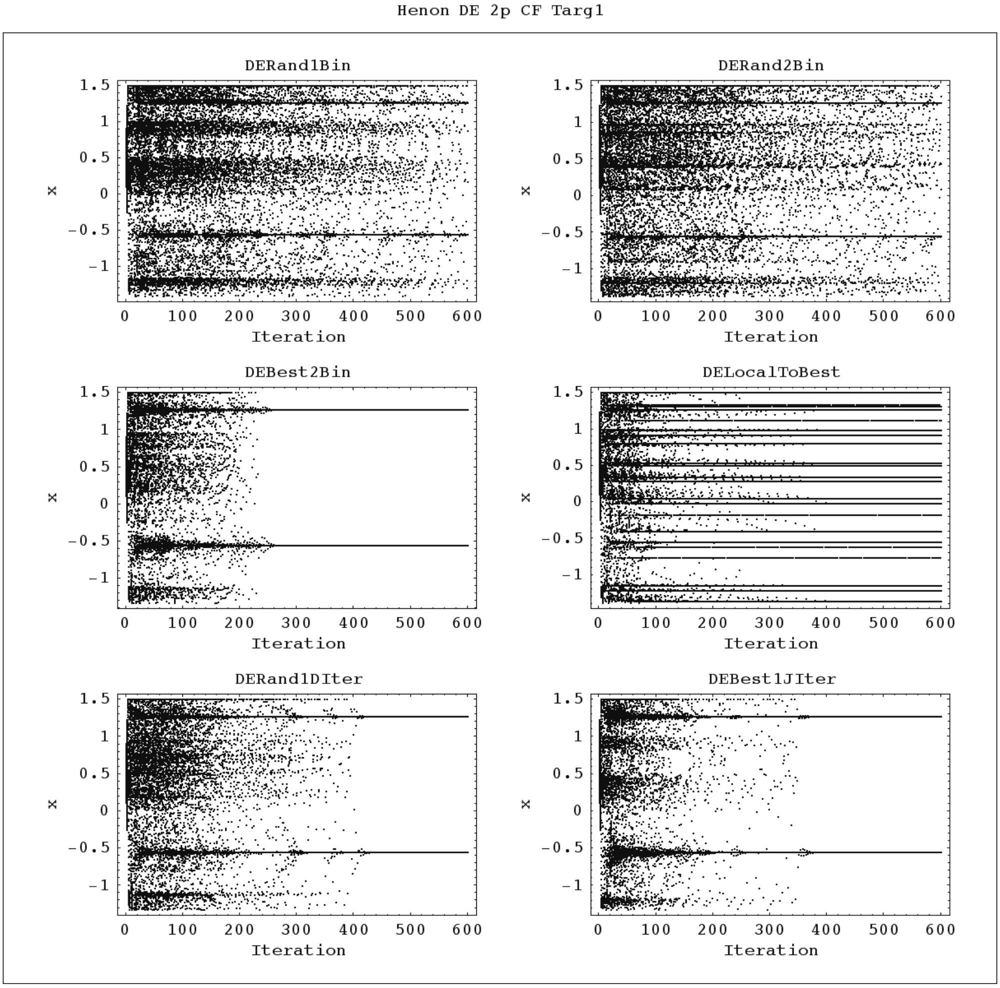


Henon SOMA 2p CF Targ1



HENON DE 2p CF Targ1




Henon DE 2p CF Targ1
DERand1Bin
DERand2Bin
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DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration
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-0.5
-1
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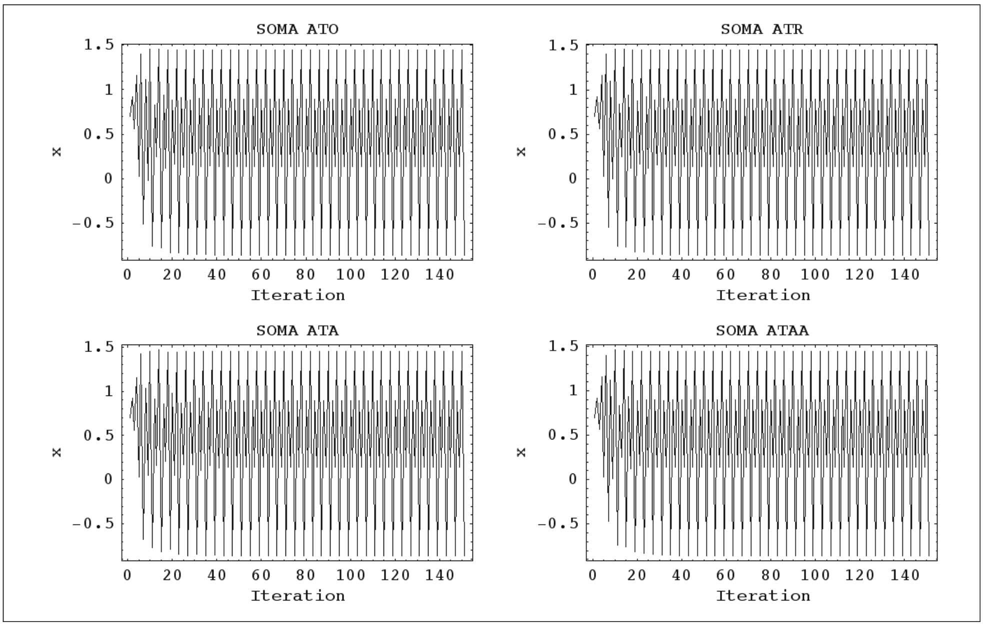
HENON SOMA 4p CF Targ1



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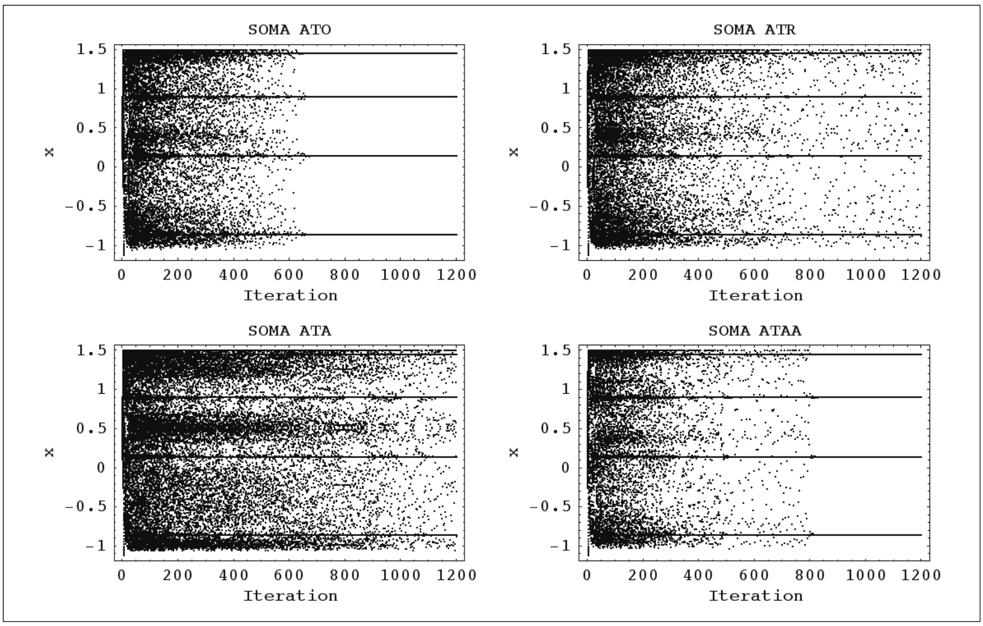

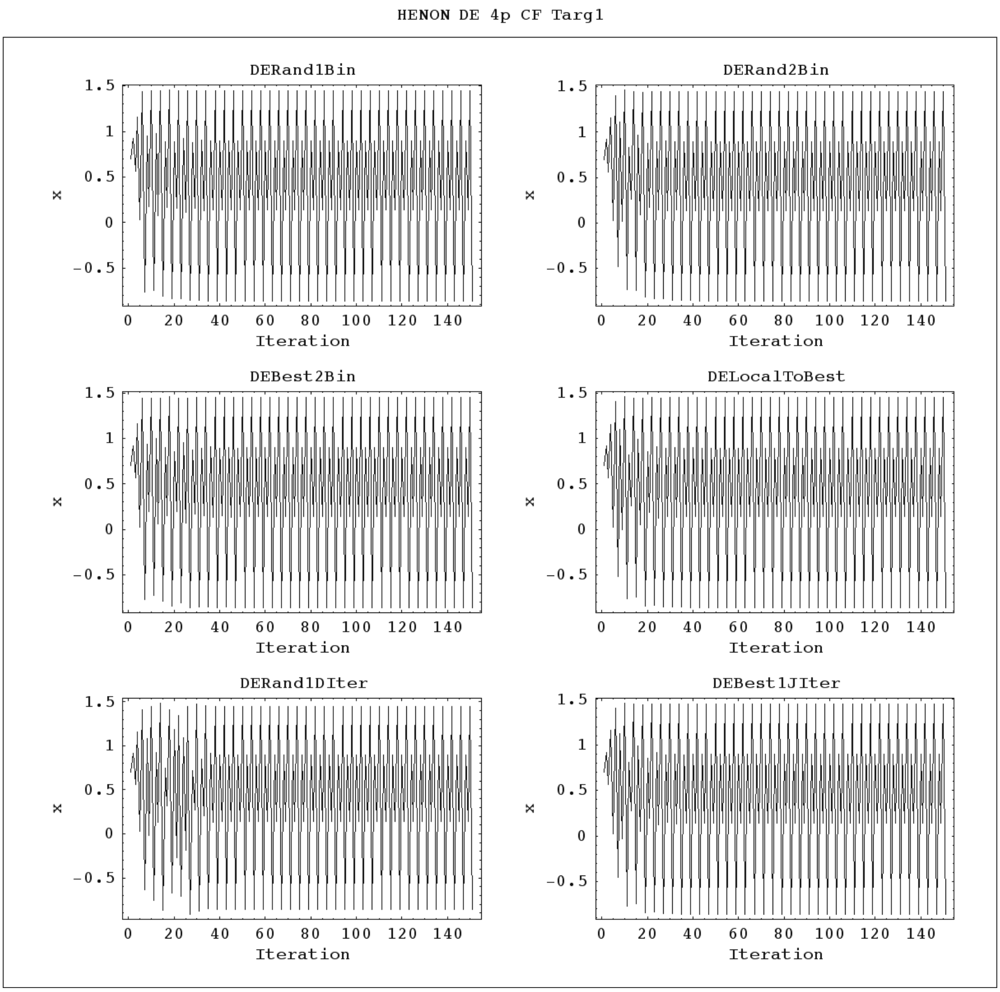
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DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration
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-0.5
0
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80
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120
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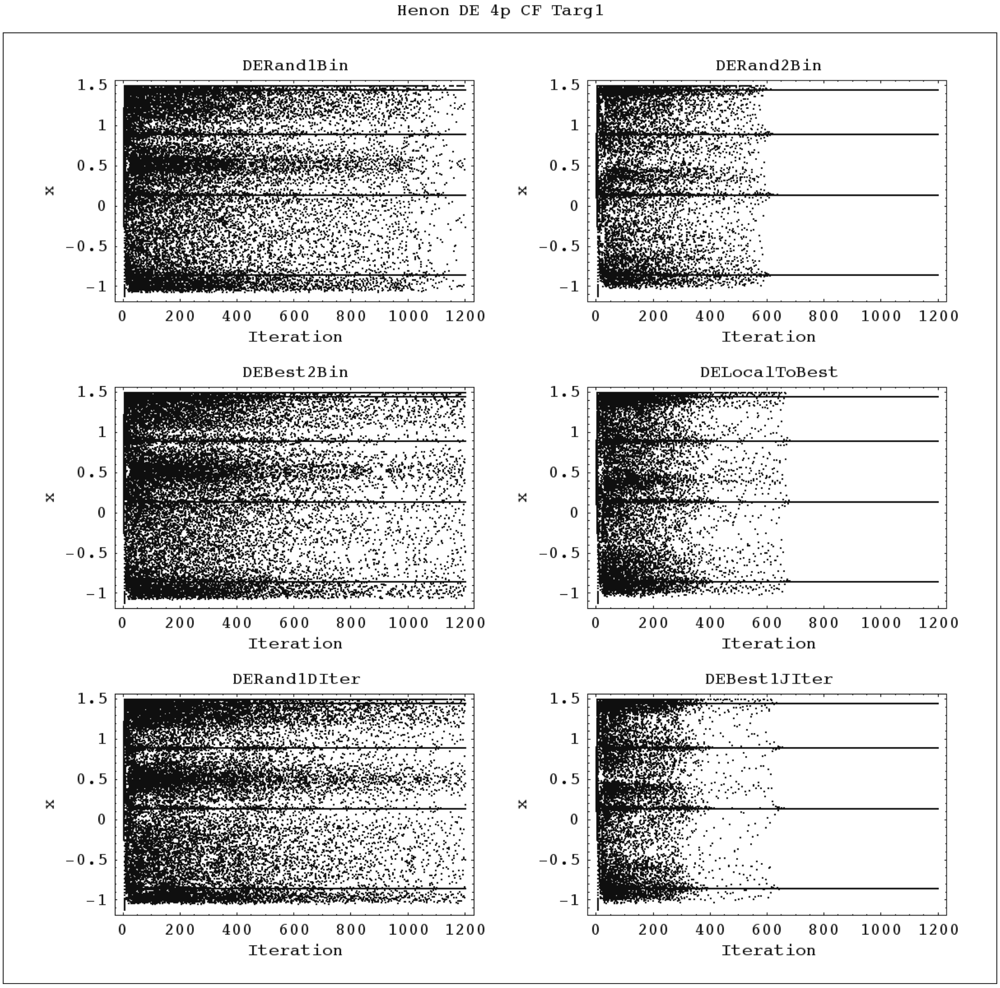
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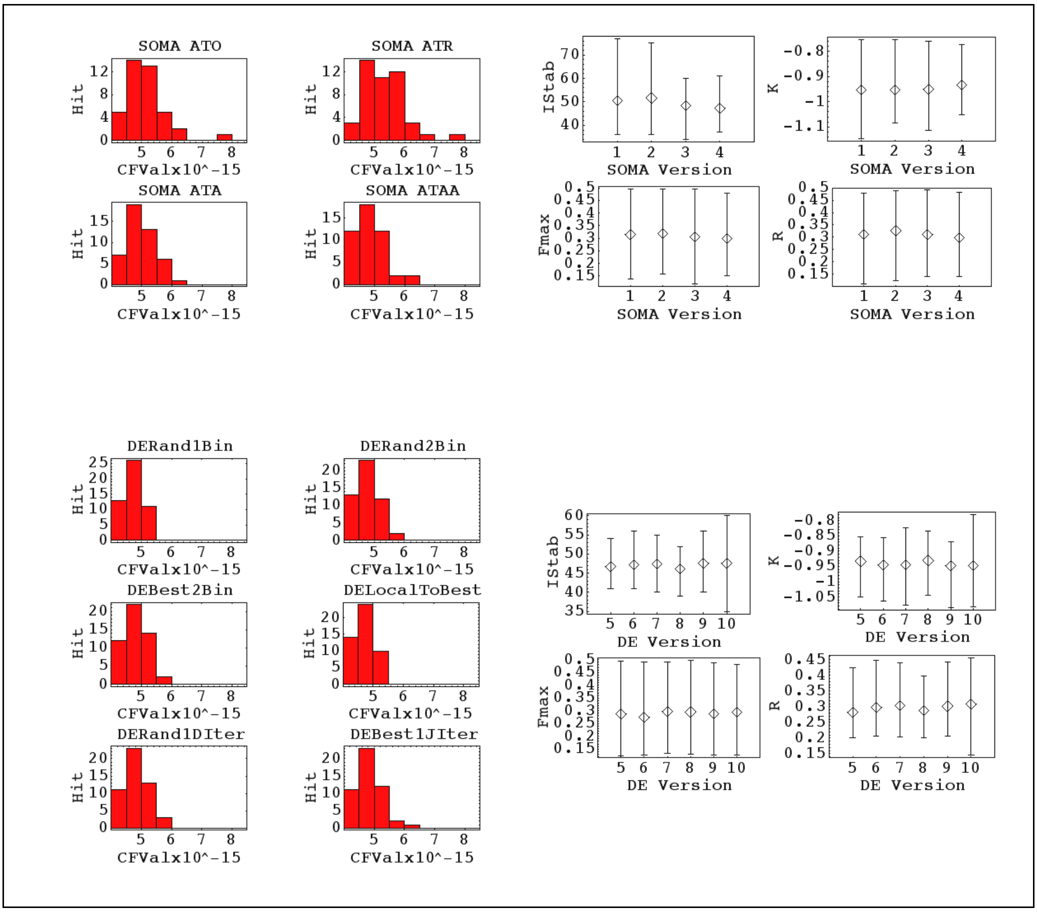
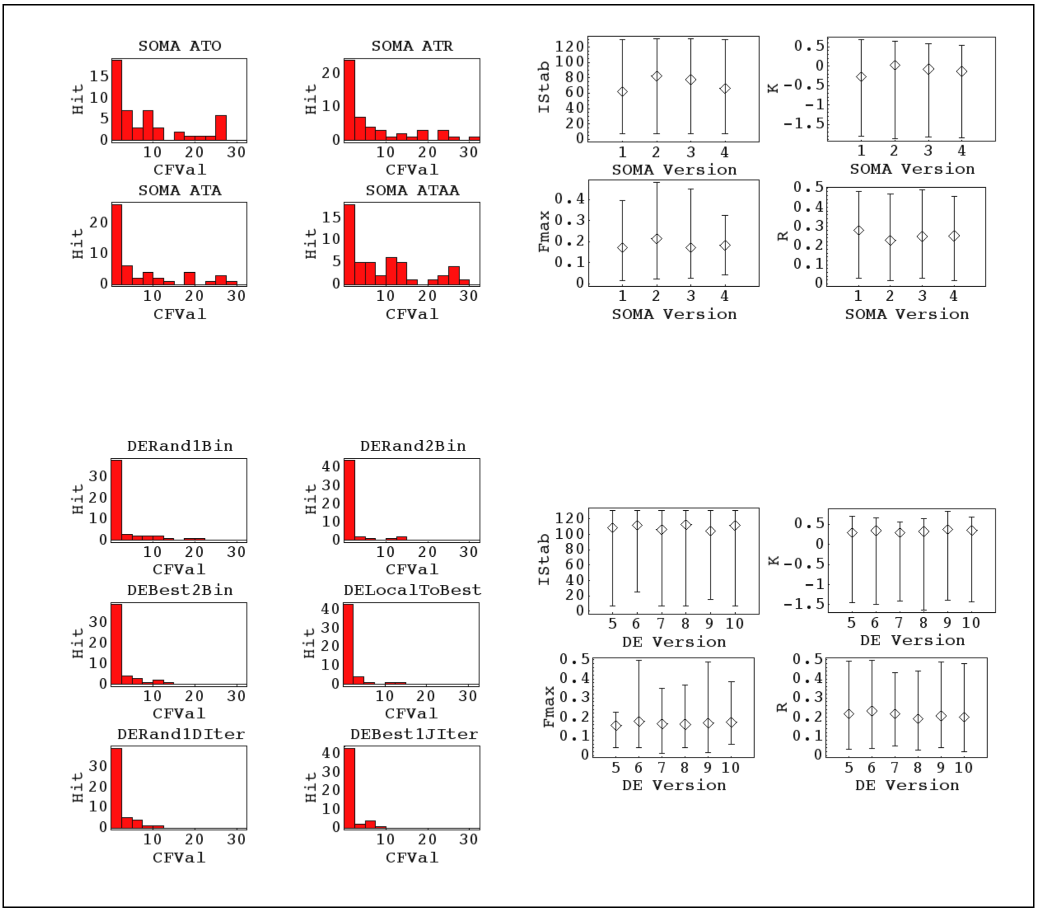
200
400
600
800
1000
1200

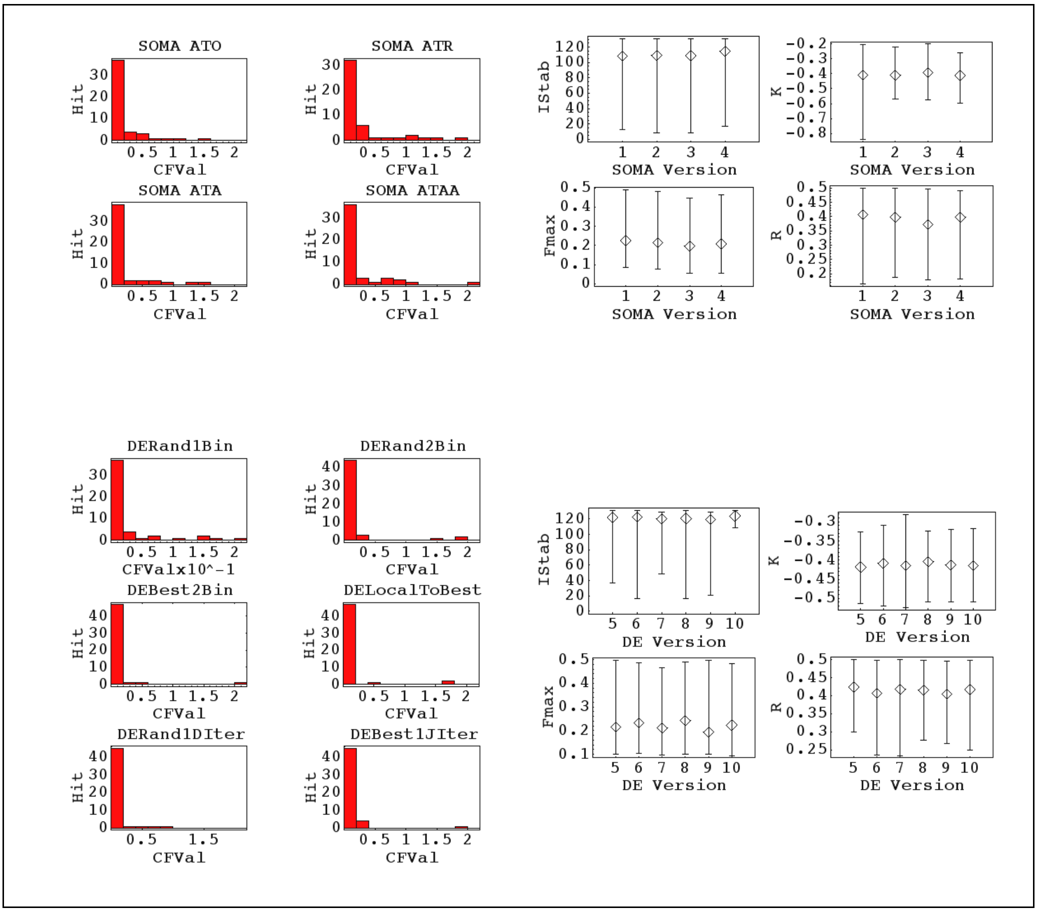
HENON SOMA & DE 1p CF Targ1



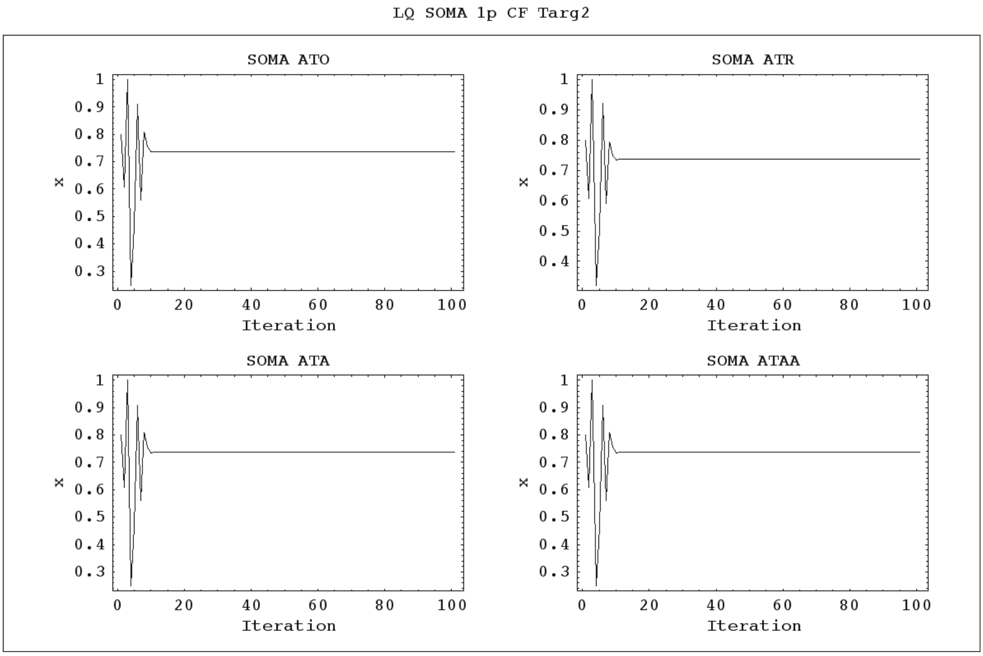
HENON SOMA & DE 2p CF Targ1

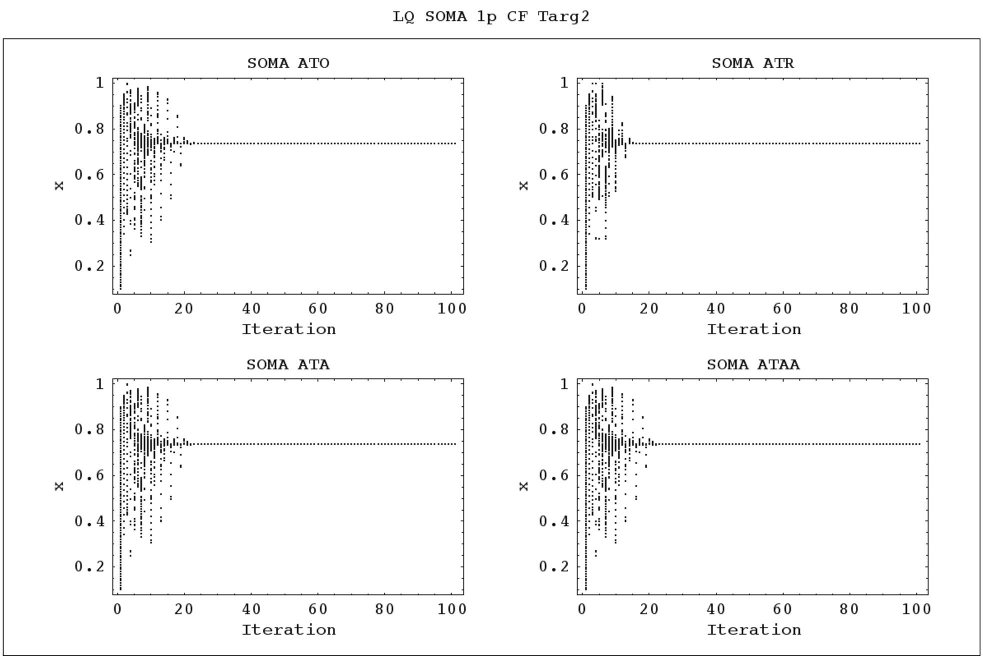


HENON SOMA & DE 4p CF Targ1

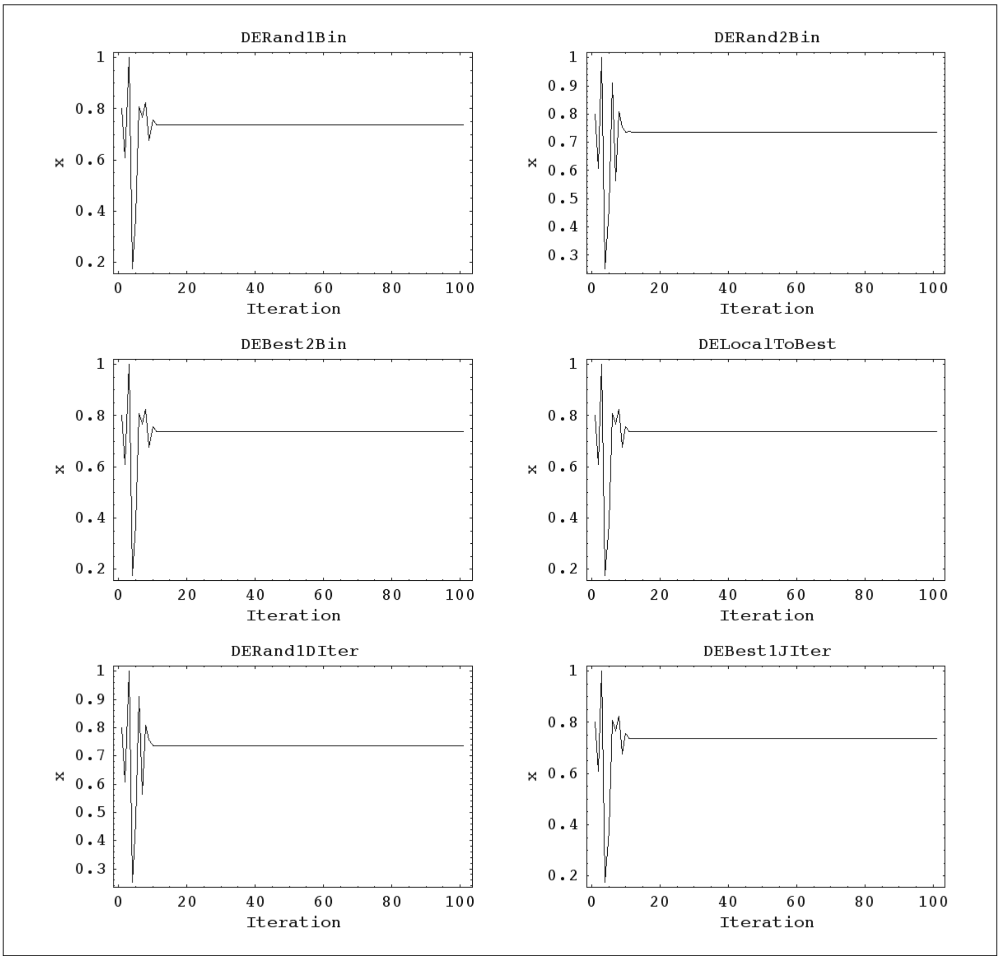


8.15 Summary of results, Case study 5, CF Targ2, LQ

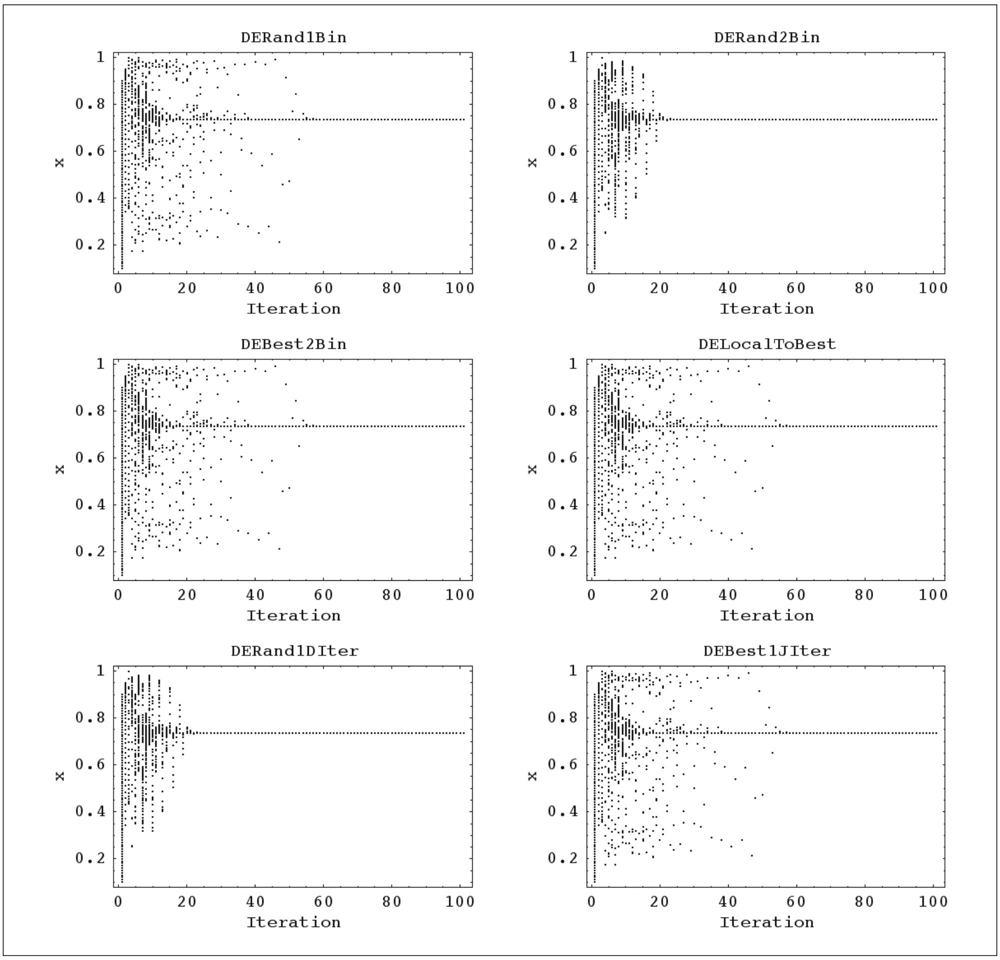


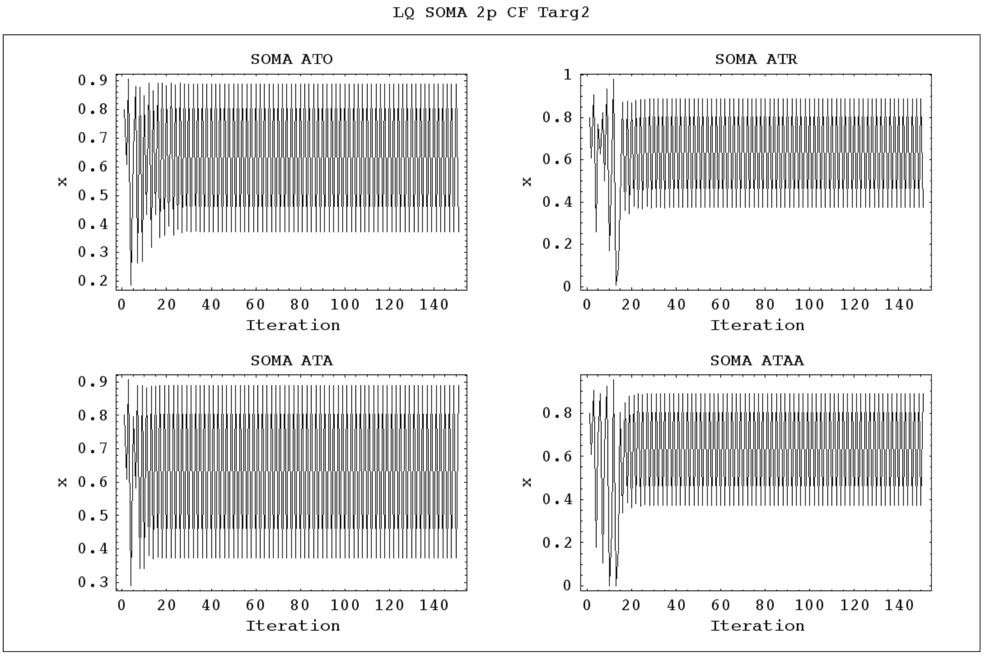


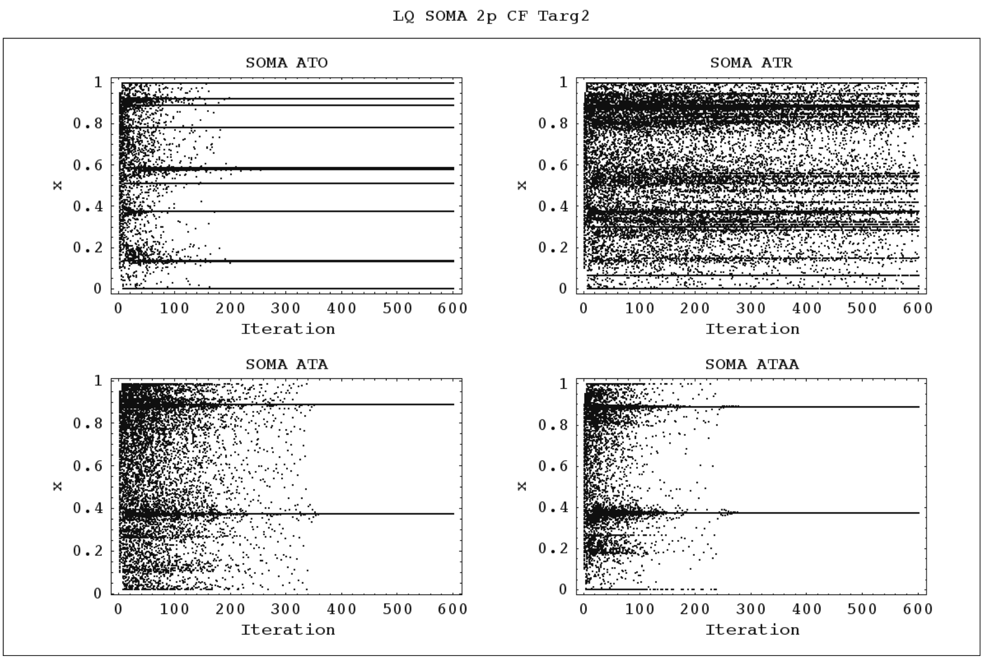
LQ DE 1p CF Targ2

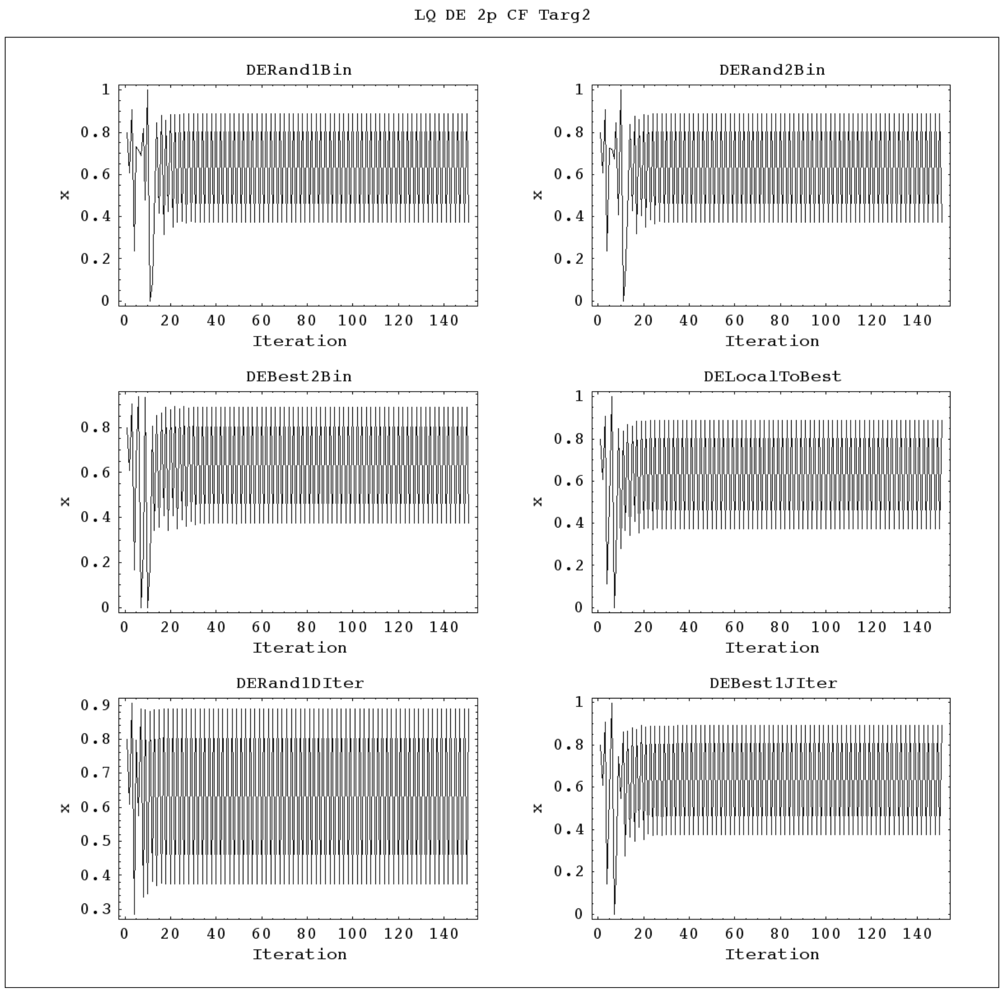


LQ DE 1p CF Targ2



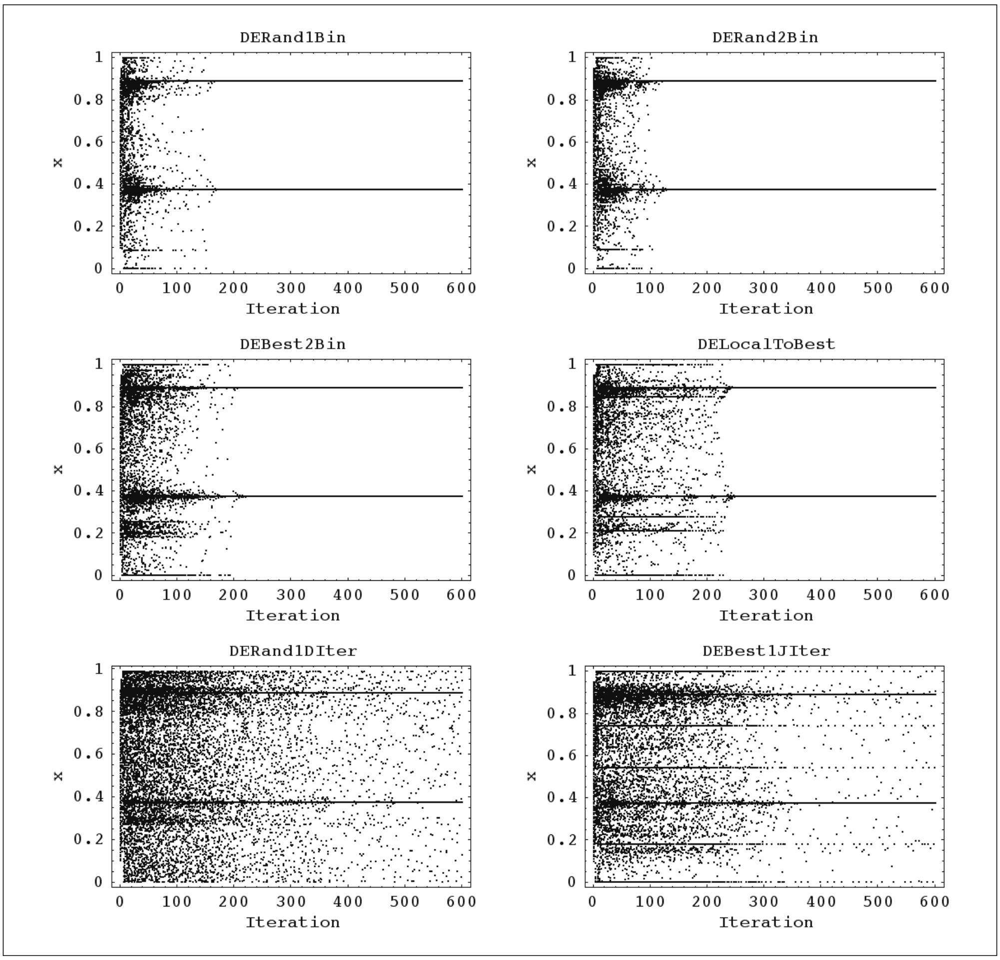

LQ SOMA 2p CF Targ2
SOMA ATO
SOMA ATR
SOMA ATA
SOMA ATAA
x
Iteration


LQ SOMA 2p CF Targ2
SOMA ATO
SOMA ATR
SOMA ATA
SOMA ATAA
x
Iteration

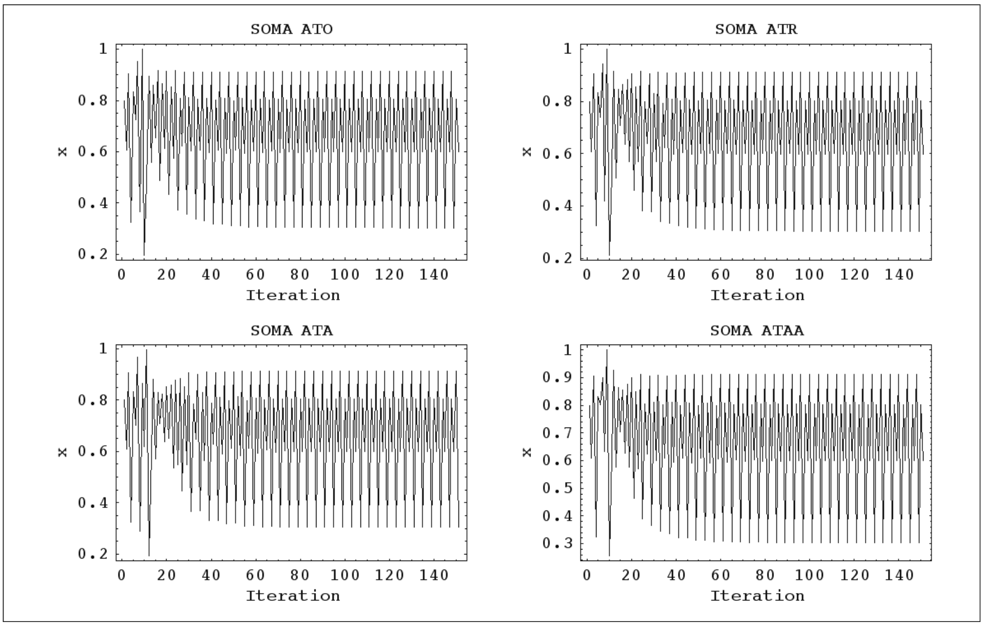


LQ DE 2p CF Targ2
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration

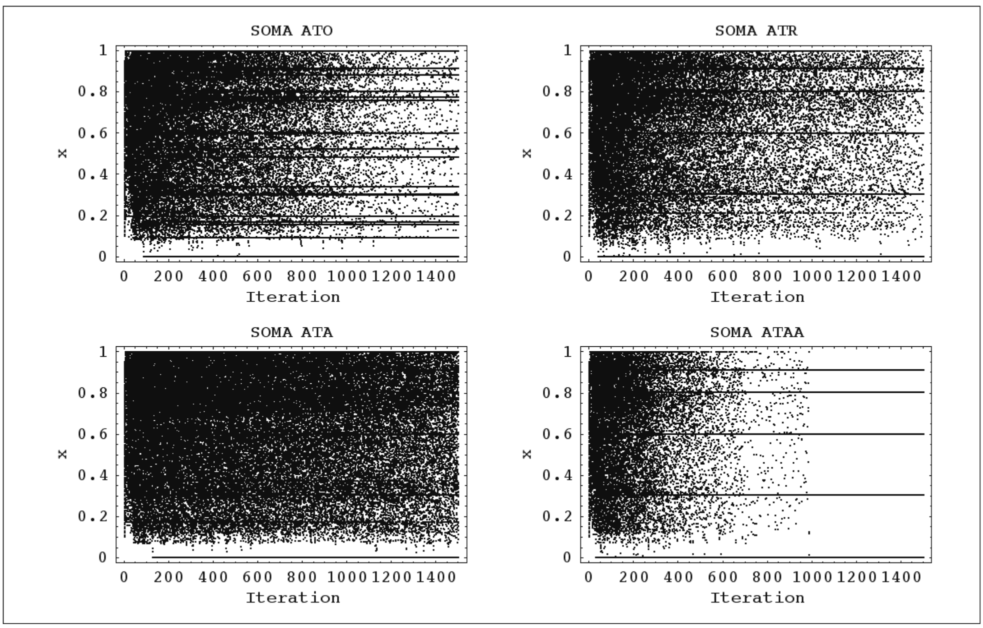
LQ DE 2p CF Targ2



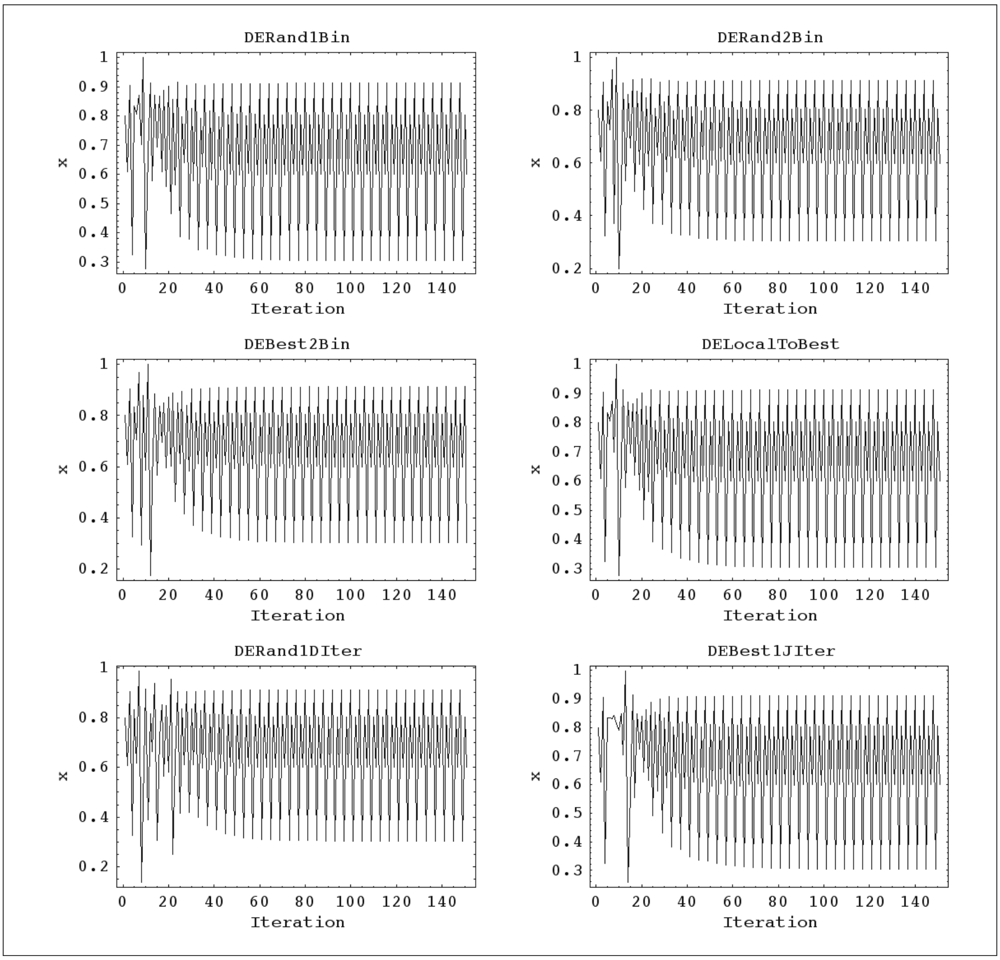
LQ SOMA 4p CF Targ2



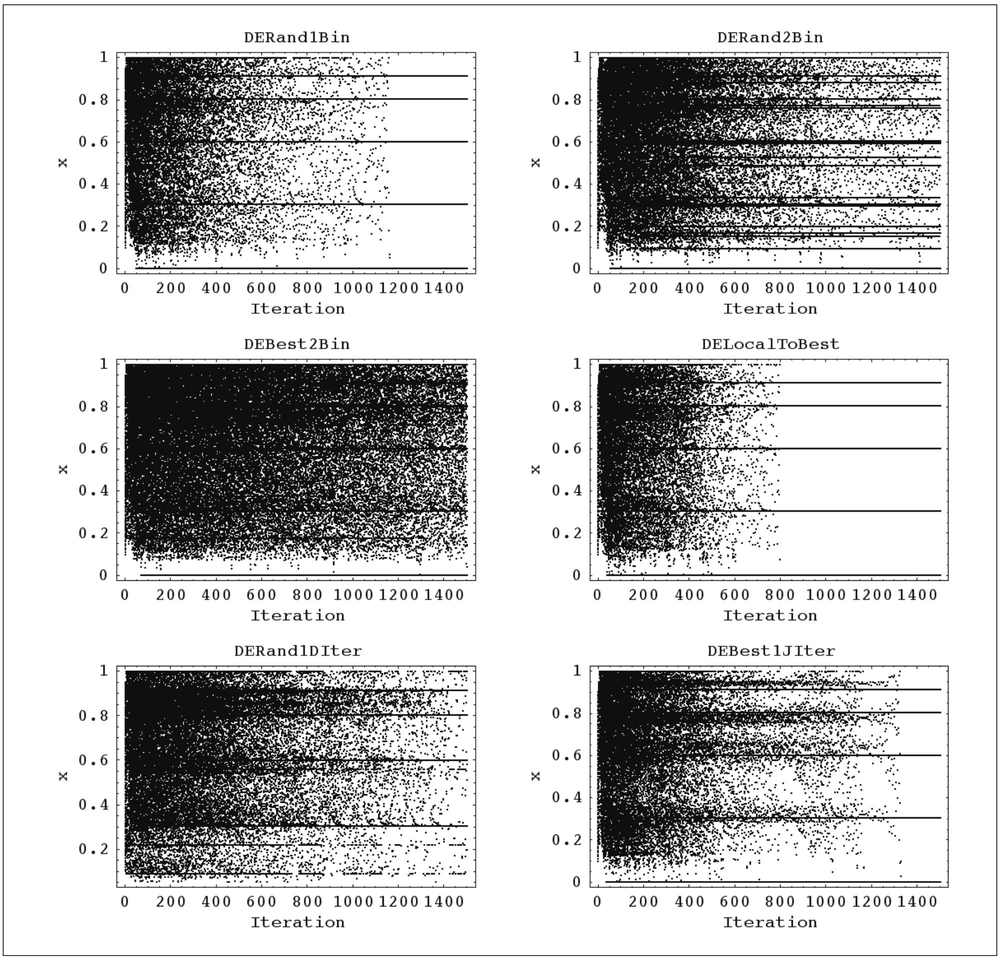
LQ SOMA 4p CF Targ2



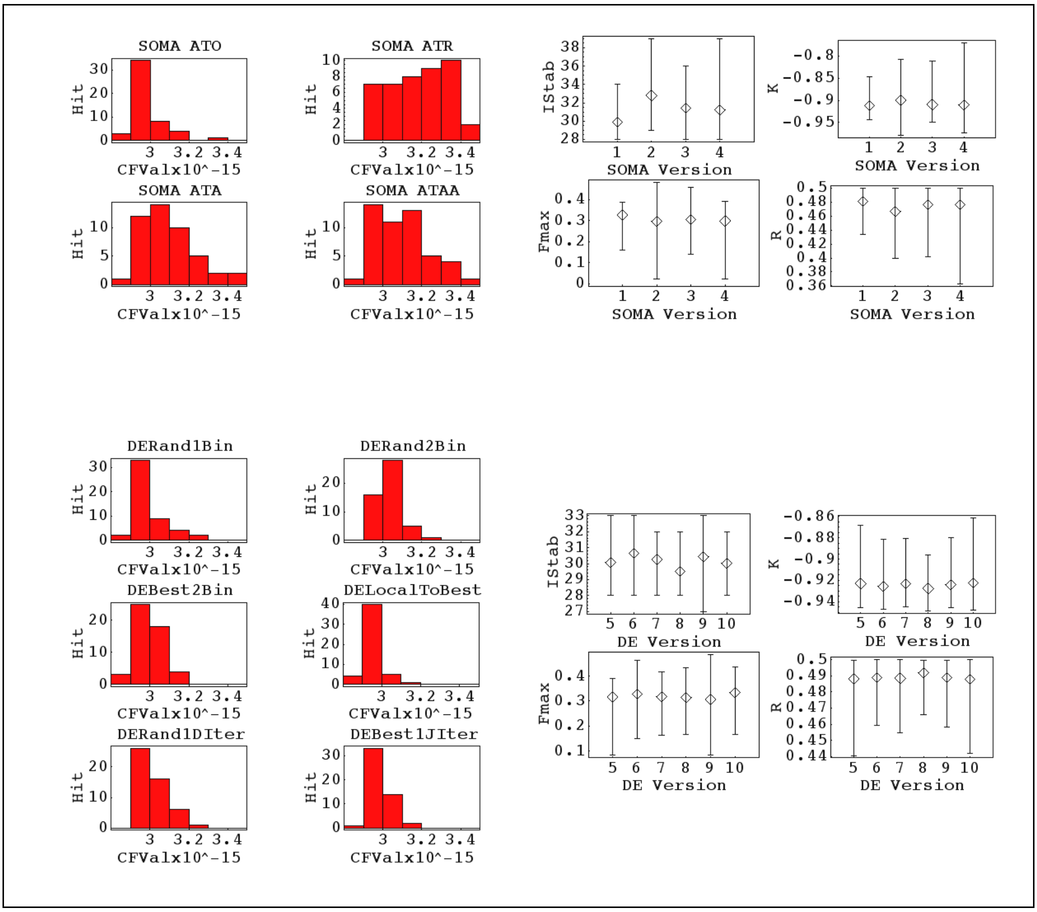
LQ DE 4p CF Targ2



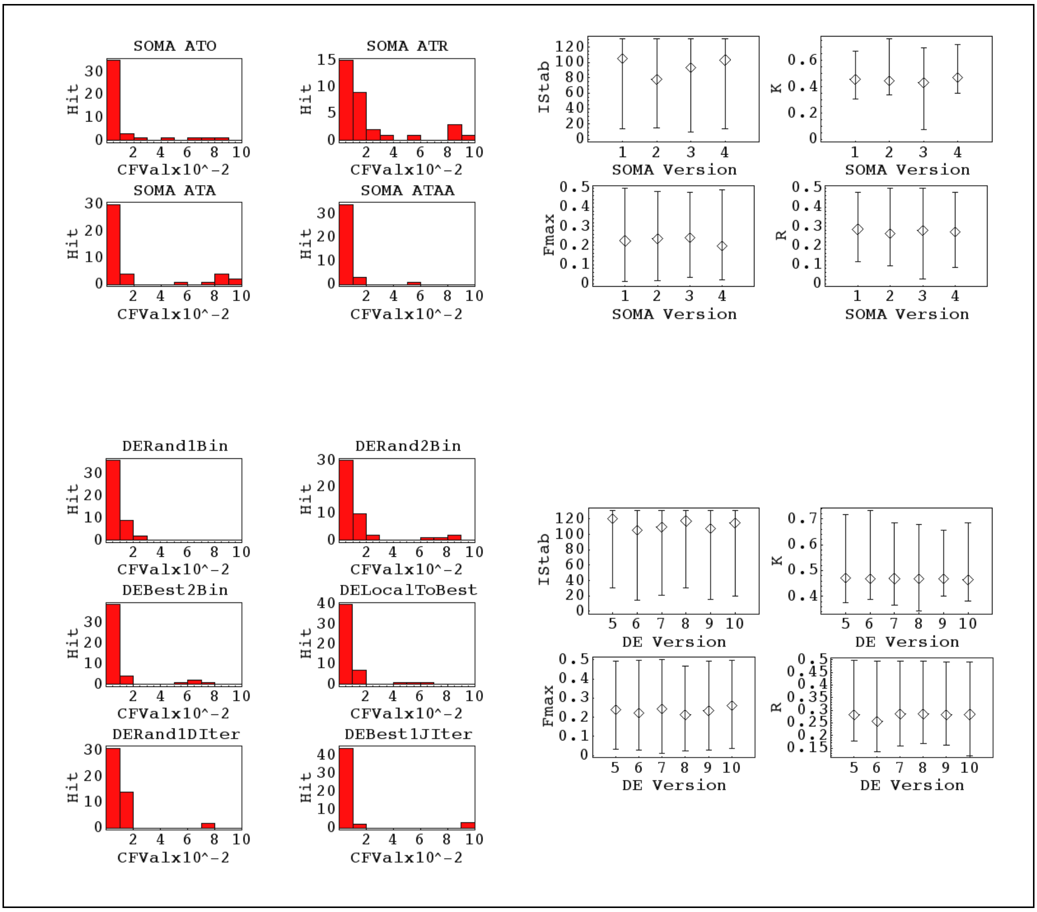
LQ DE 4p CF Targ2



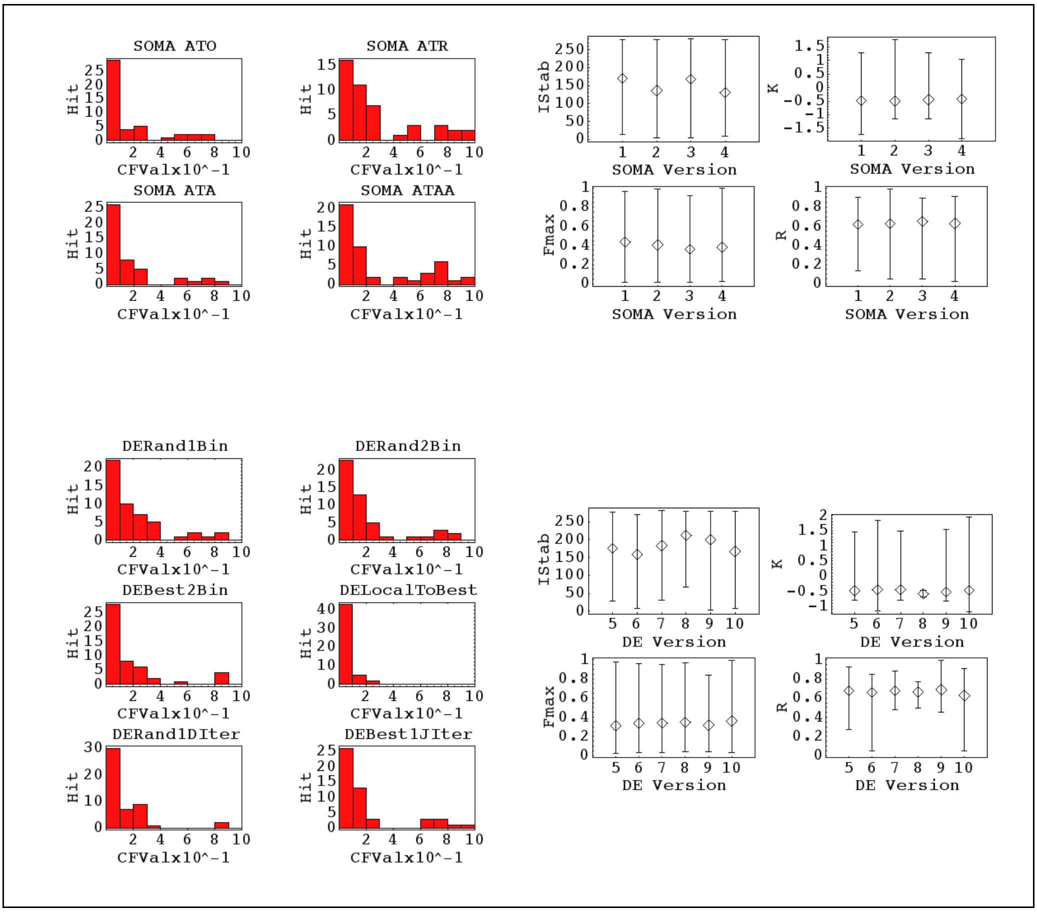
LQ SOMA & DE 1p CF Targ2



LQ SOMA & DE 2p CF Targ2

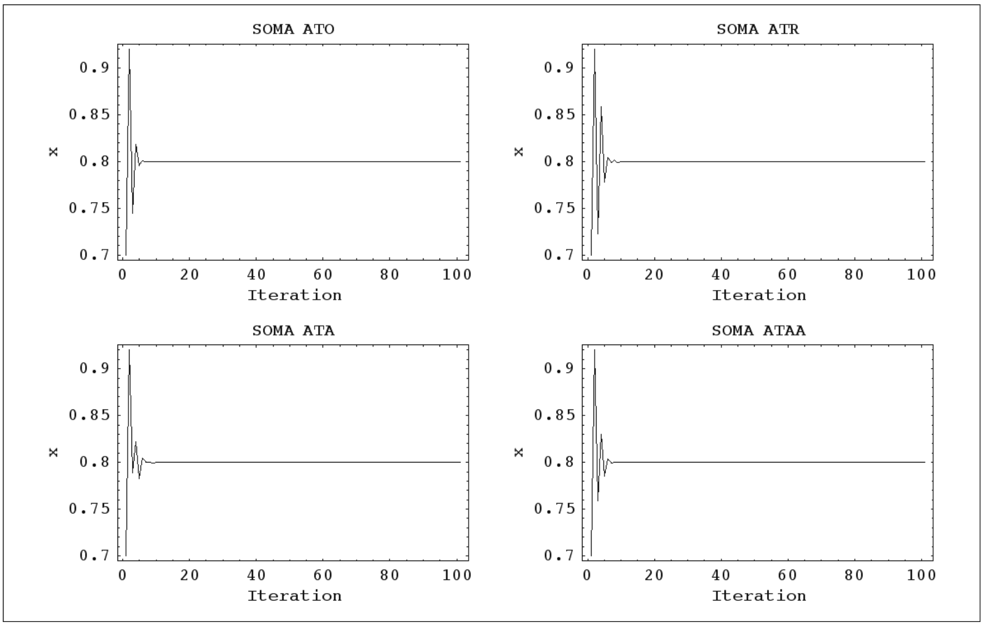


LQ SOMA & DE 4p CF Targ2

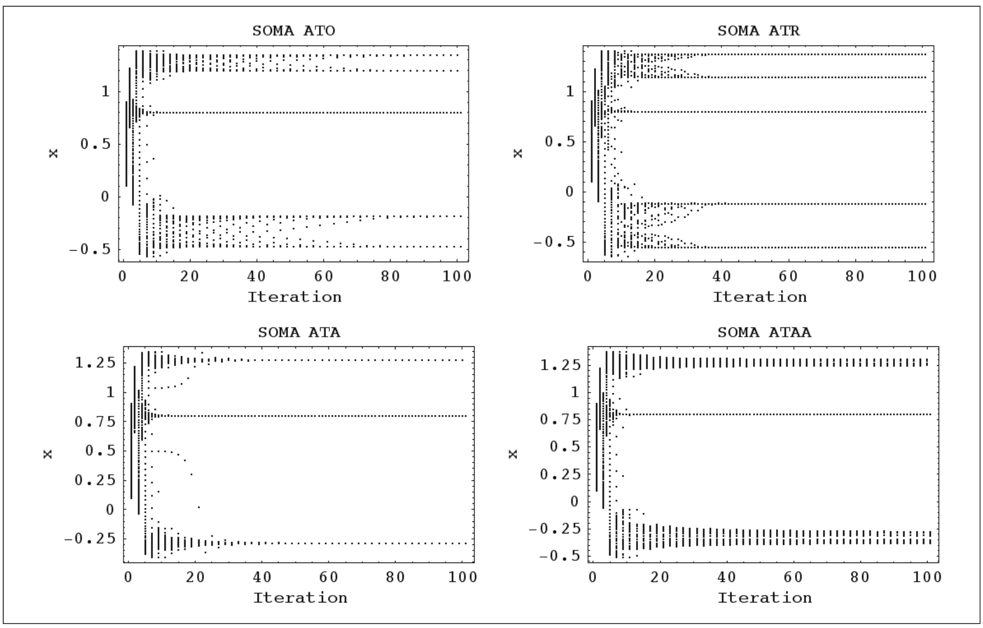


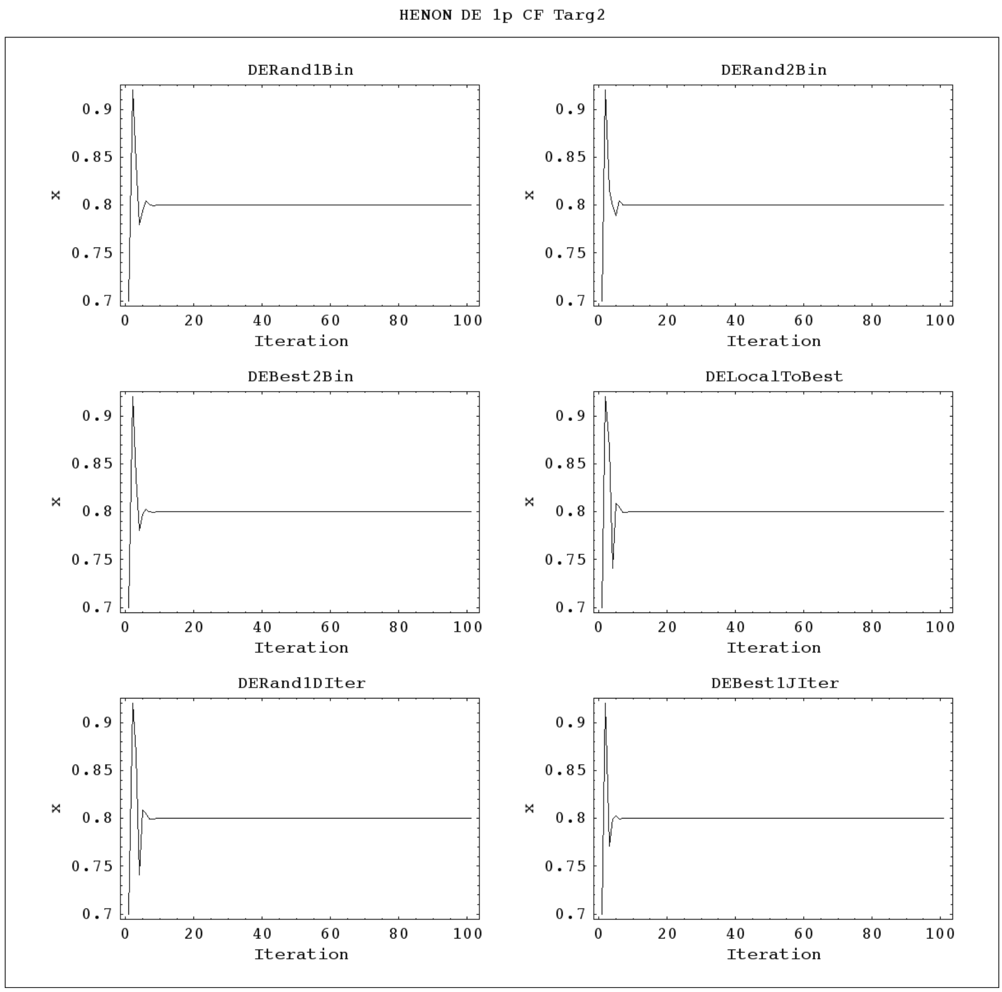
8.16 Summary of results, Case study 5, CF Targ2, HENON

HENON SOMA 1p CF Targ2

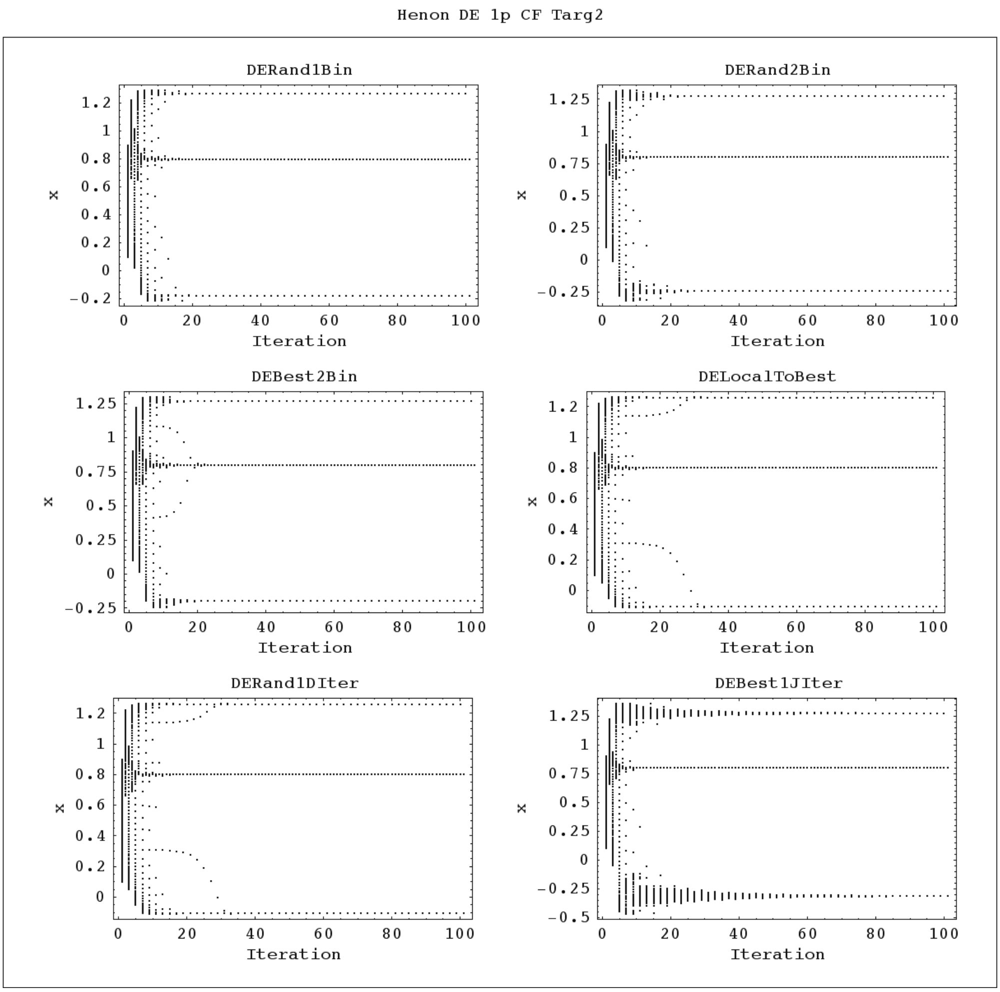


Henon SOMA 1p CF Targ2

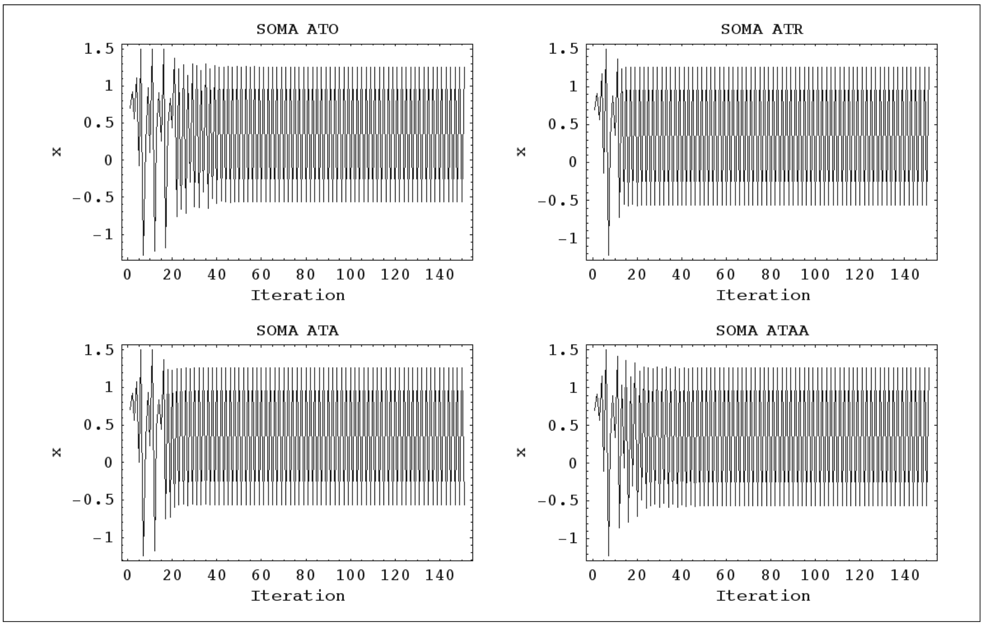




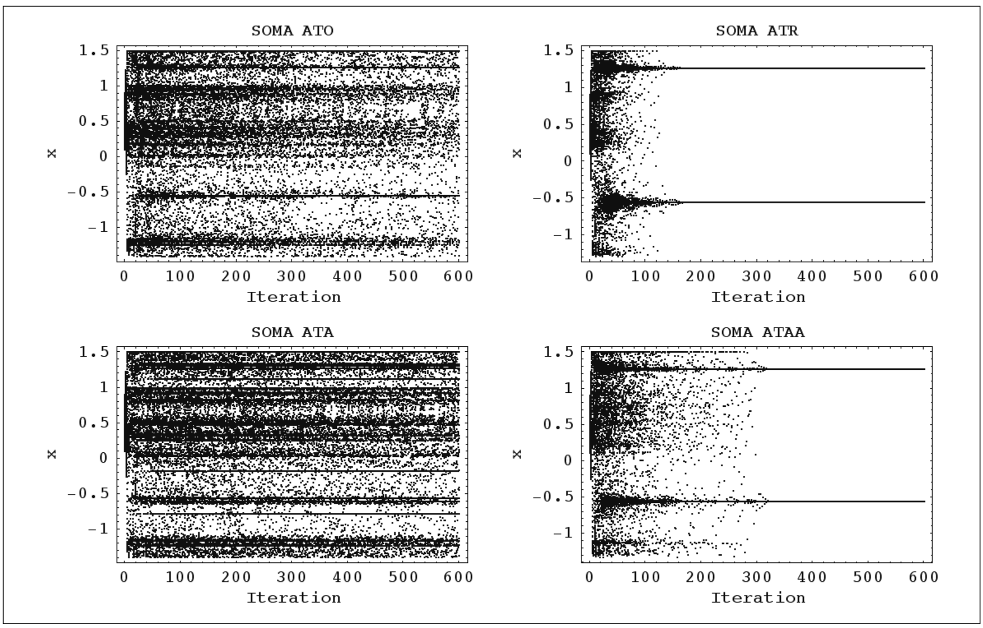
HENON DE 1p CF Targ2
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration
0.7
0.75
0.8
0.85
0.9
0
20
40
60
80
100


Henon DE 1p CF Targ2
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration

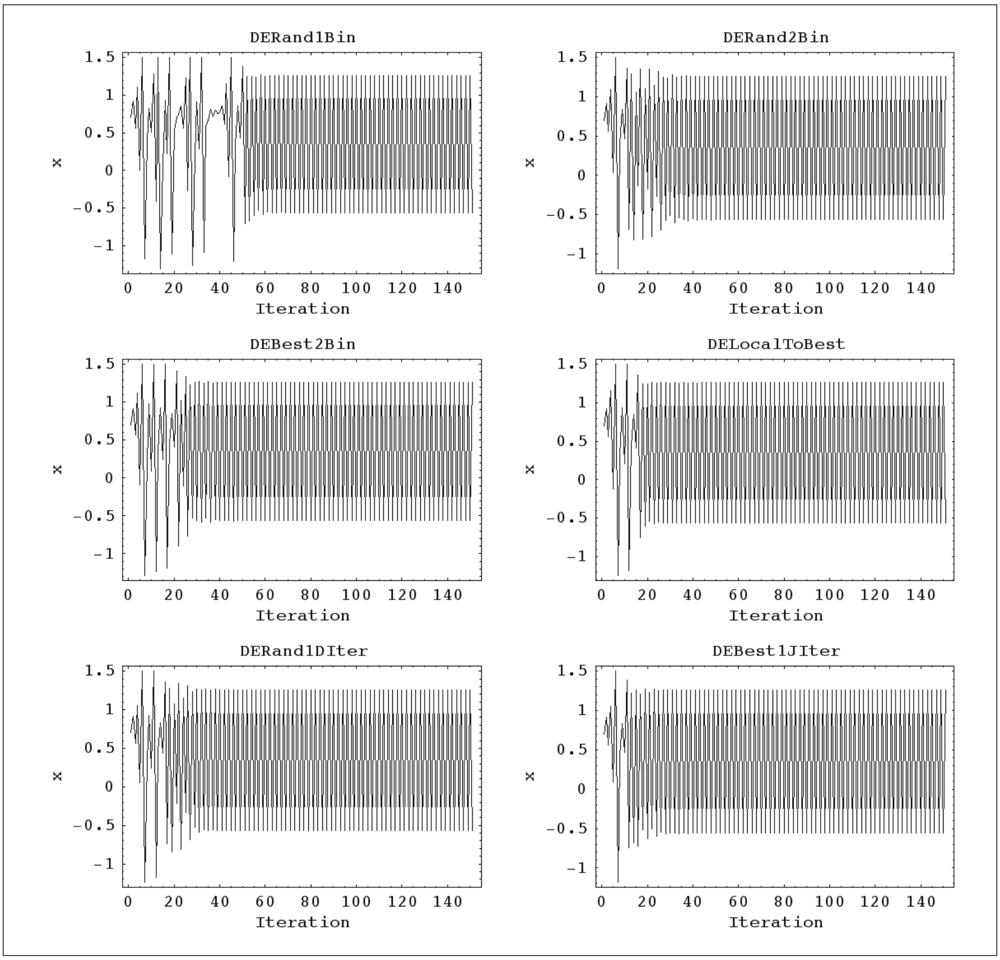
HENON SOMA 2p CF Targ2

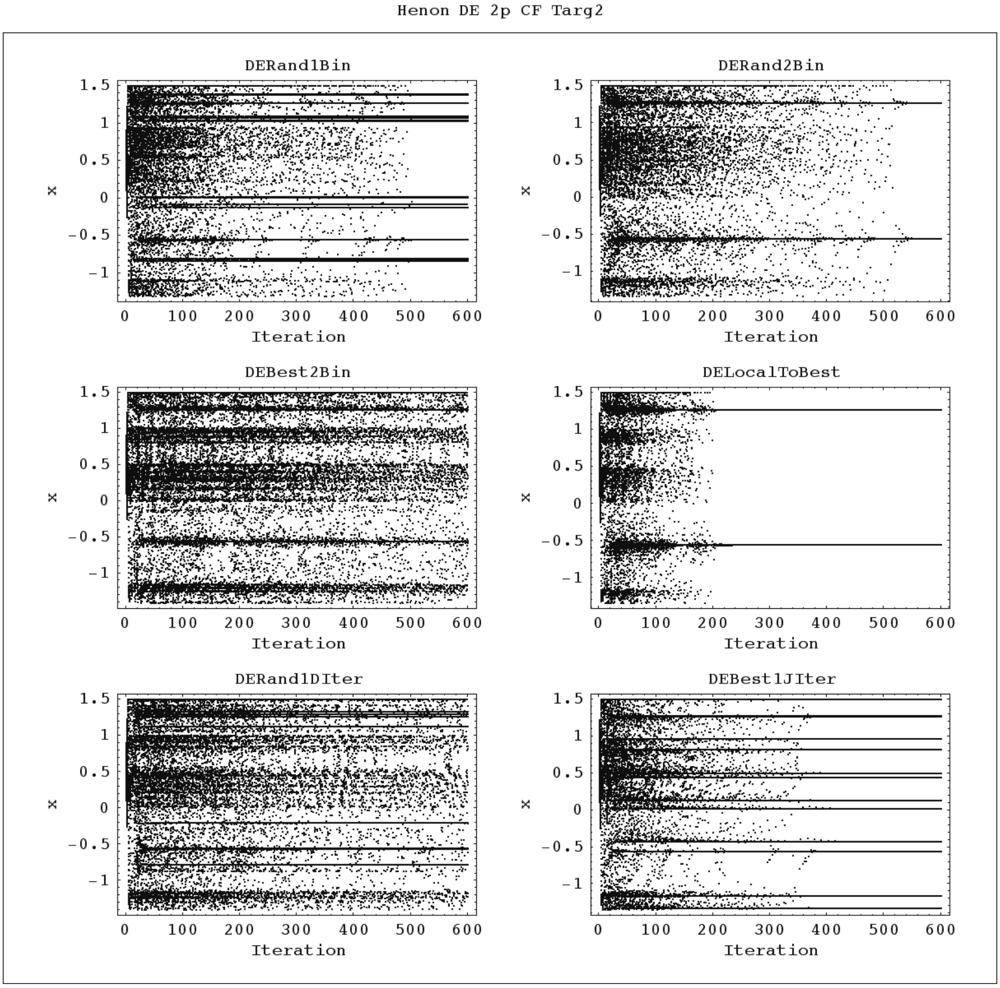


Henon SOMA 2p CF Targ2



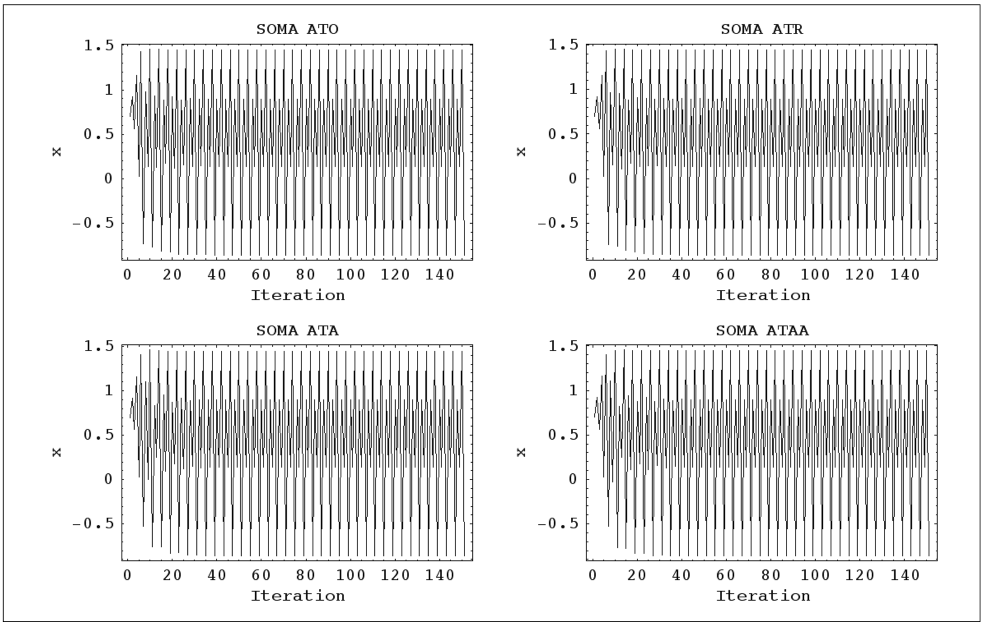
HENON DE 2p CF Targ2



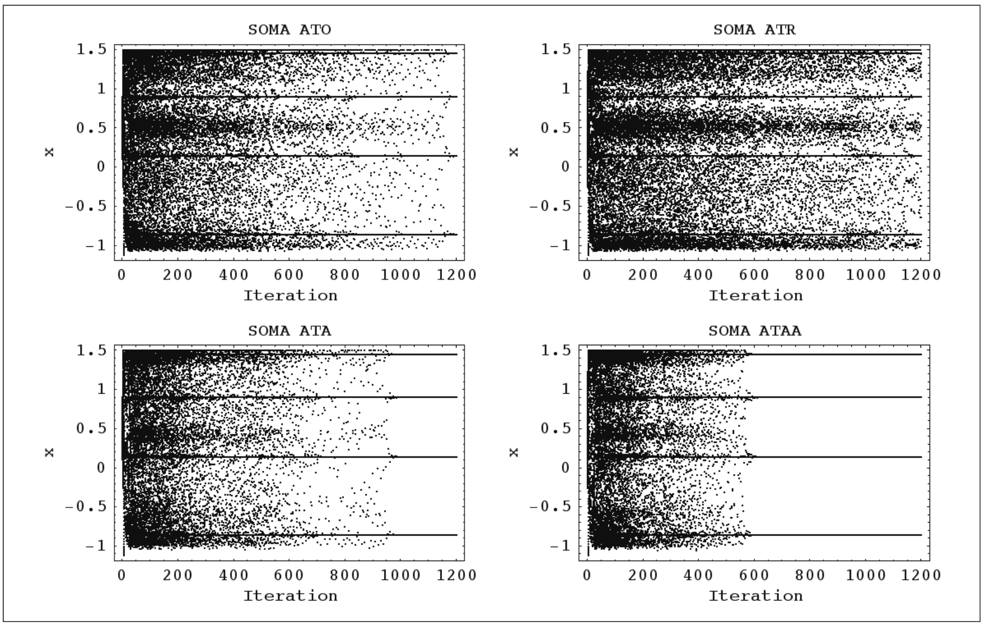


Henon DE 2p CF Targ2
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration
1.5
1
0.5
0
-0.5
-1
0
100
200
300
400
500
600

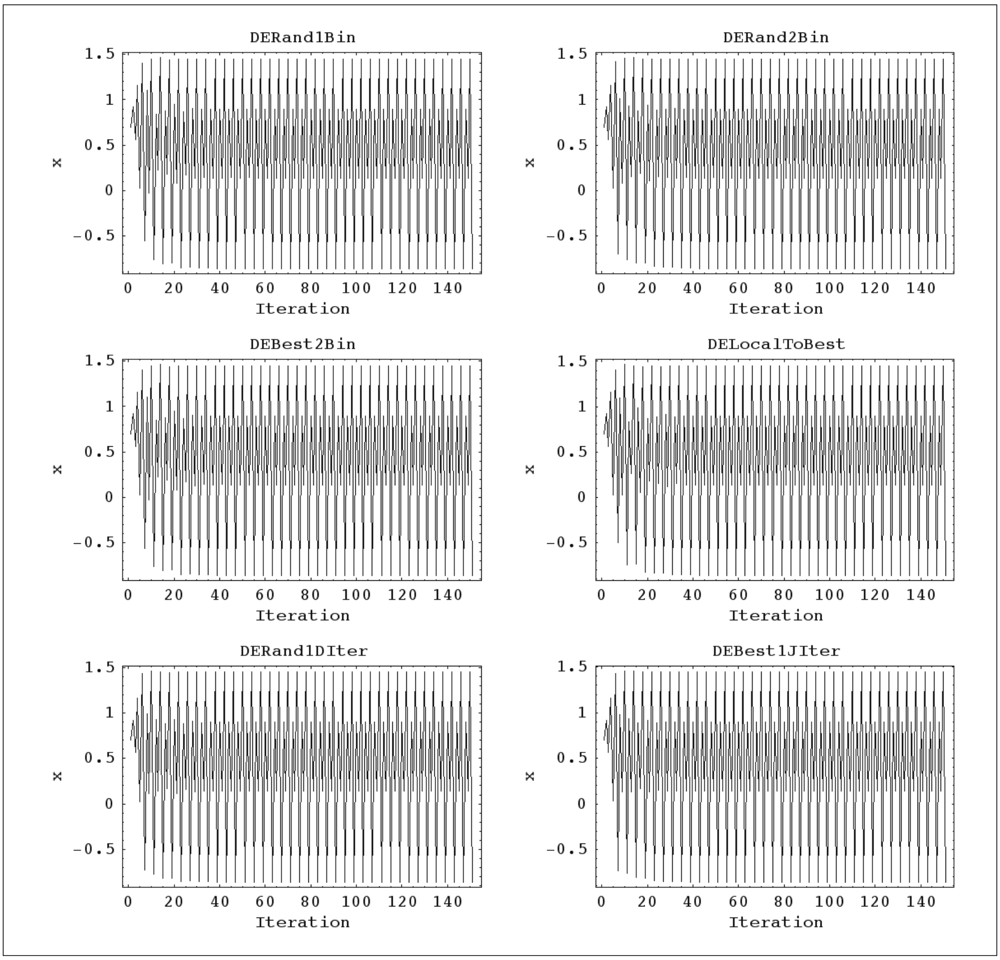
HENON SOMA 4p CF Targ2



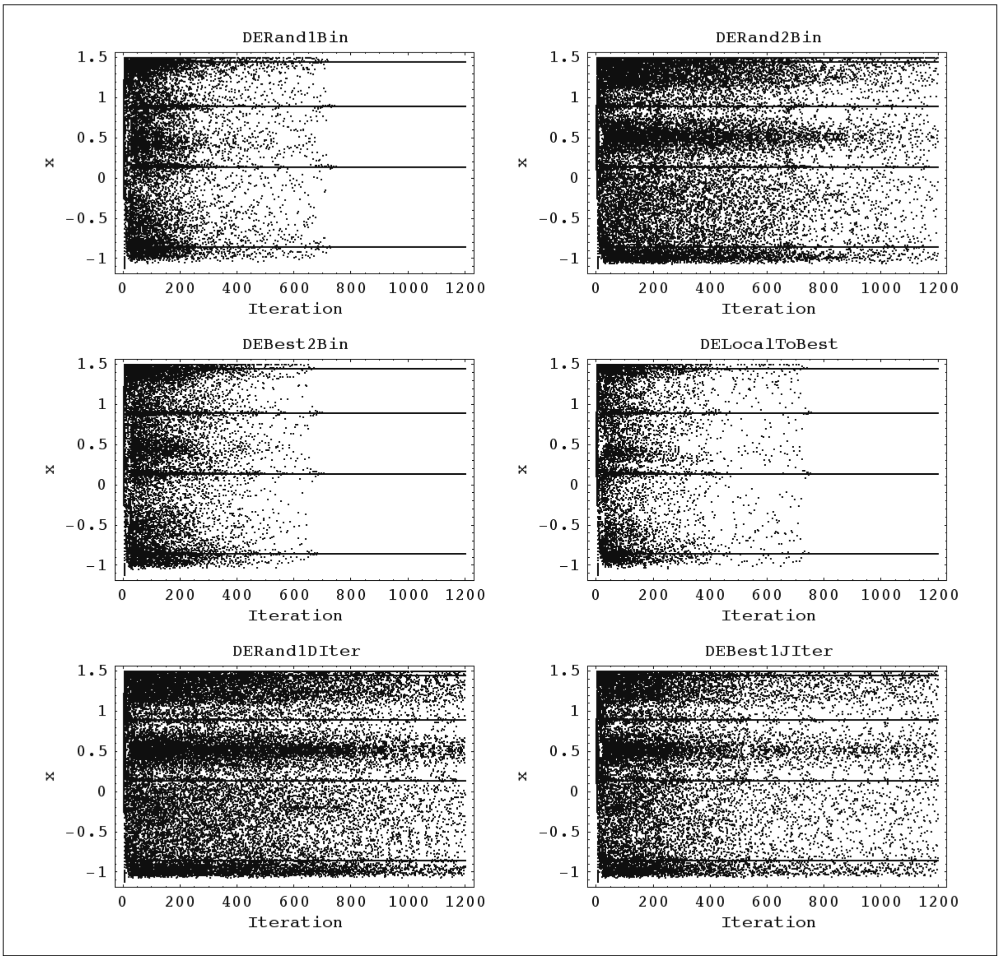
Henon SOMA 4p CF Targ2



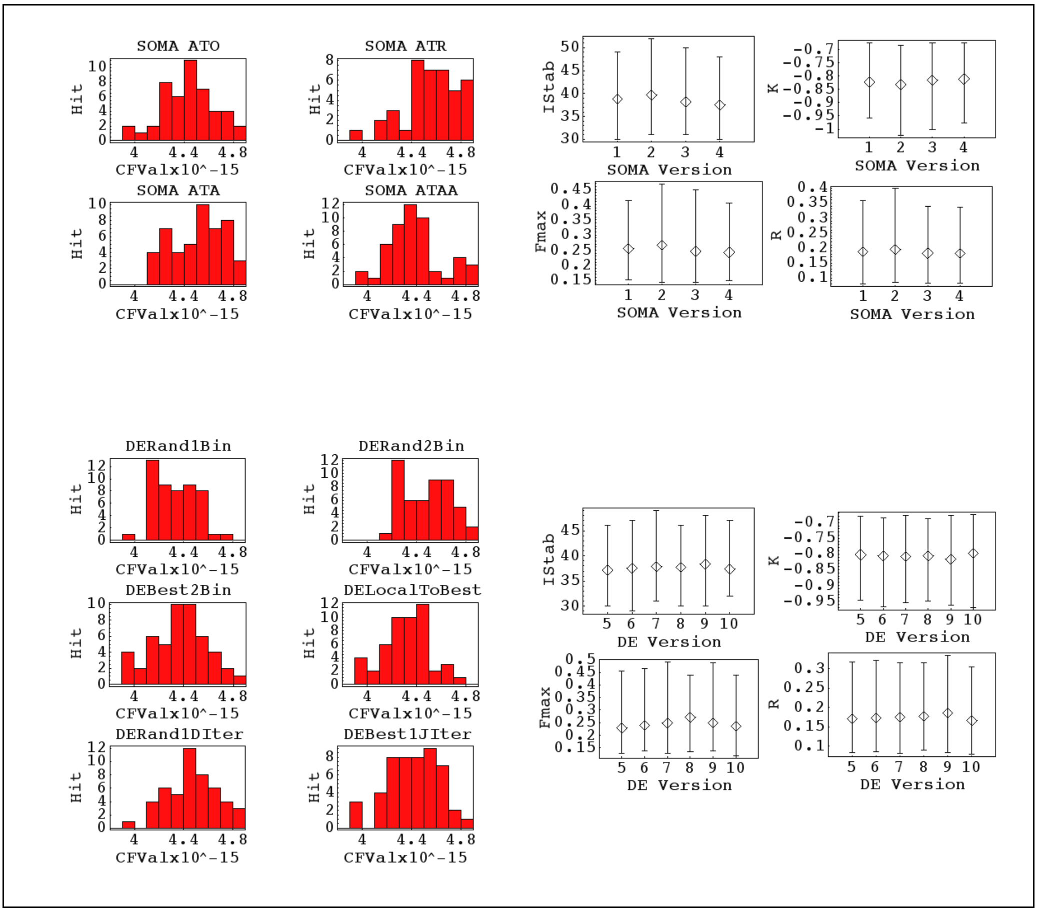
HENON DE 4p CF Targ2



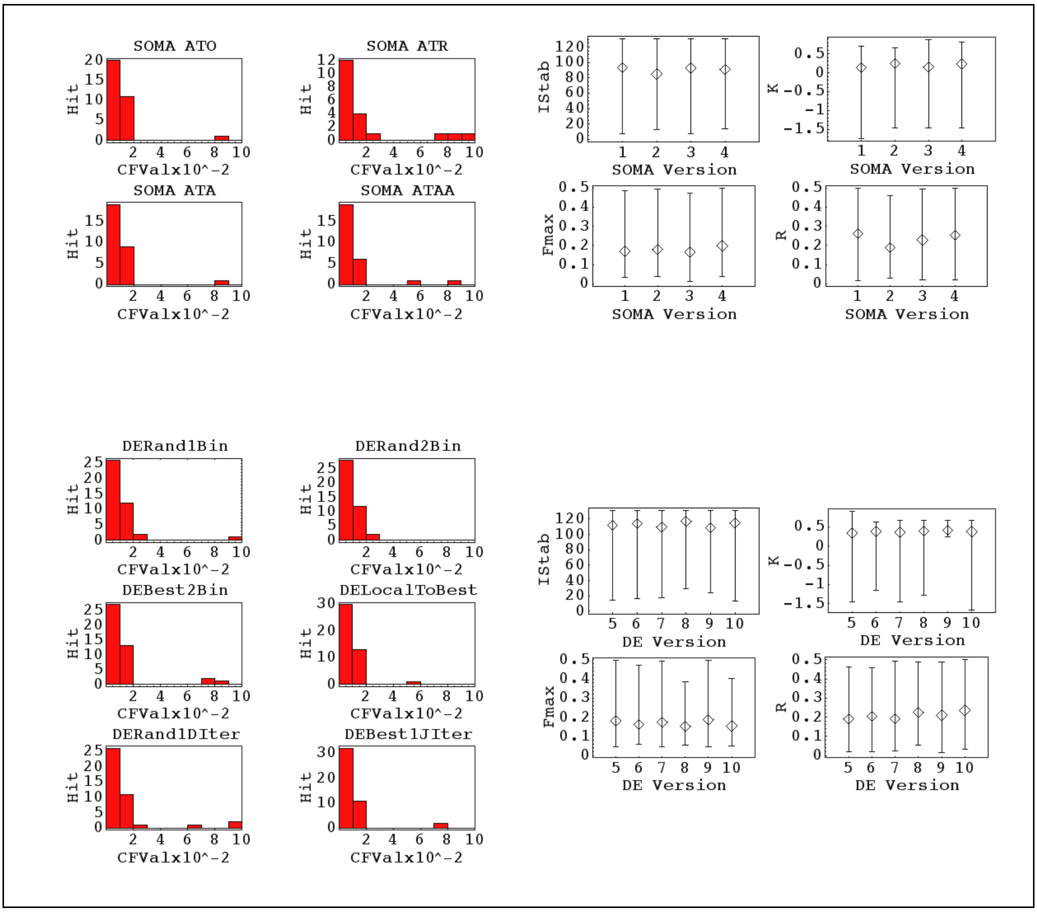
Henon DE 4p CF Targ2



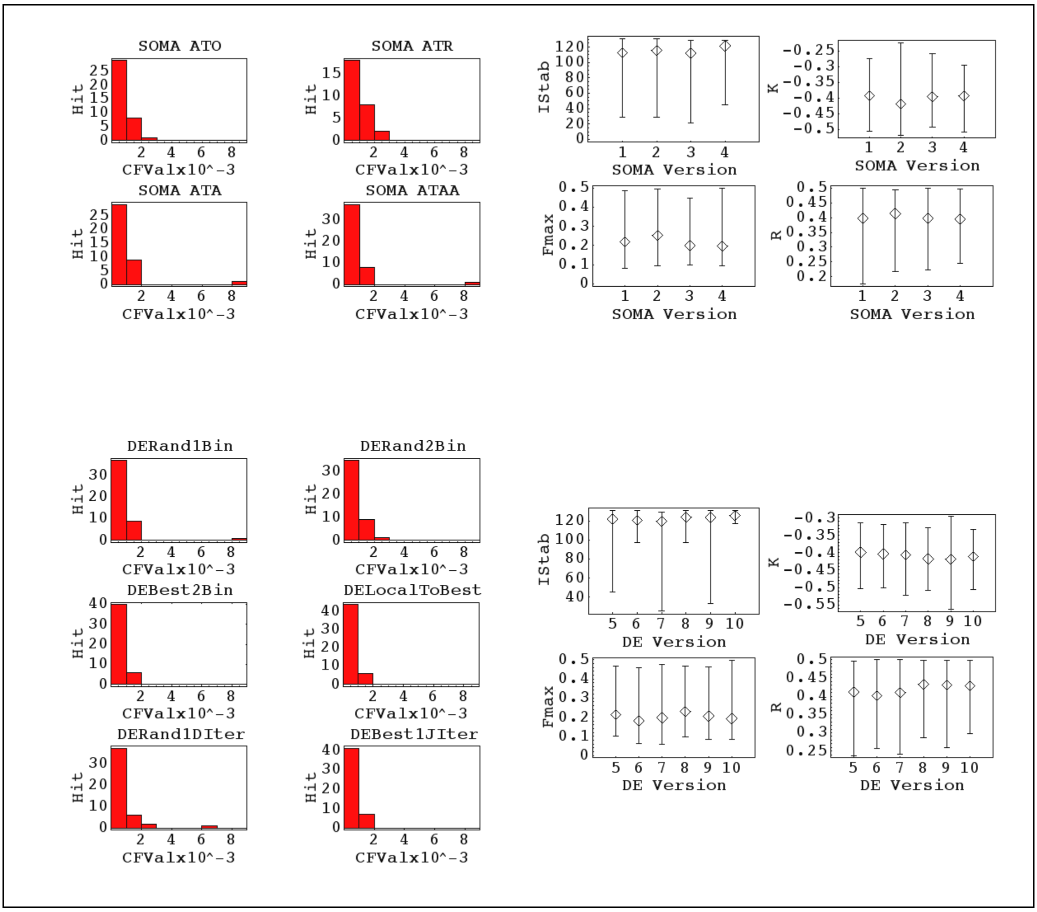
HENON SOMA & DE 1p CF Targ2



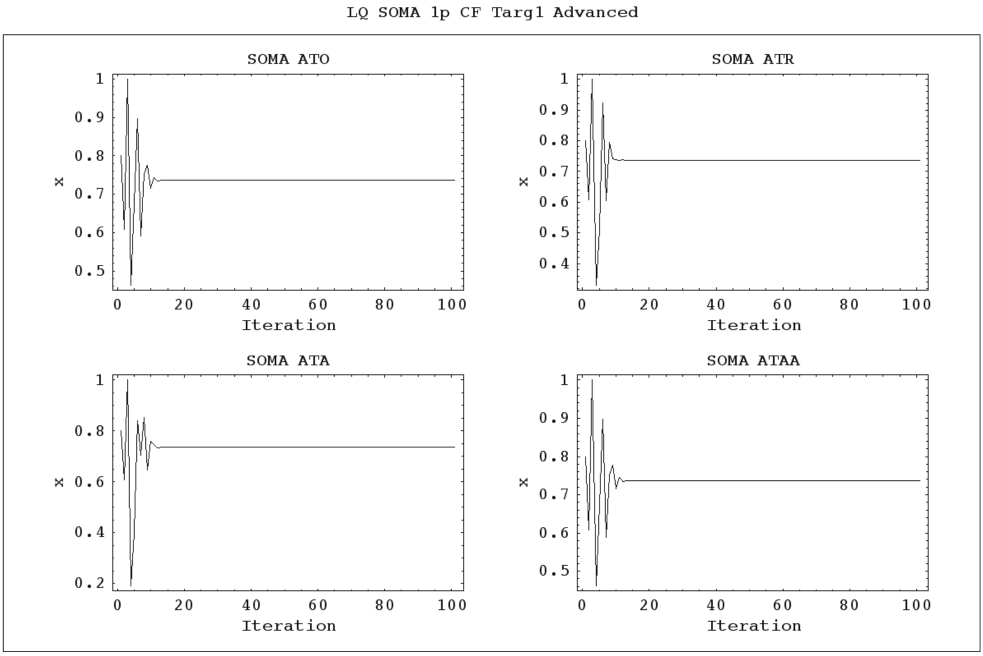
HENON SOMA & DE 2p CF Targ2

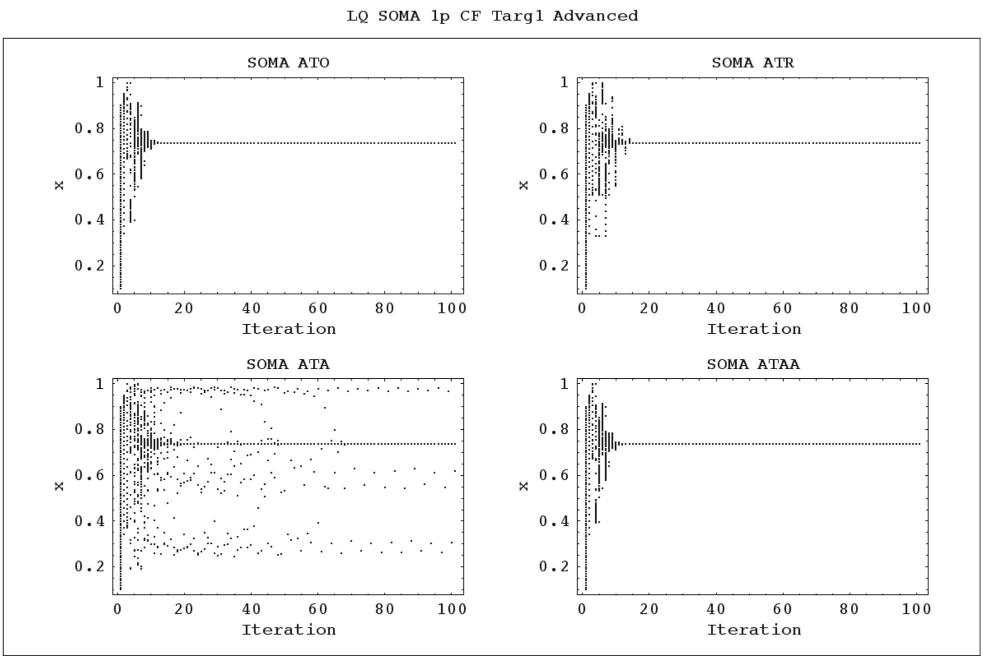


HENON SOMA & DE 4p CF Targ2

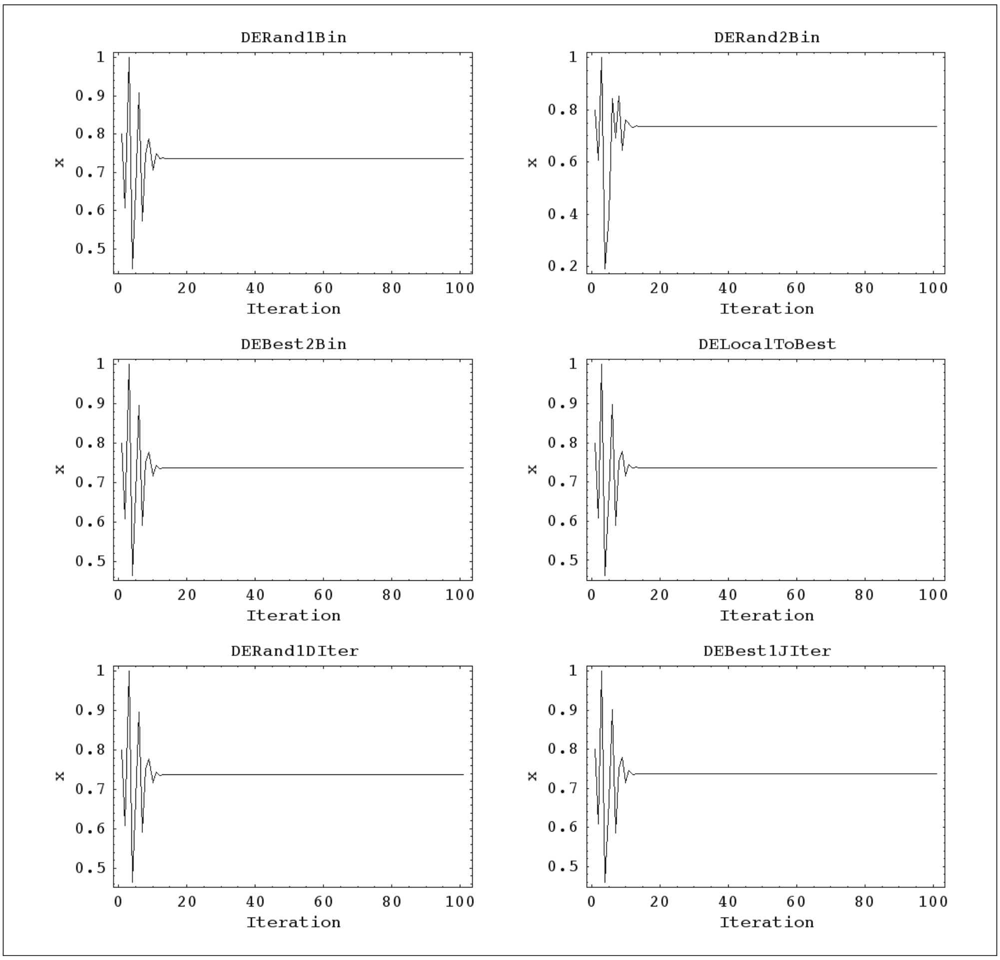


8.17 Summary of results, Case study 6, CF Targ1 Advanced, LQ

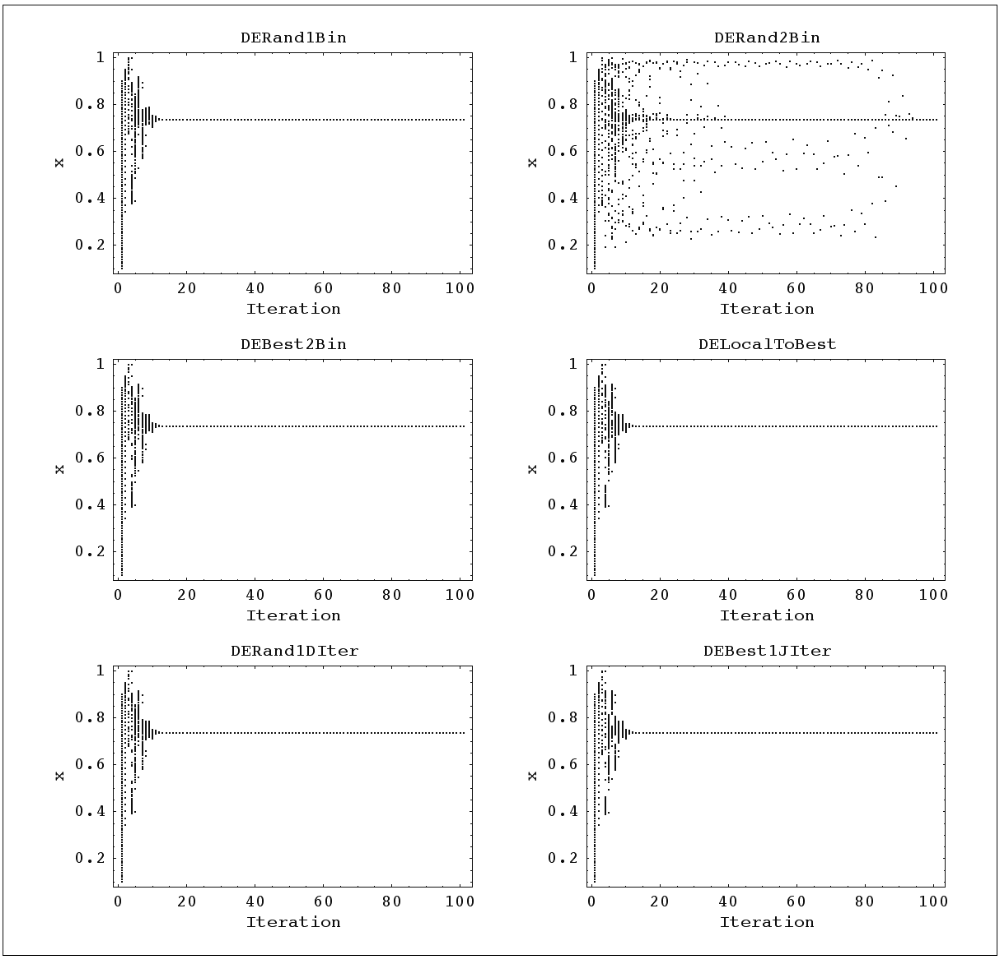




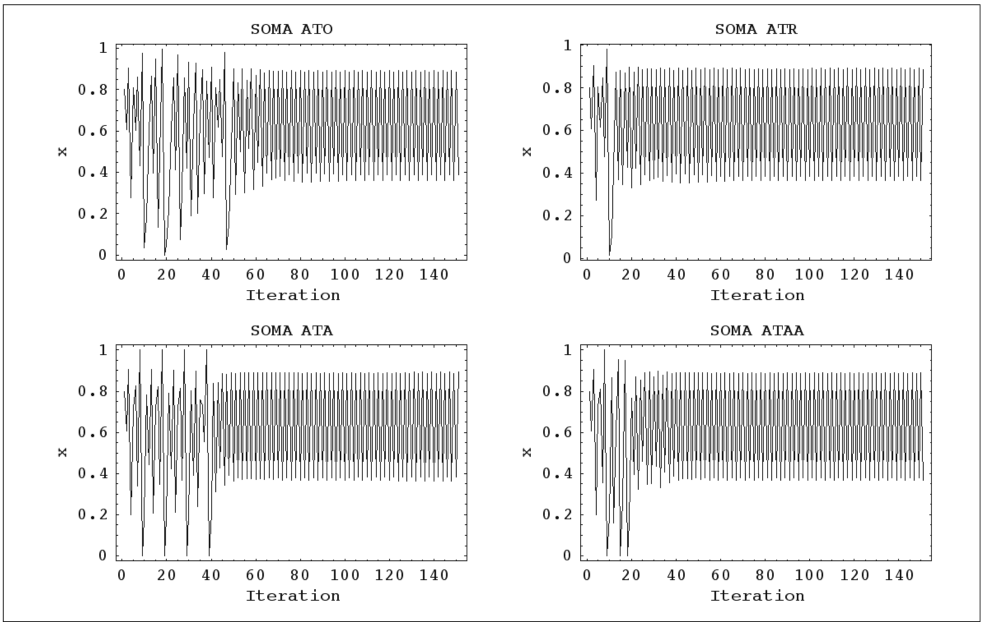
LQ DE 1p CF Targ1 Advanced



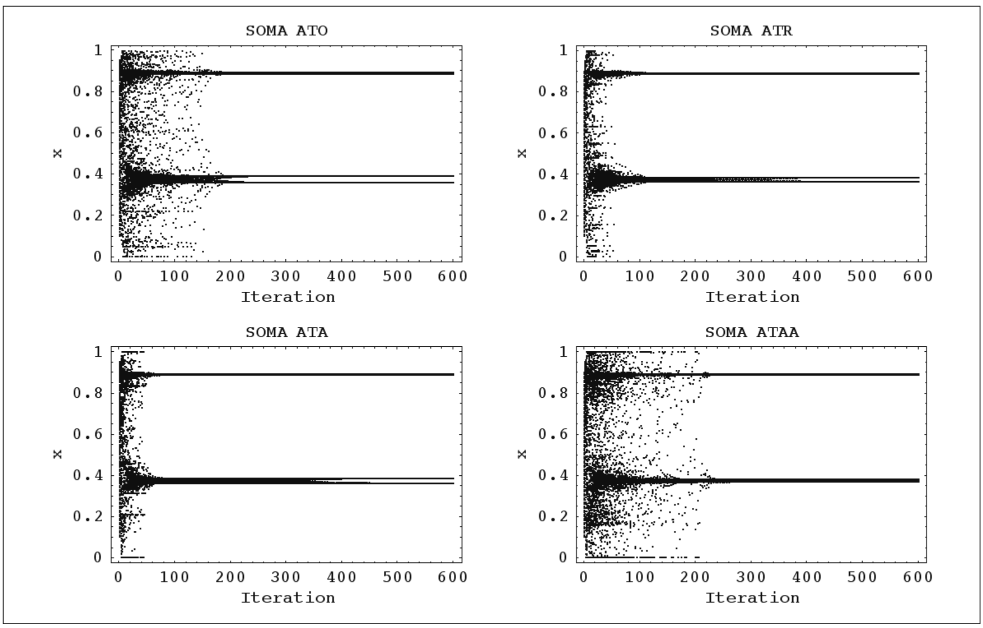
LQ DE 1p CF Targ1 Advanced

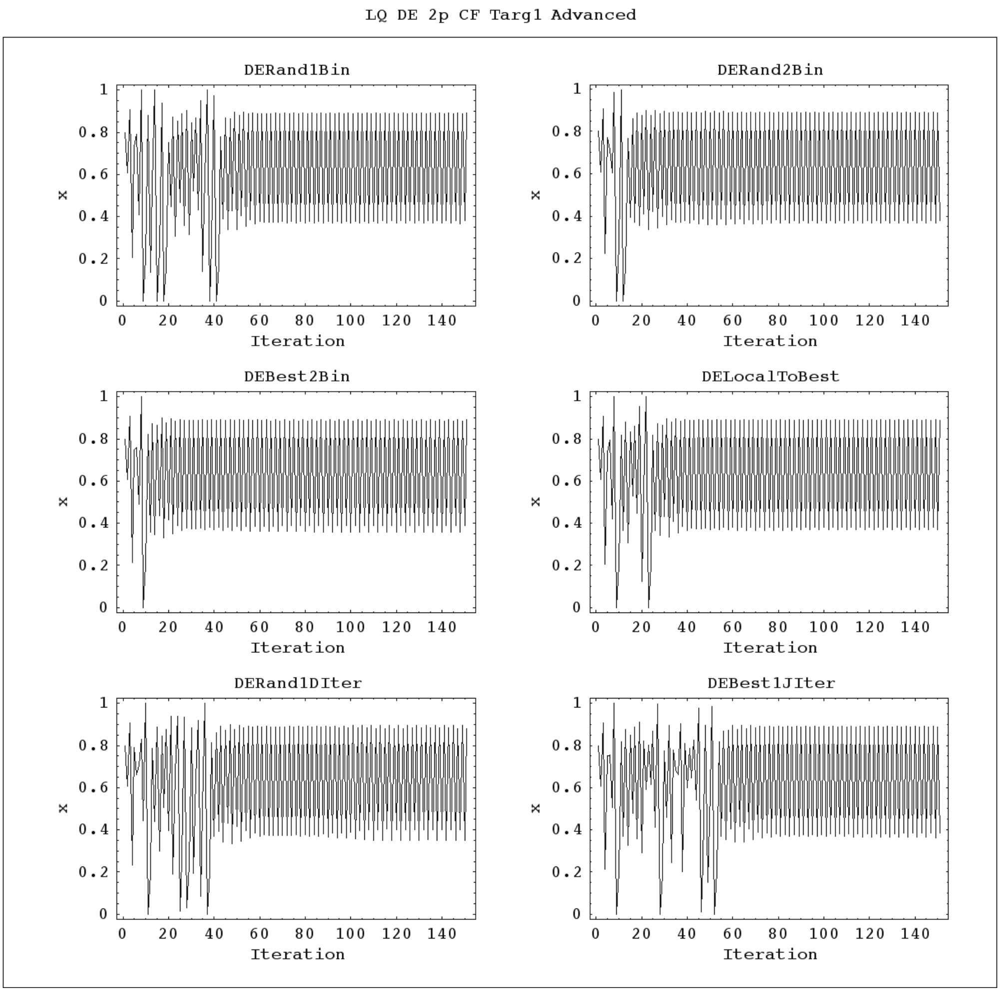


LQ SOMA 2p CF Targ1 Advanced

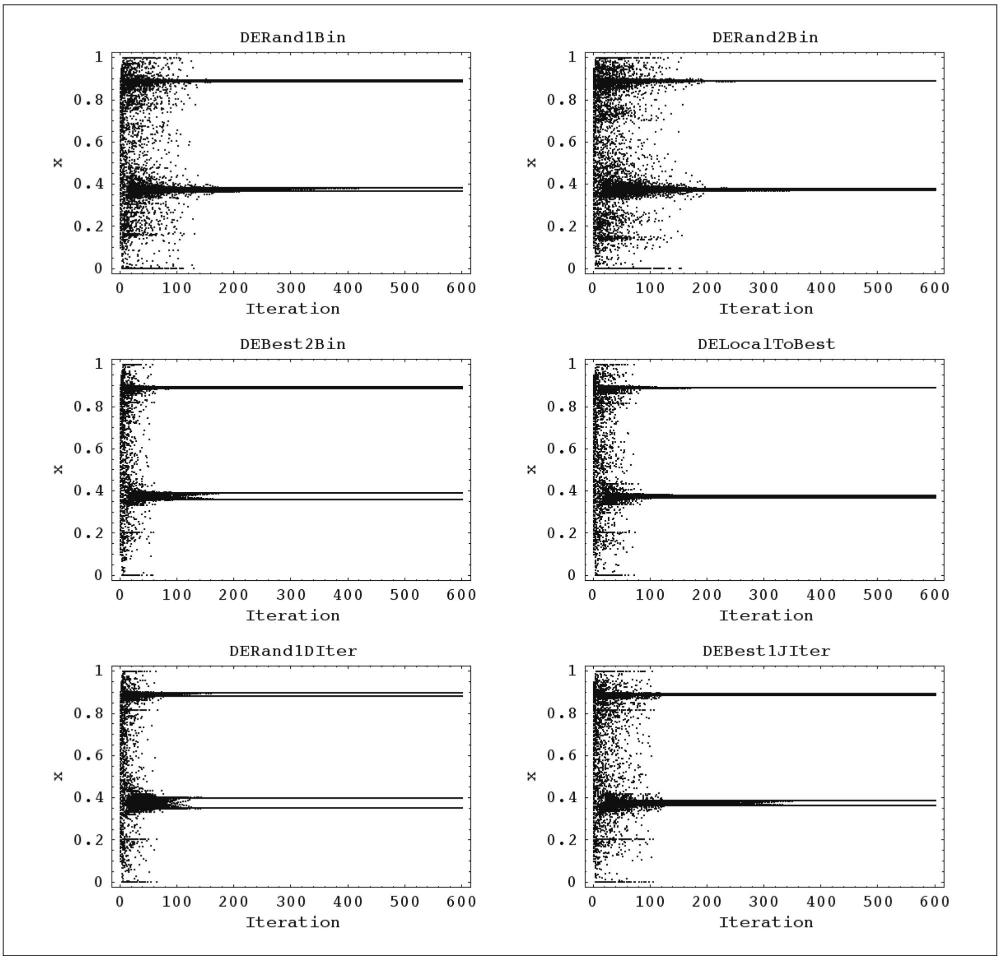


LQ SOMA 2p CF Targ1 Advanced

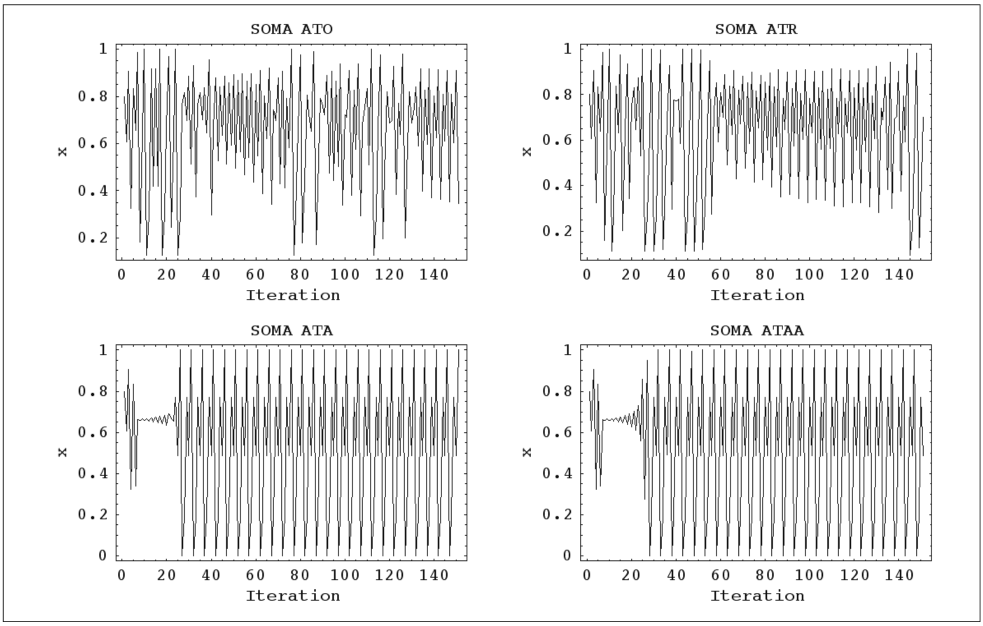



LQ DE 2p CF Targ1 Advanced
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration
0
0.2
0.4
0.6
0.8
1
0 20 40 60 80 100 120 140

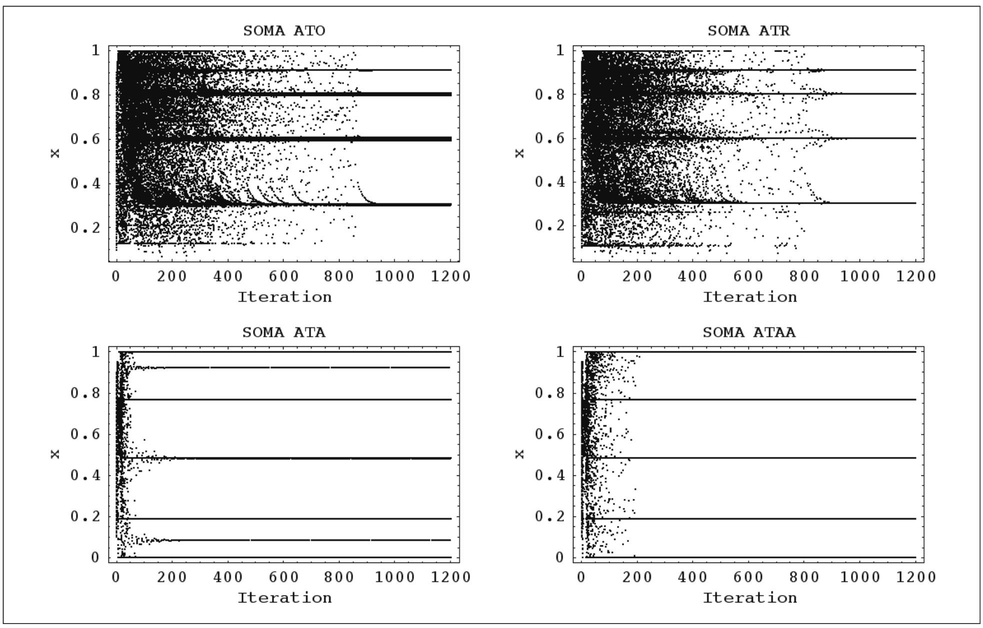
LQ DE 2p CF Targ1 Advanced

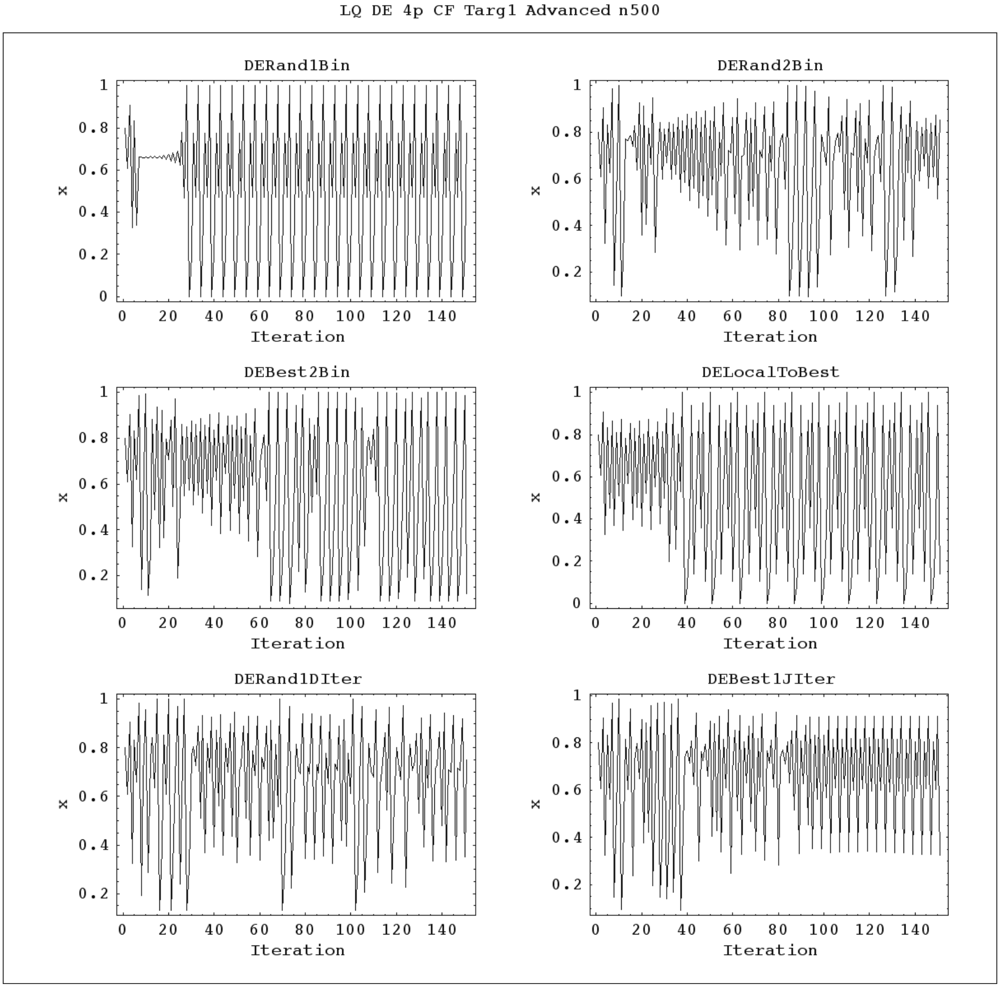


LQ SOMA 4p CF Targ1 Advanced n500

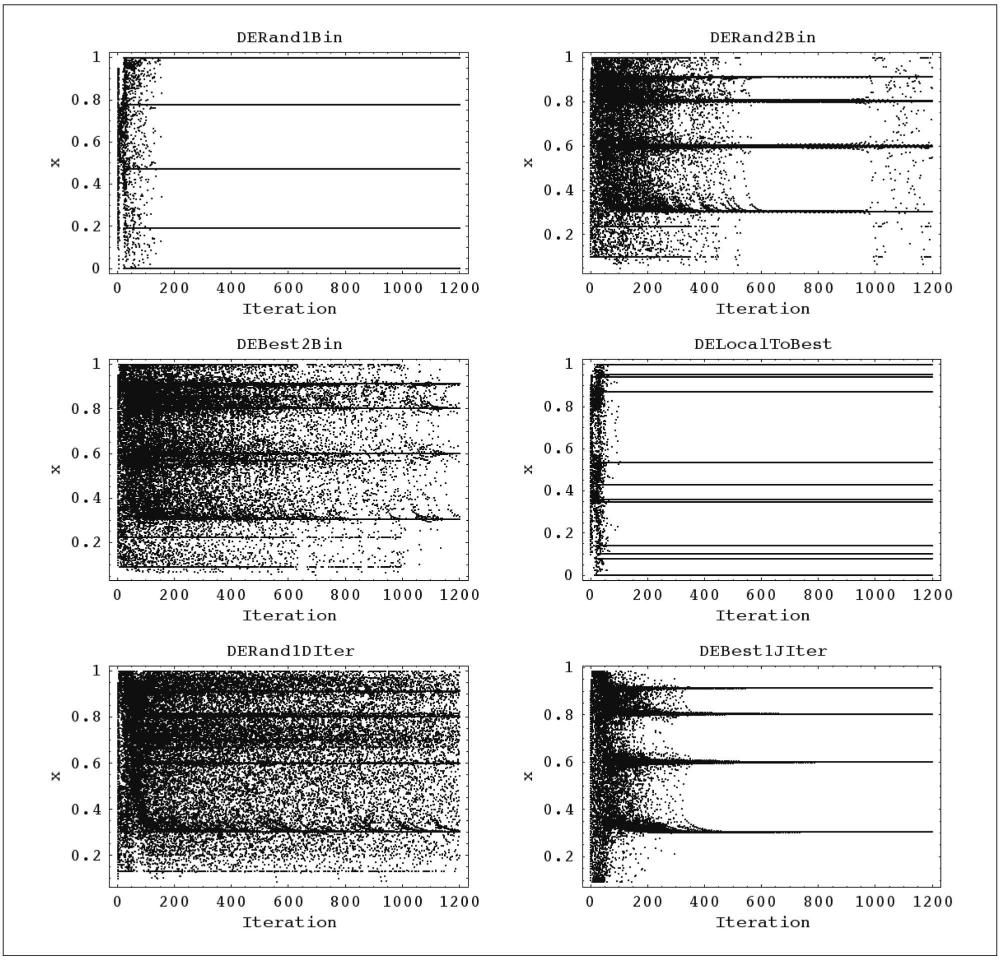


LQ SOMA 4p CF Targ1 Advanced n500

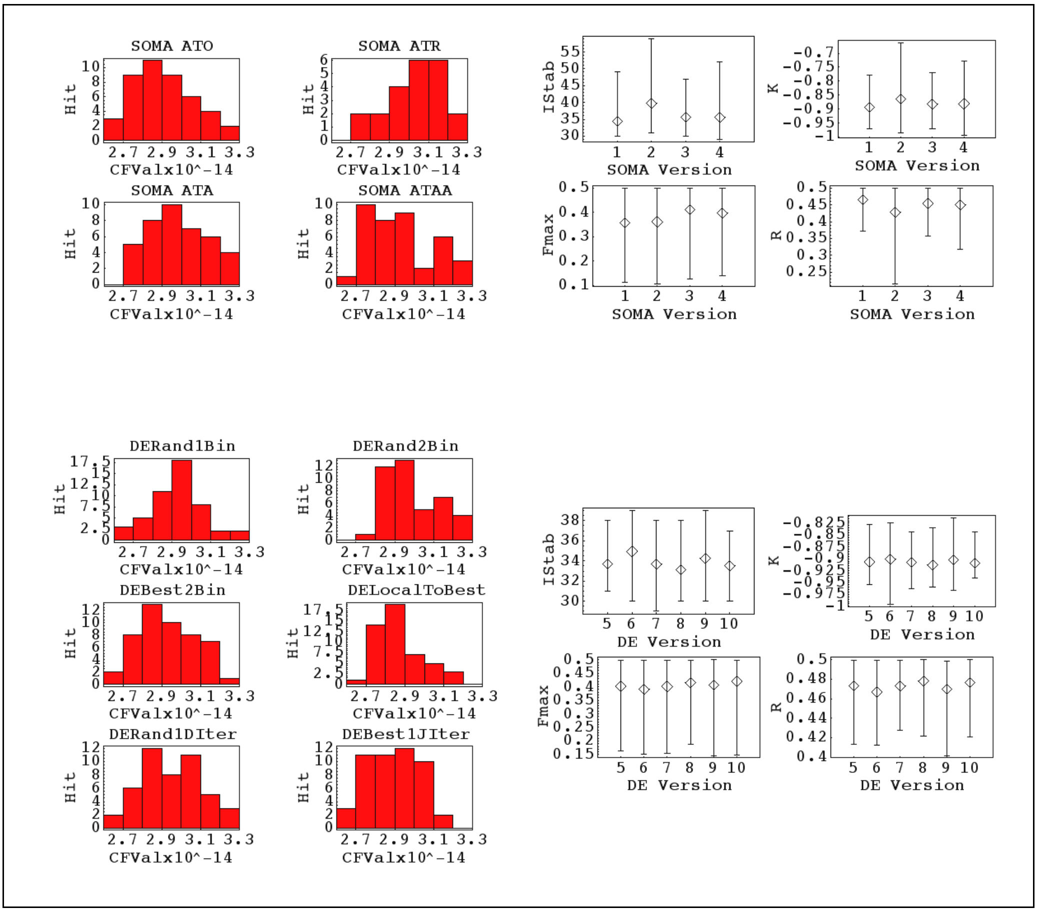



LQ DE 4p CF Targ1 Advanced n500
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration

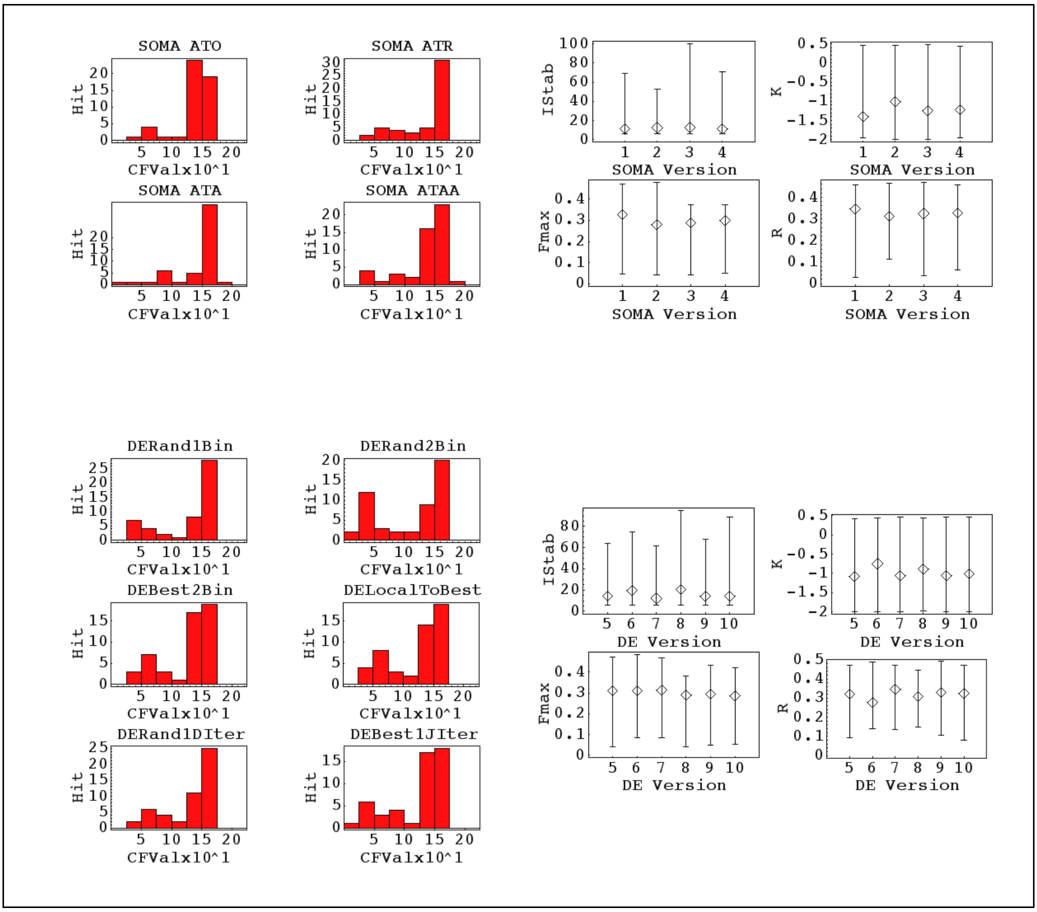
LQ DE 4p CF Targ1 Advanced n500



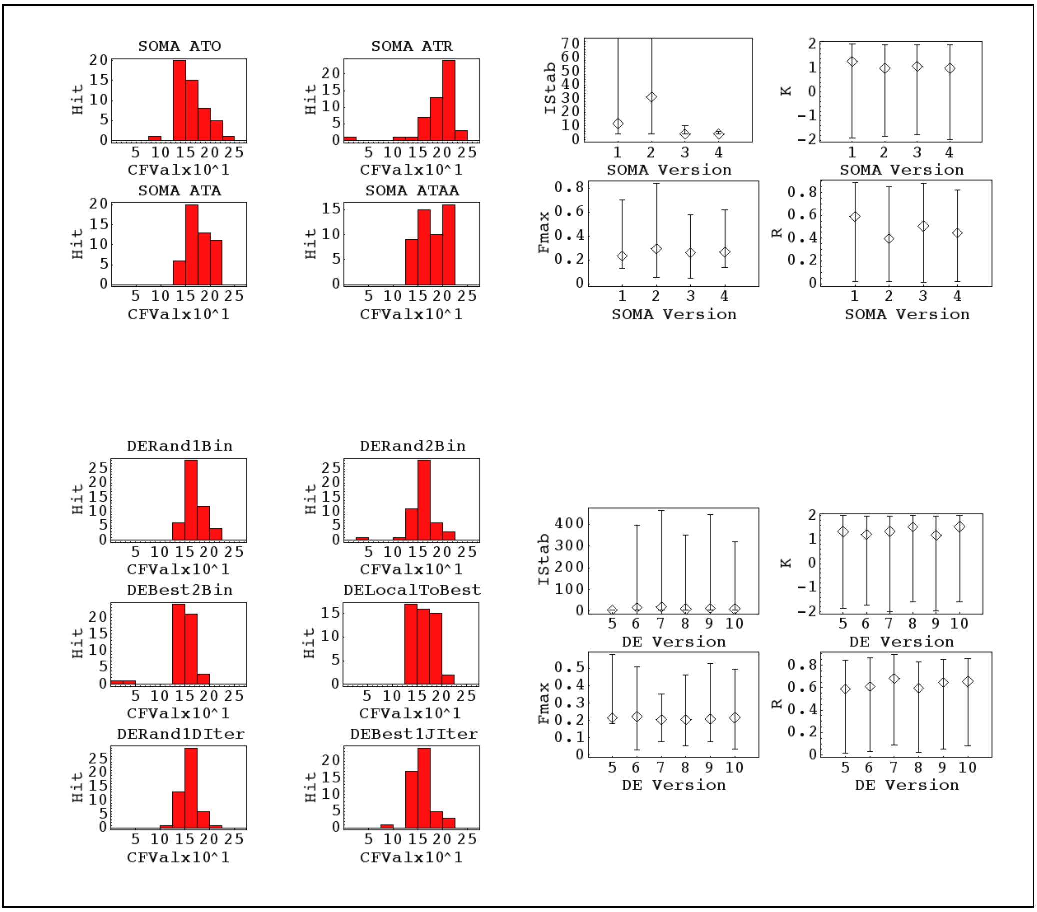
LQ SOMA & DE 1p CF Targ1 Advanced



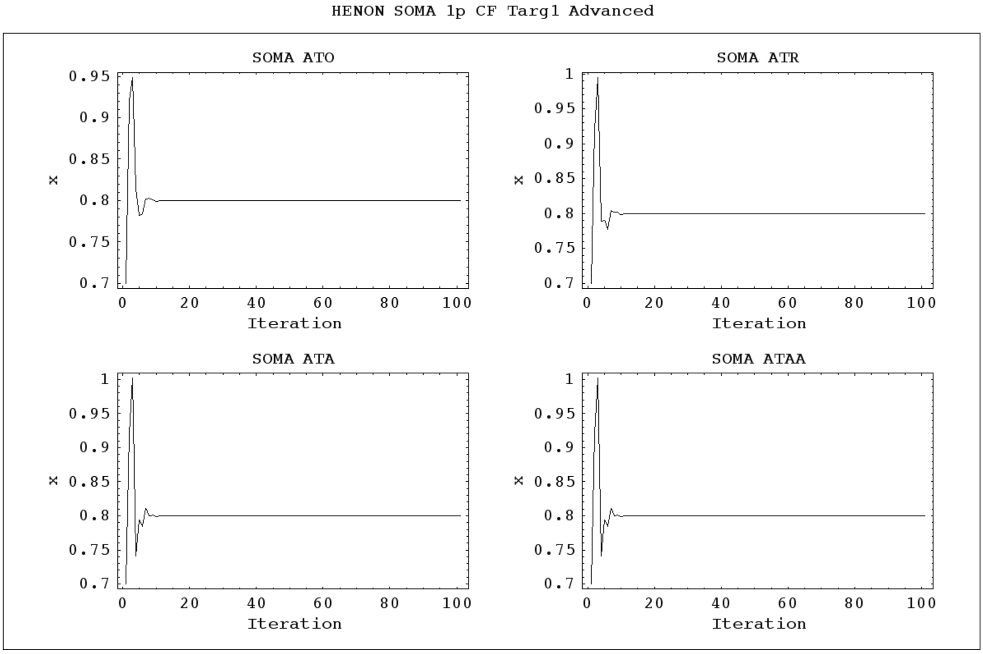
LQ SOMA & DE 2p CF Targ1 Advanced

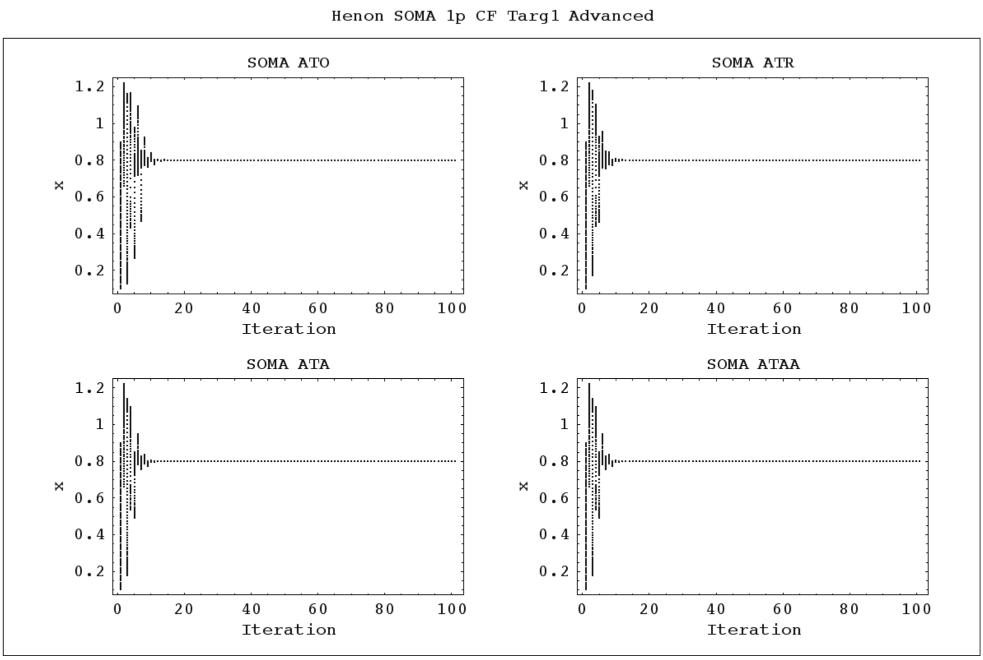


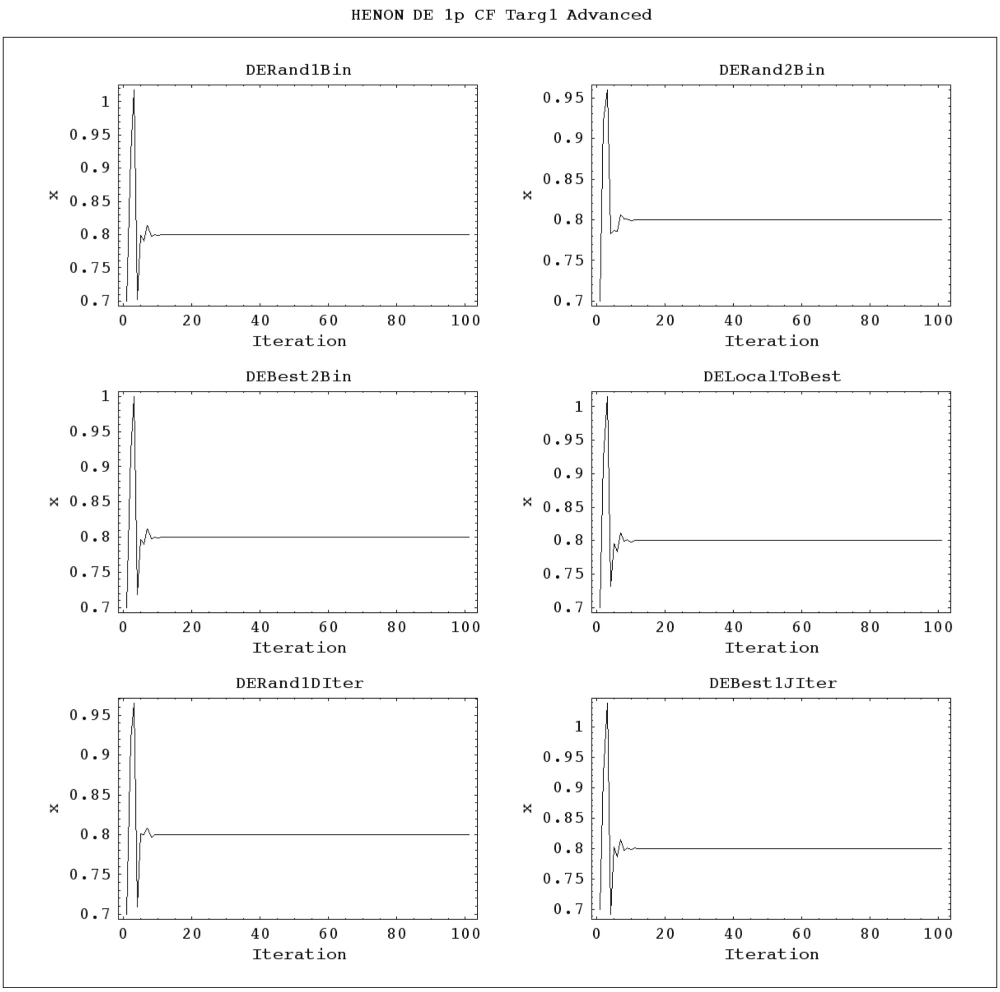
LQ SOMA & DE 4p CF Targ1 Advanced



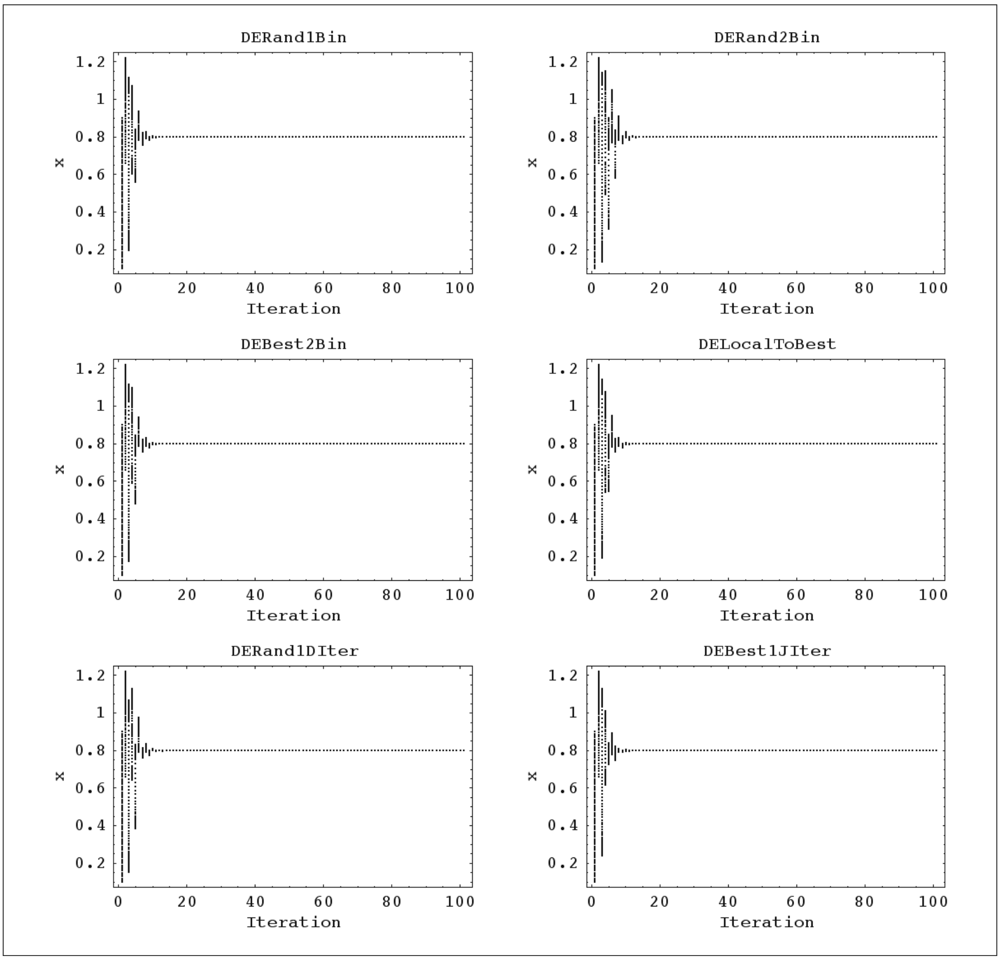
8.18 Summary of results, Case study 6, CF Targ1 Advanced, HENON



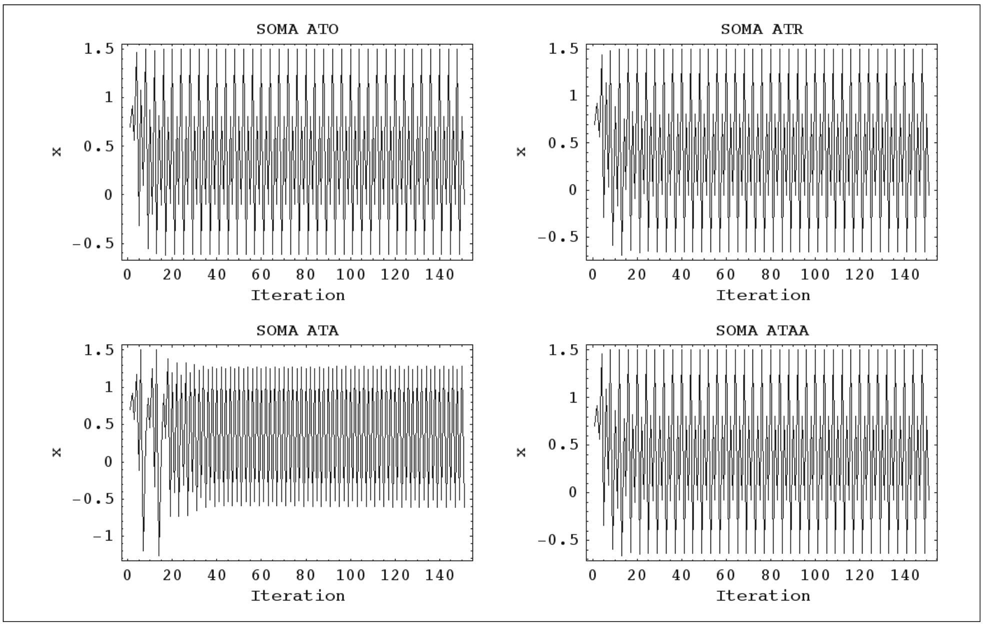



HENON DE 1p CF Targ1 Advanced
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration
0 20 40 60 80 100
0.7 0.75 0.8 0.85 0.9 0.95 1

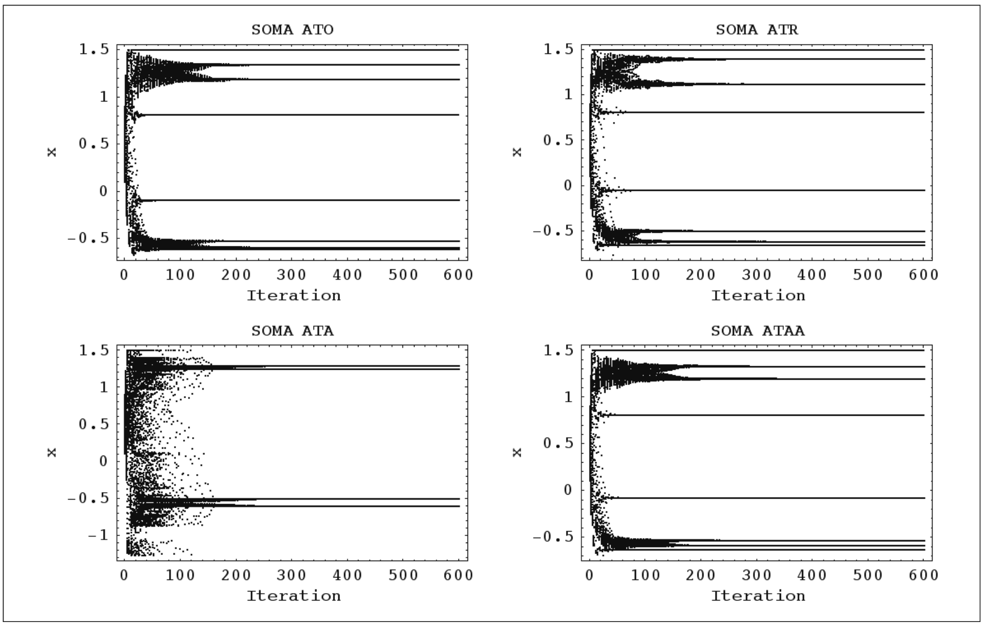
Henon DE 1p CF Targ1 Advanced

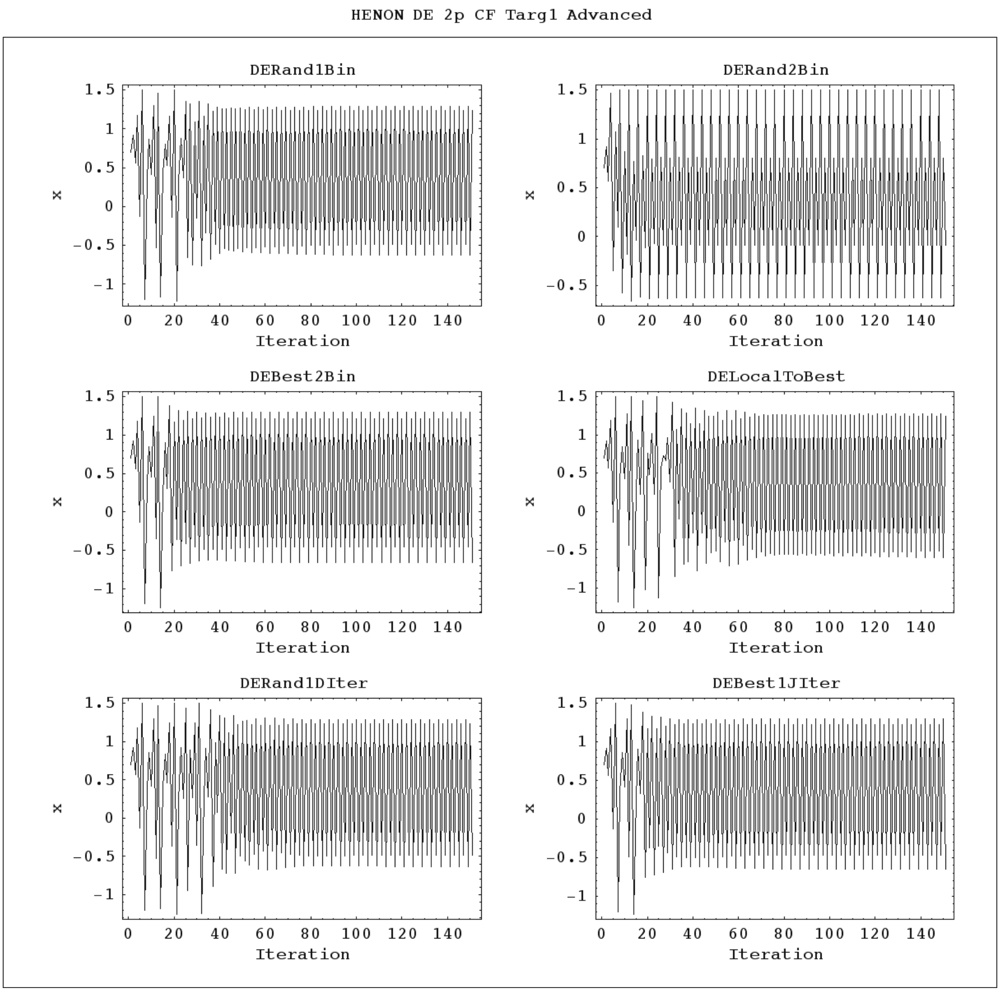


HENON SOMA 2p CF Targ1 Advanced



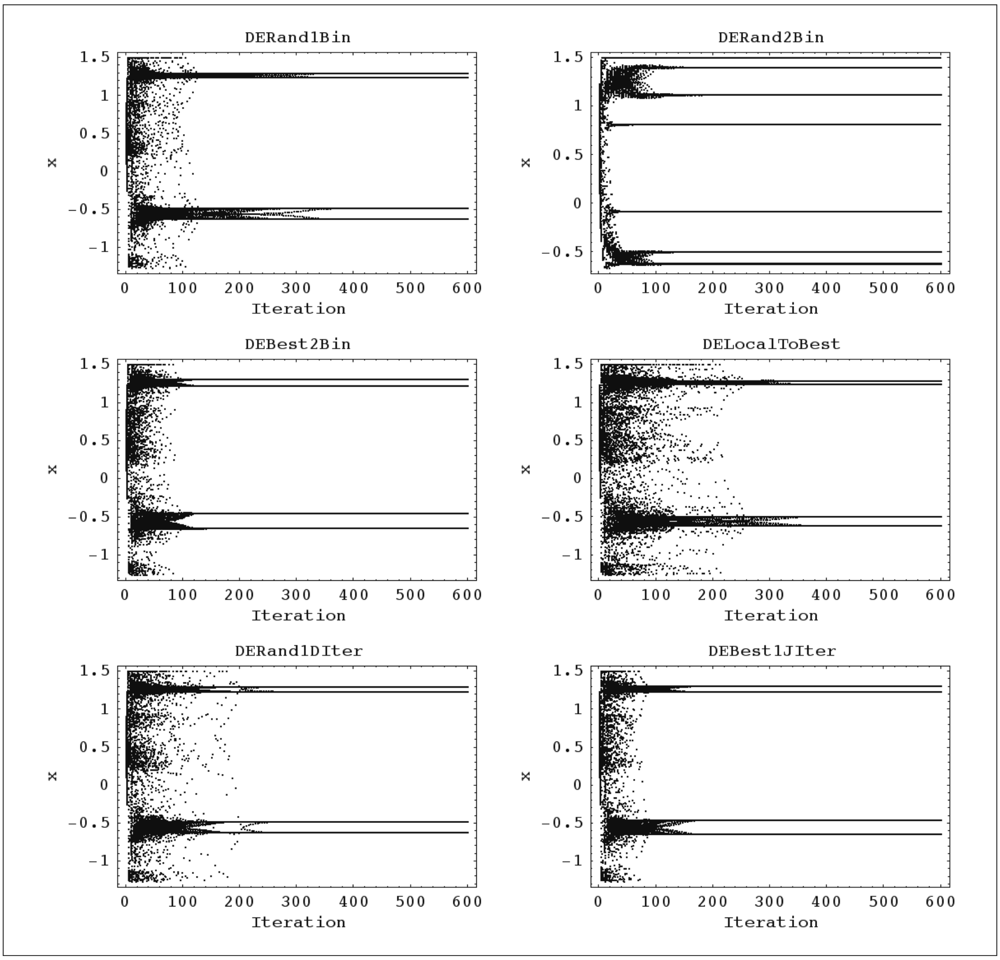
Henon SOMA 2p CF Targ1 Advanced



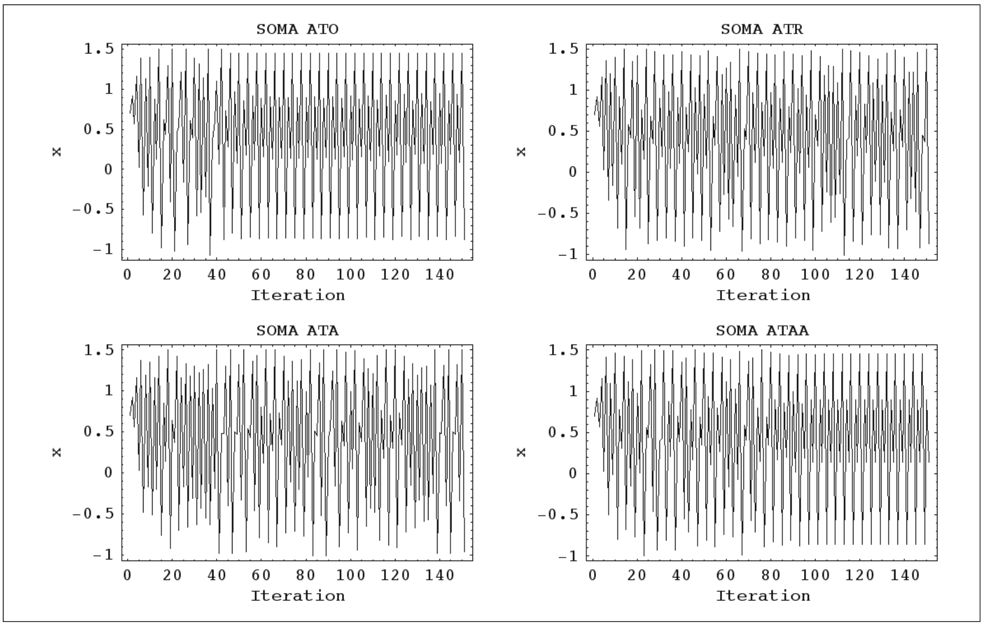


HENON DE 2p CF Targ1 Advanced
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration
1.5
1
0.5
0
-0.5
-1
0
20
40
60
80
100
120
140

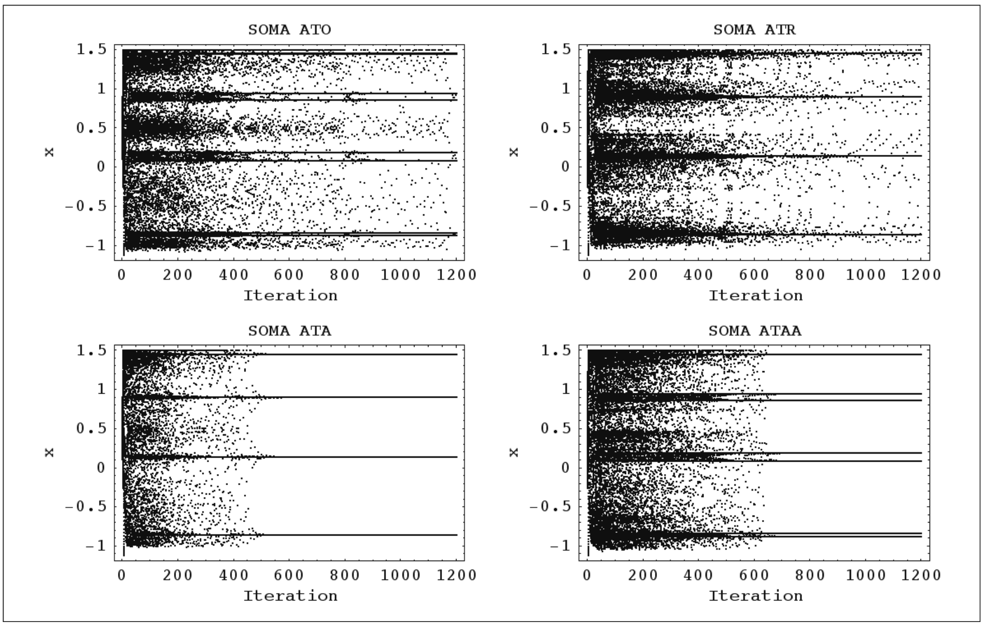
Henon DE 2p CF Targ1 Advanced



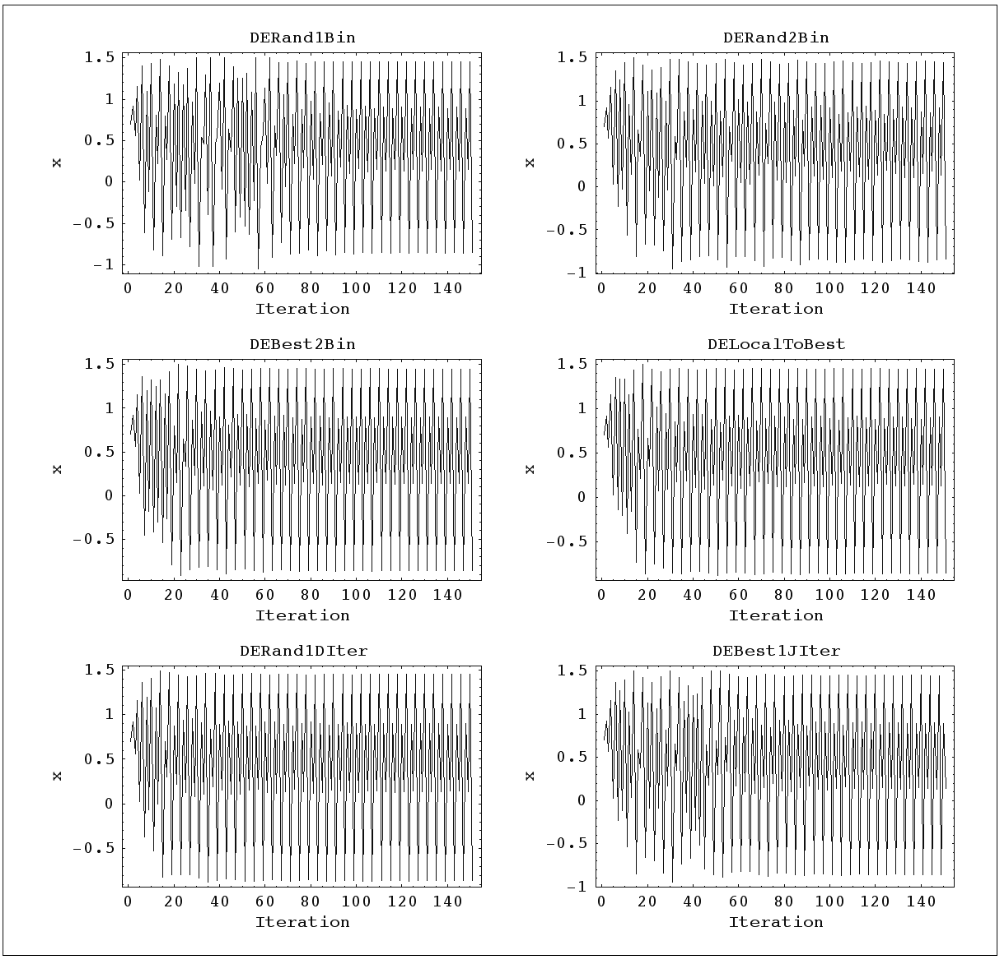
HENON SOMA 4p CF Targ1 Advanced



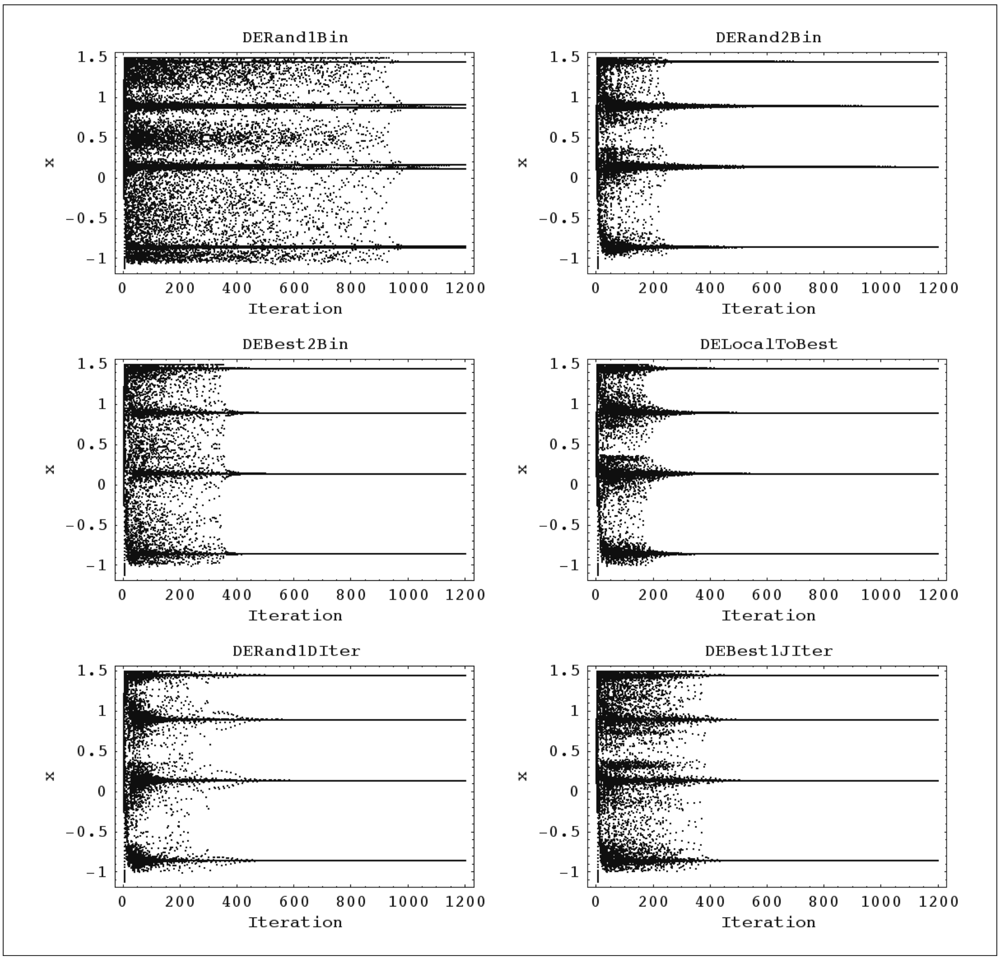
Henon SOMA 4p CF Targ1 Advanced



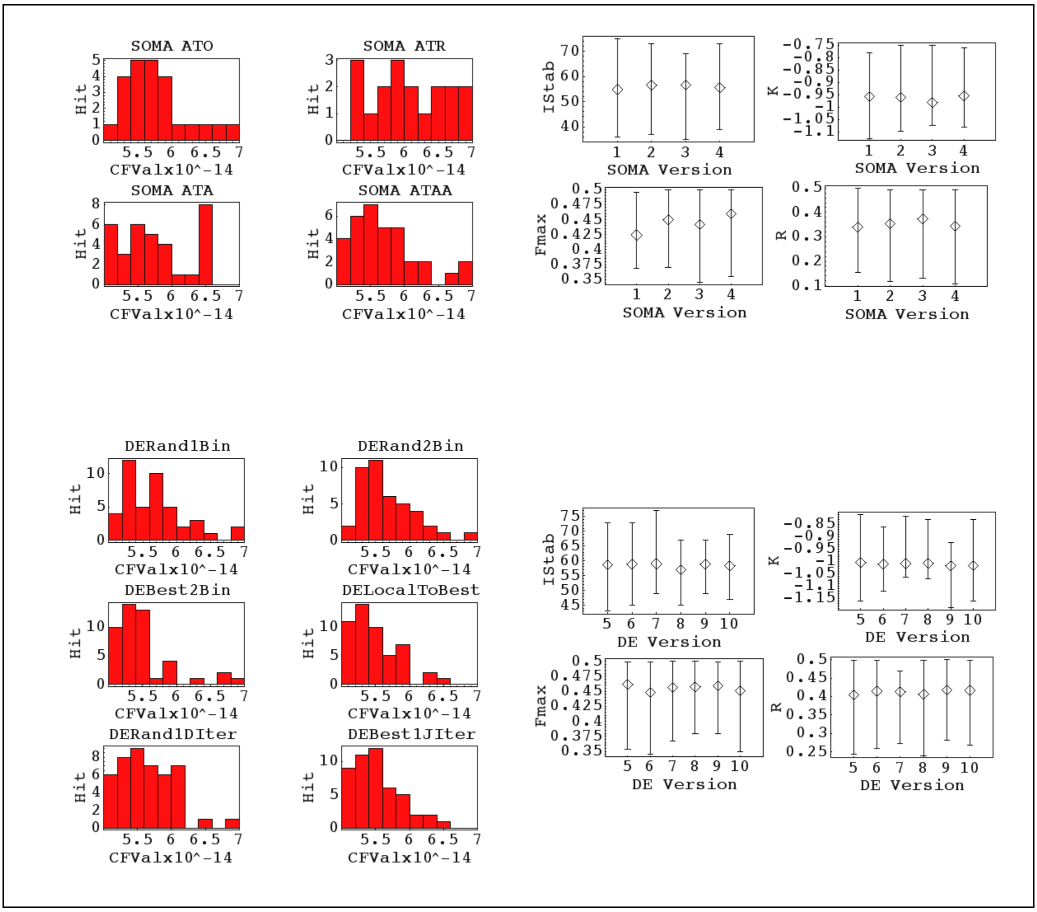
HENON DE 4p CF Targ1 Advanced



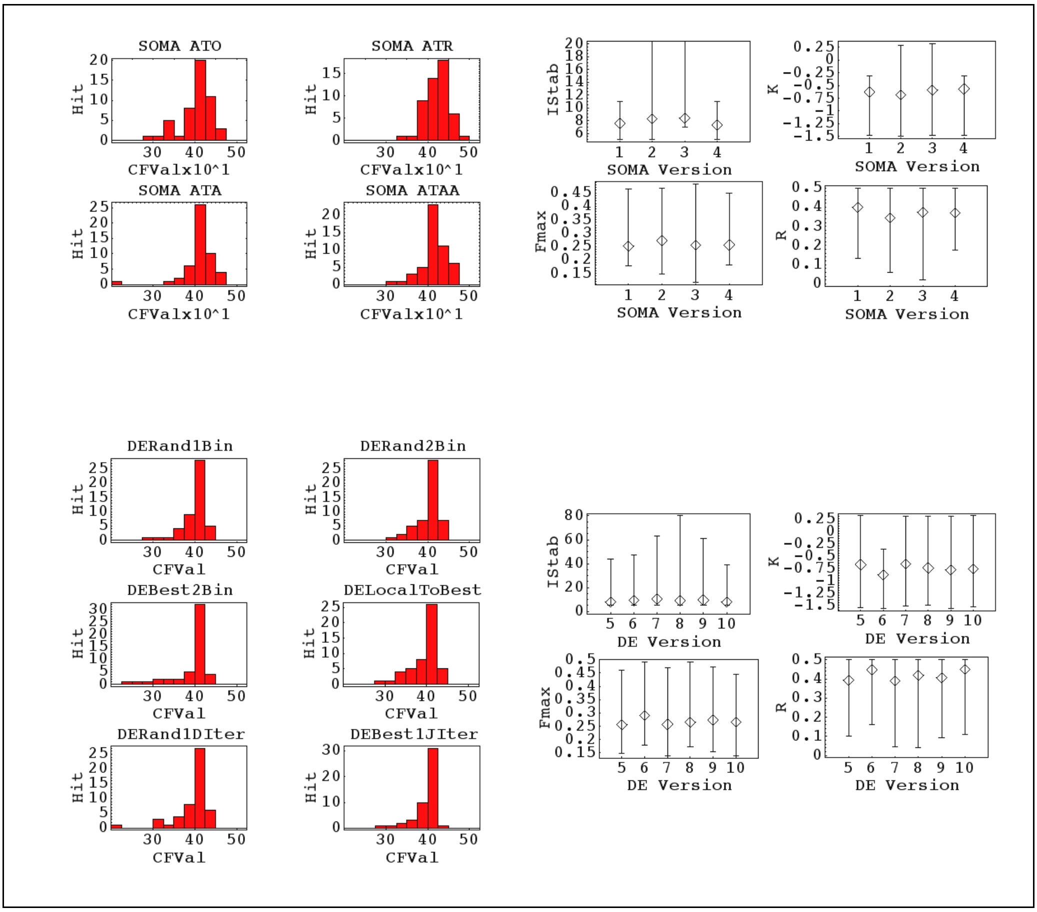
Henon DE 4p CF Targ1 Advanced



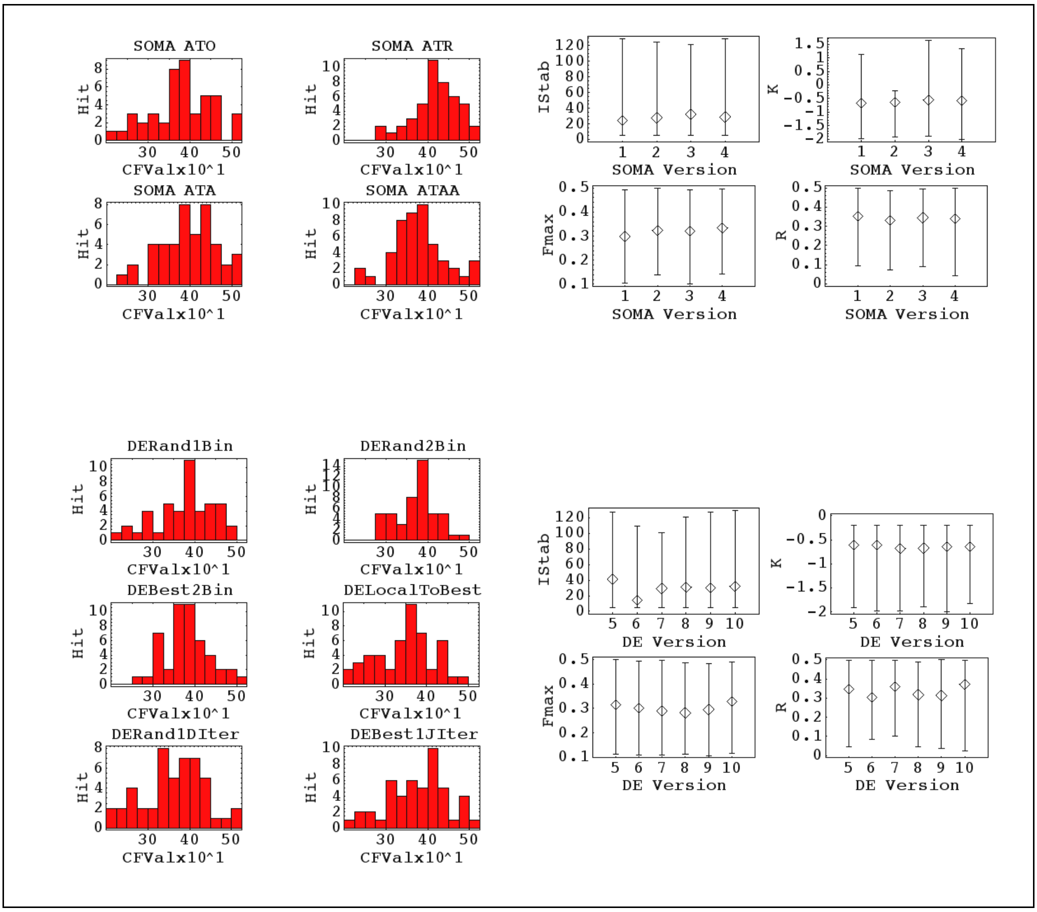
HENON SOMA & DE 1p CF Targ1 Advanced



HENON SOMA & DE 2p CF Targ1 Advanced

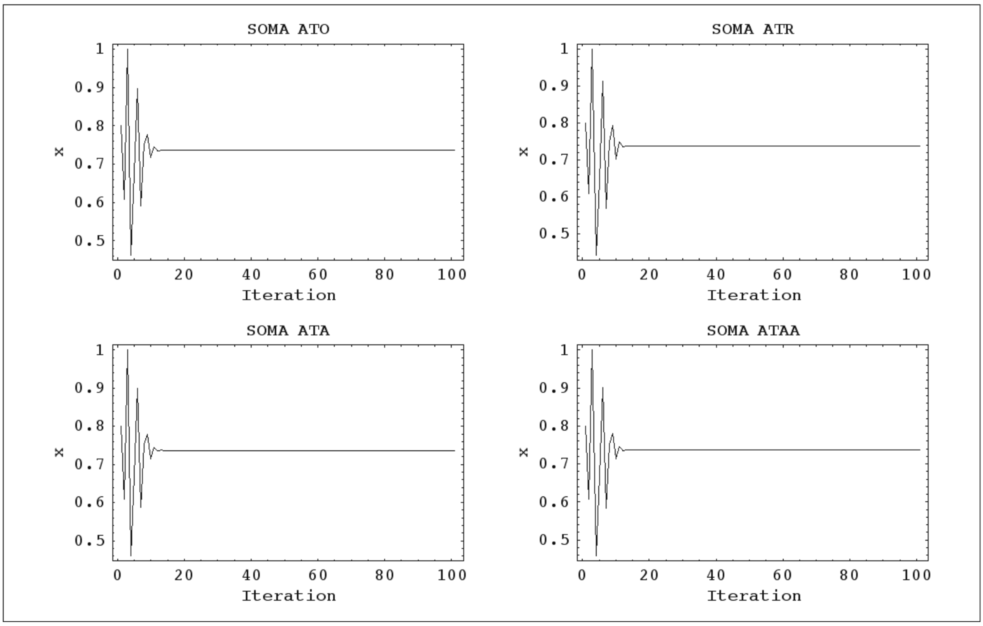


HENON SOMA & DE 4p CF Targ1 Advanced

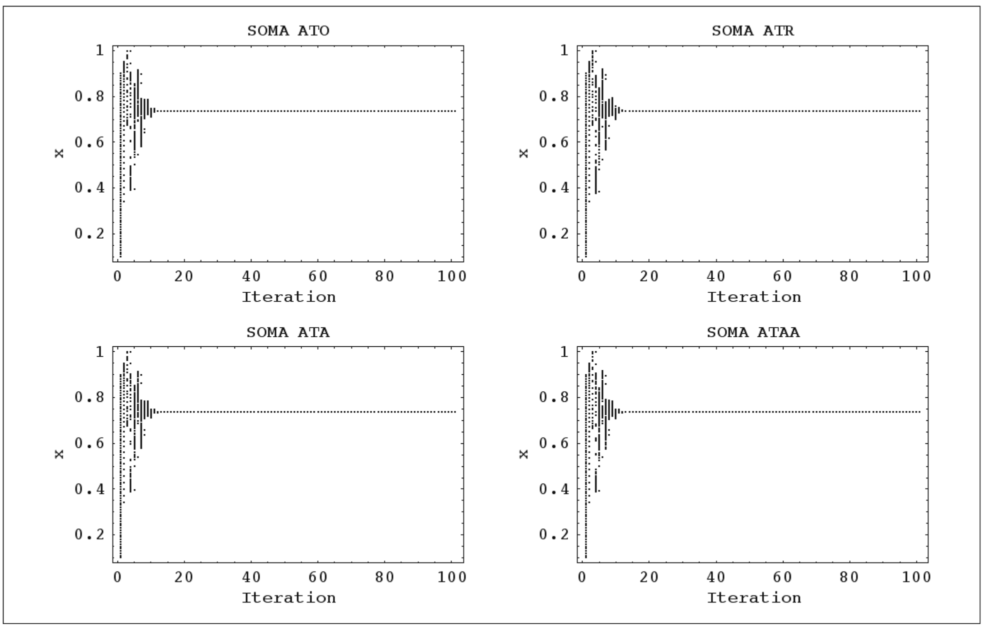


8.19 Summary of results, Case study 7, CF Targ2 Advanced, LQ

LQ SOMA 1p CF Targ2 Advanced



LQ SOMA 1p CF Targ2 Advanced



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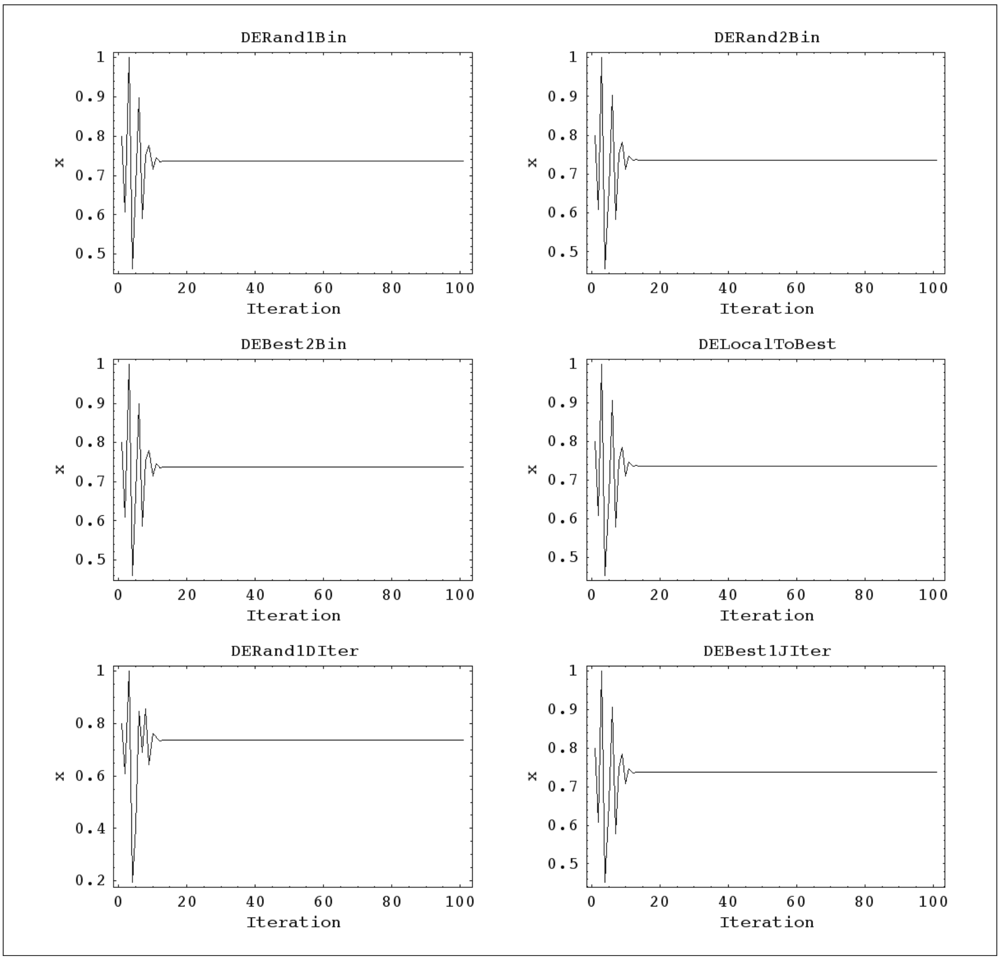
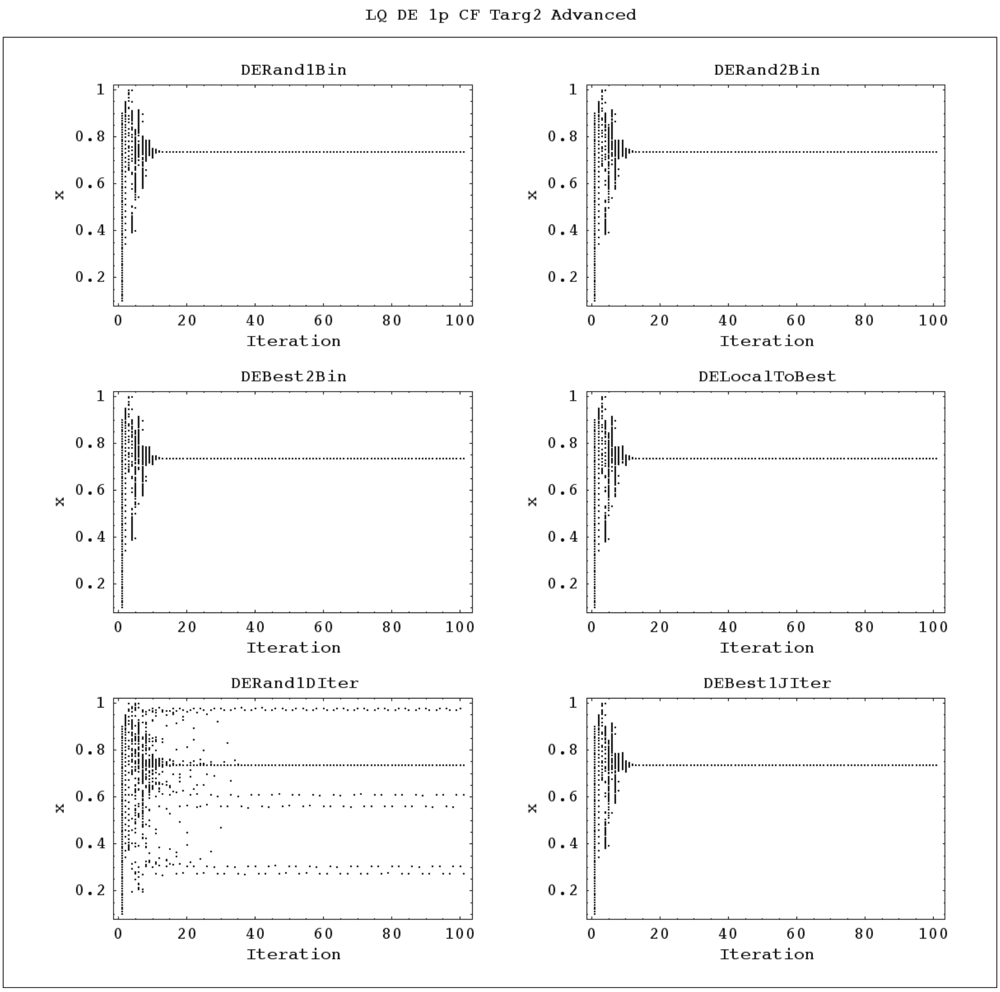
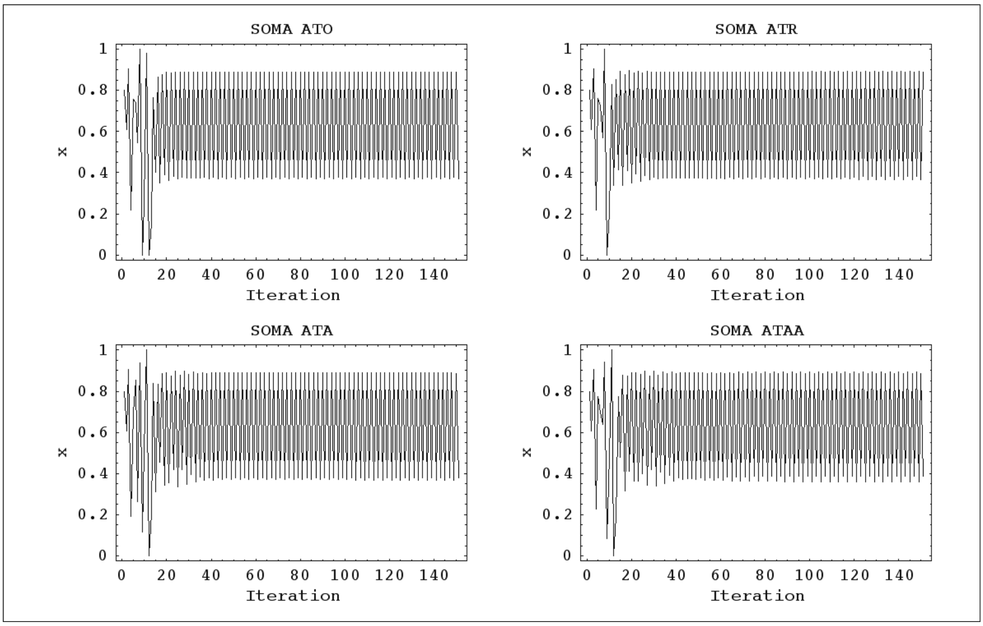




LQ DE 1p CF Targ2 Advanced
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration
0 20 40 60 80 100
0.2 0.4 0.6 0.8 1

LQ SOMA 2p CF Targ2 Advanced



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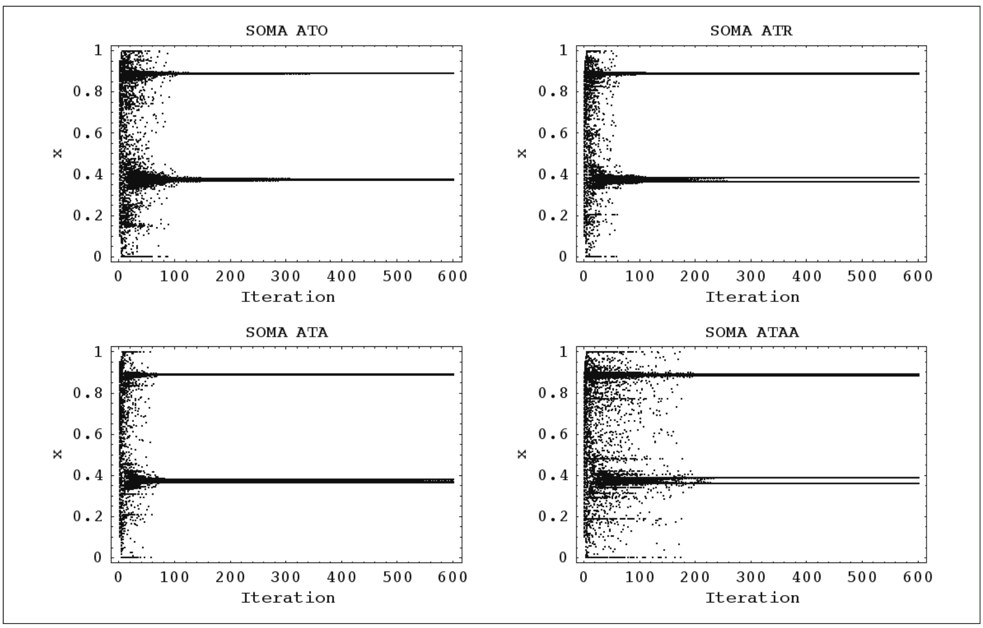

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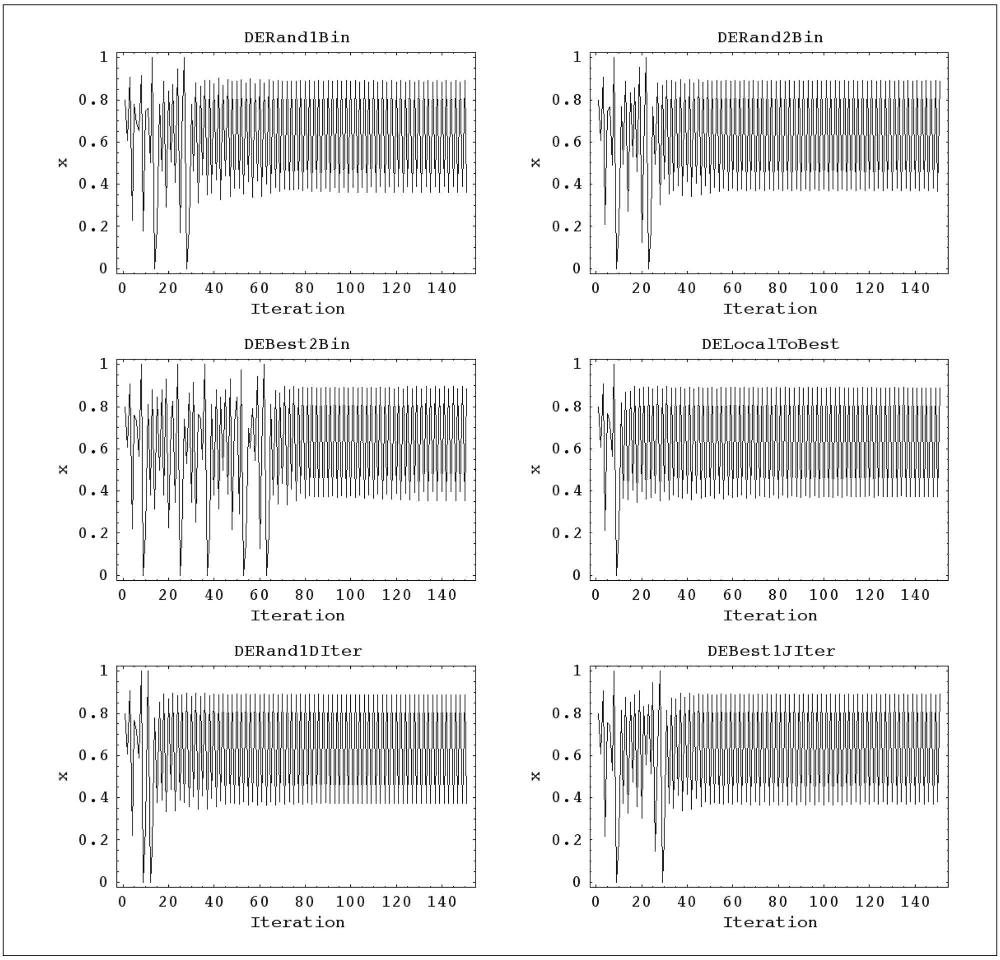
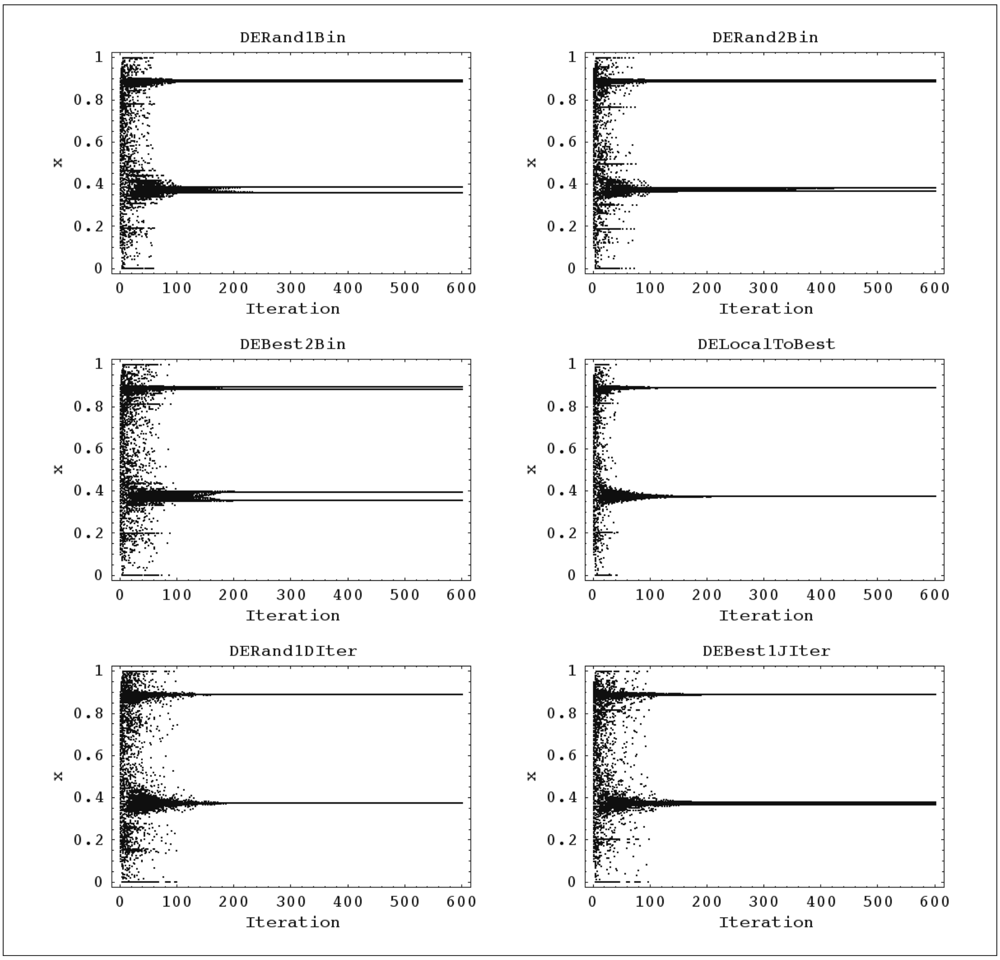


LQ DE 2p CF Targ2 Advanced



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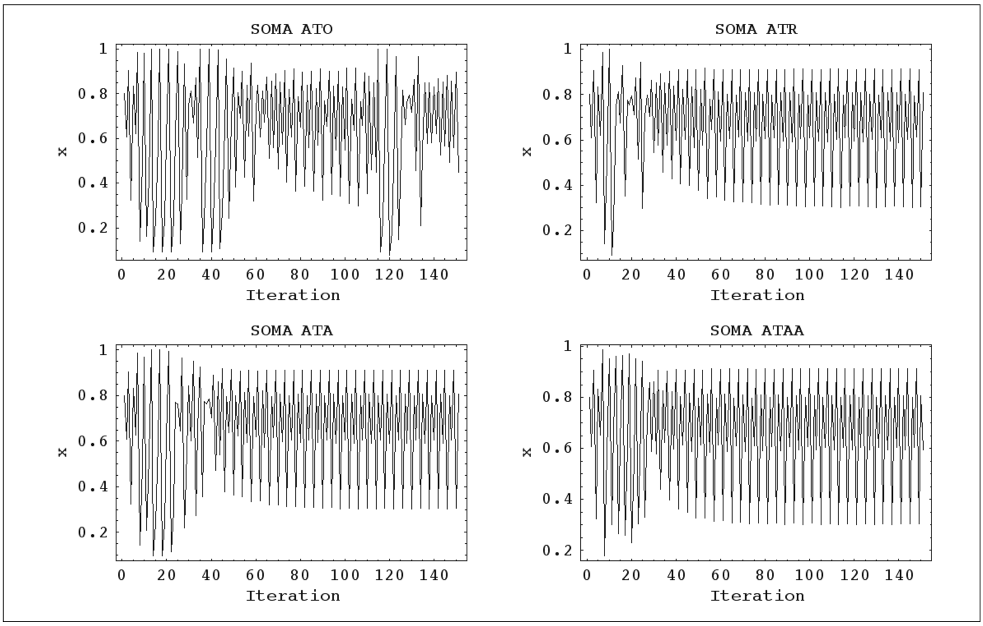

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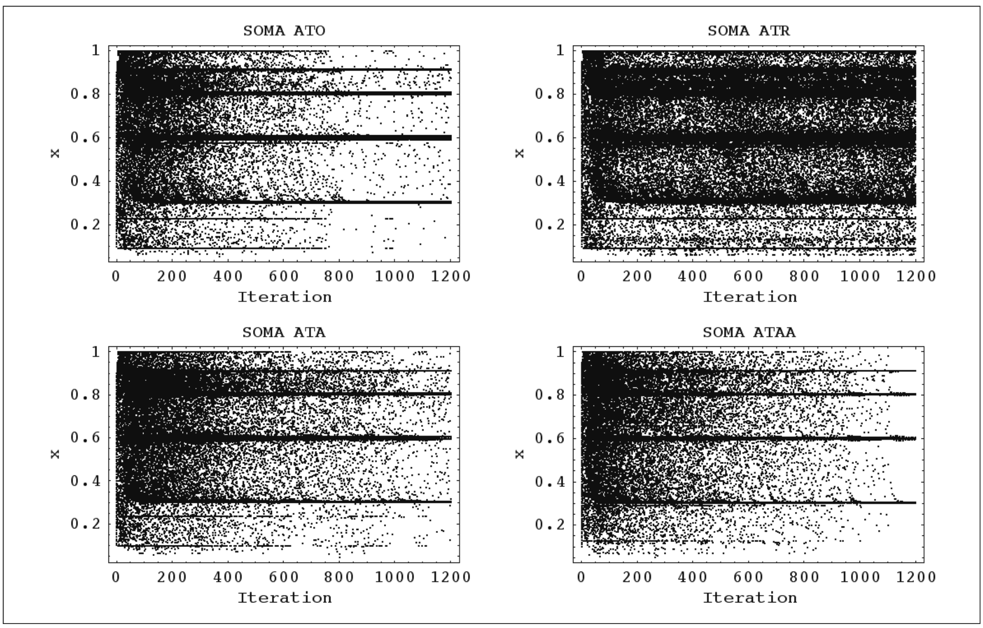

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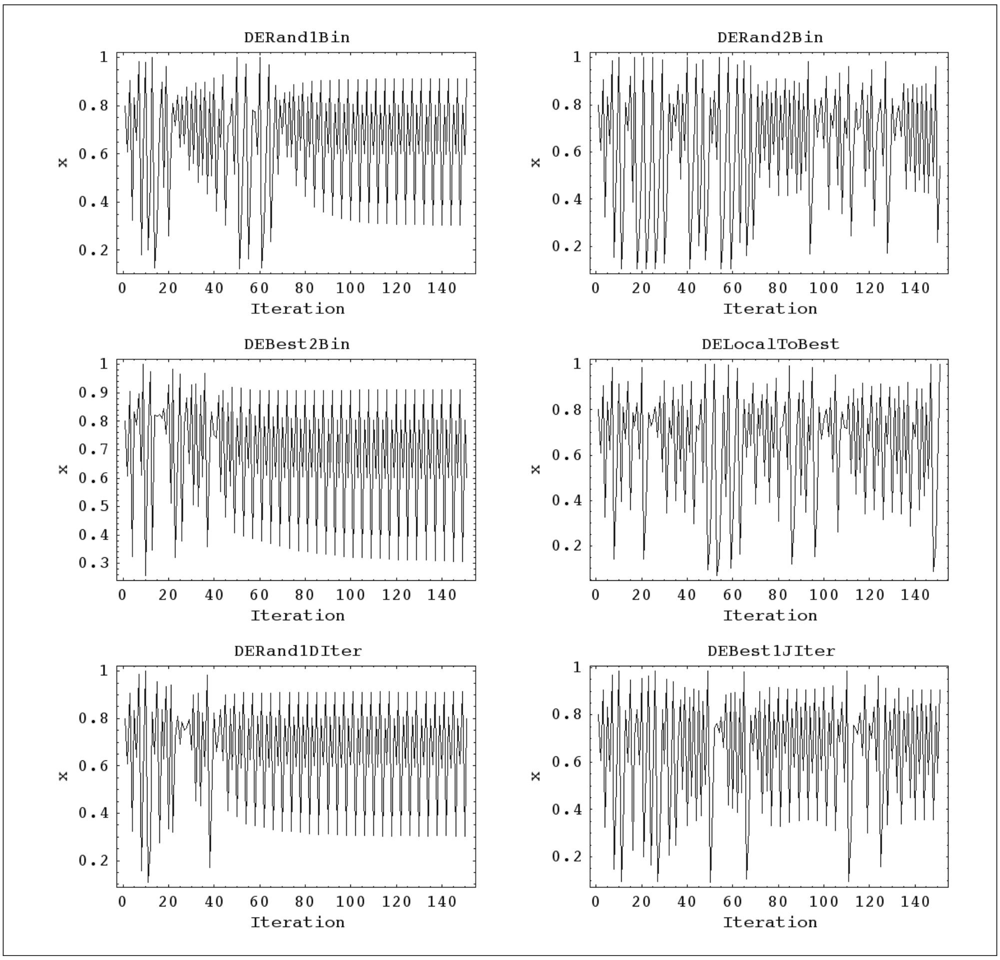
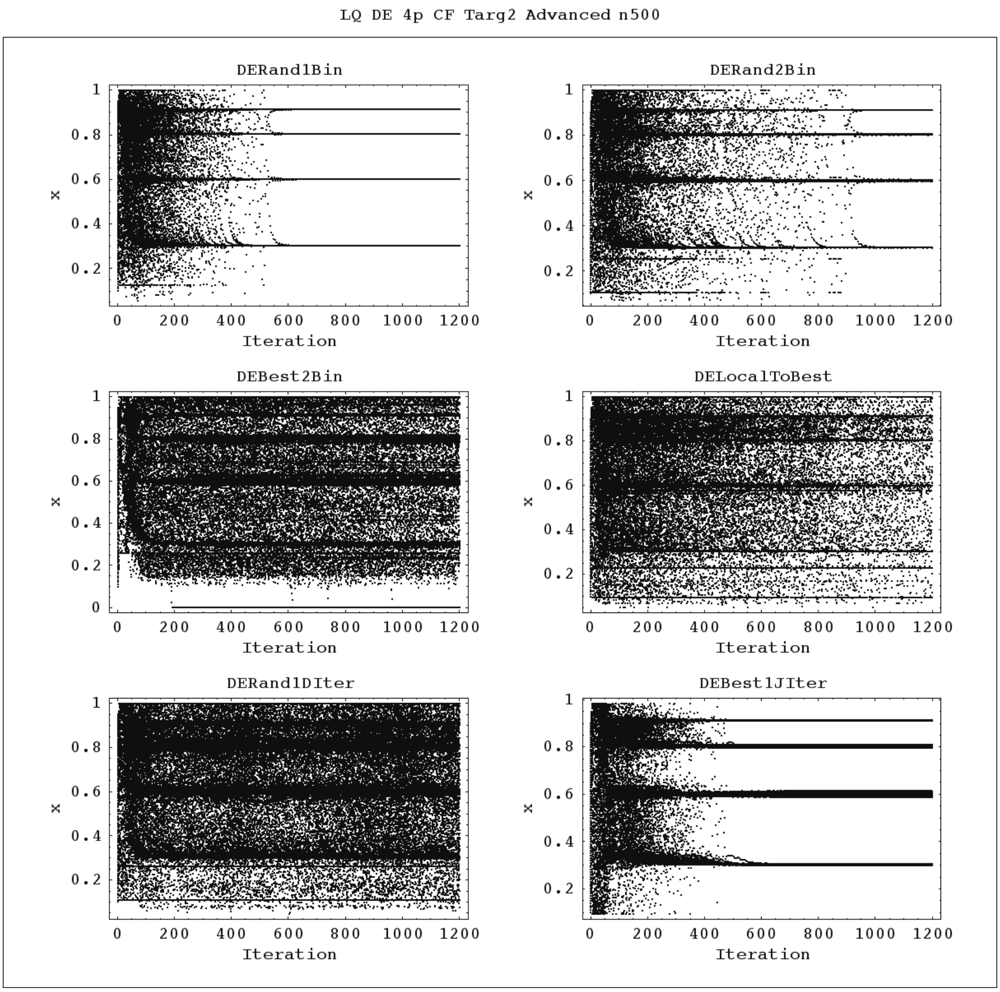
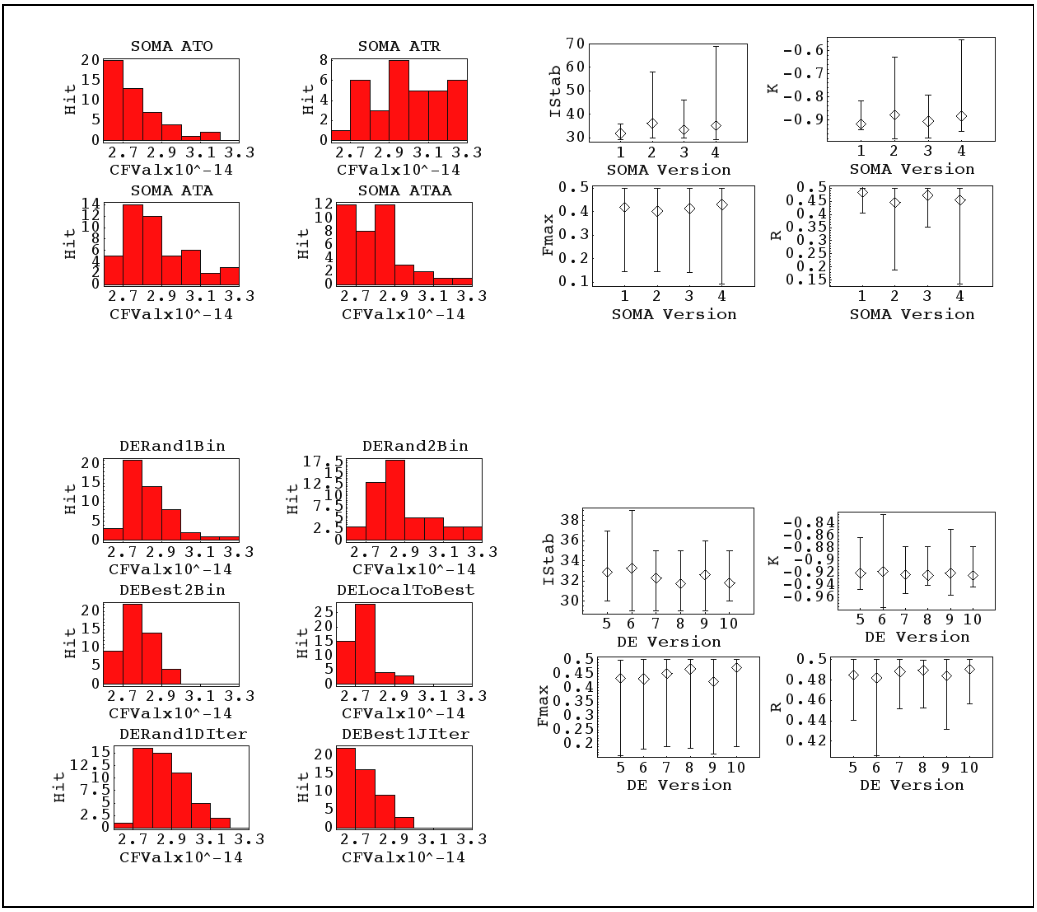
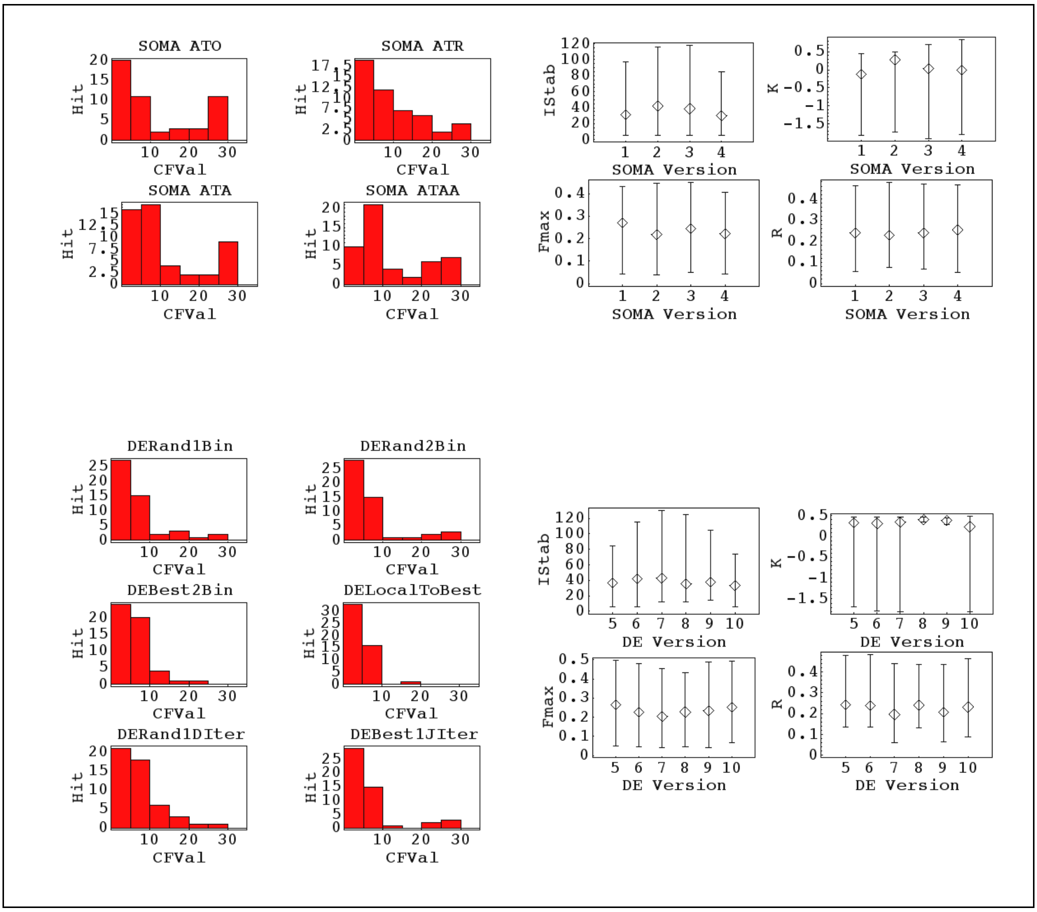



LQ DE 4p CF Targ2 Advanced n500
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration
0 200 400 600 800 1000 1200
0 0.2 0.4 0.6 0.8 1

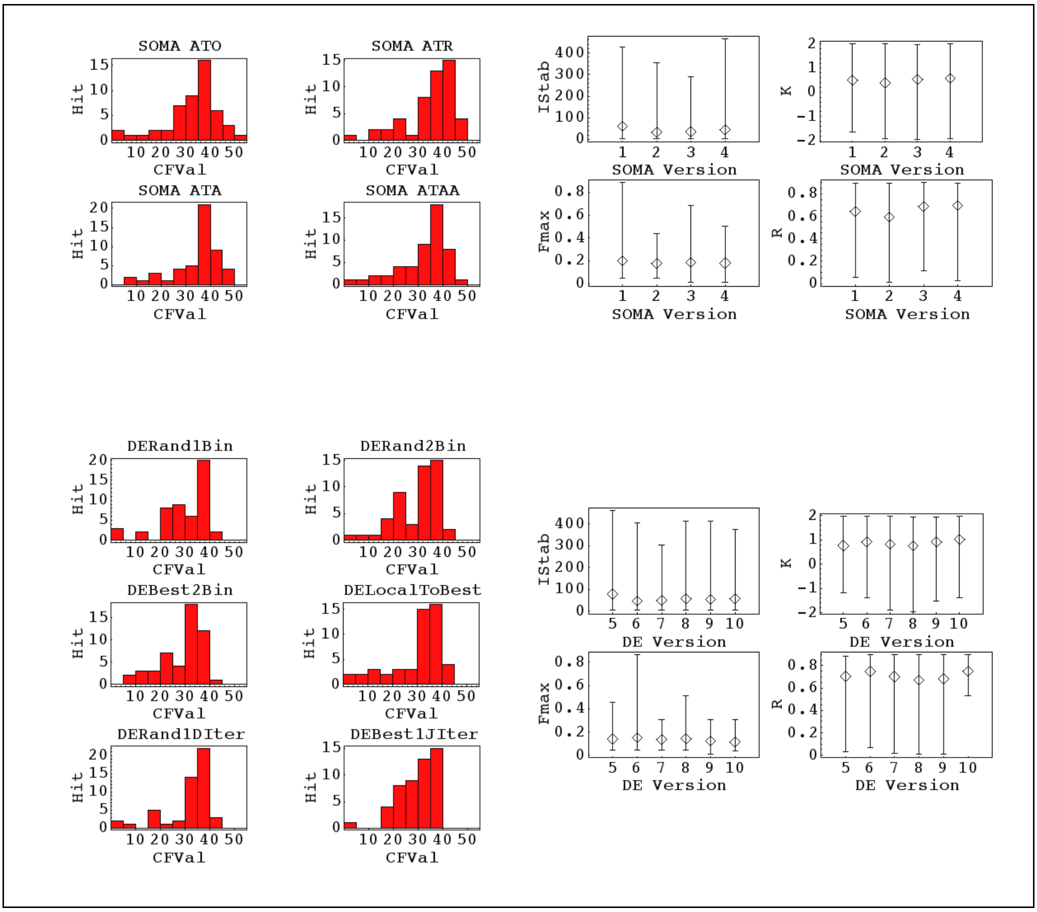
LQ SOMA & DE 1p CF Targ2 Advanced



LQ SOMA & DE 2p CF Targ2 Advanced

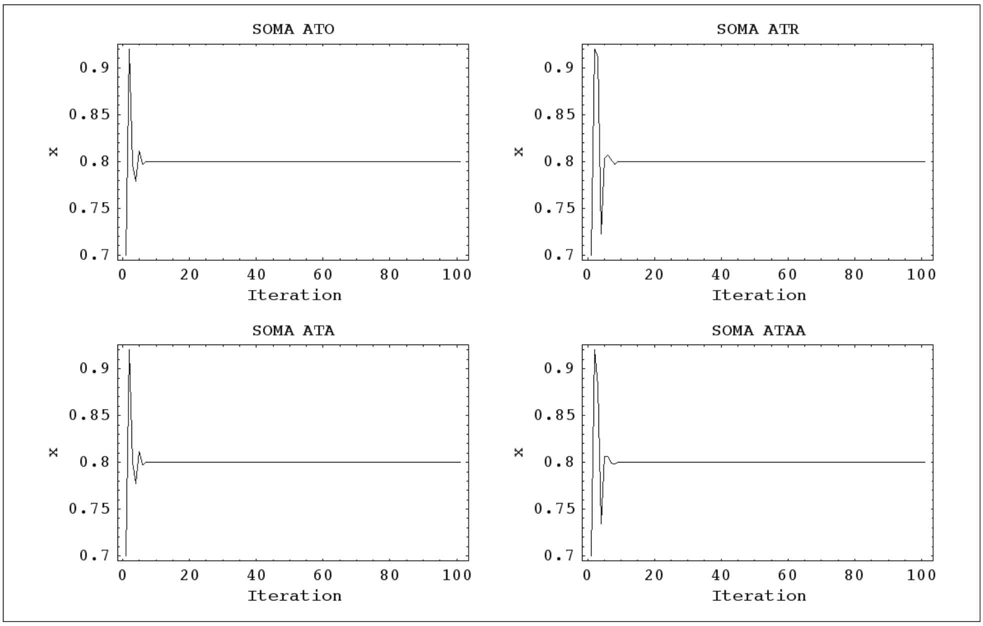


LQ SOMA & DE 4p CF Targ2 Advanced

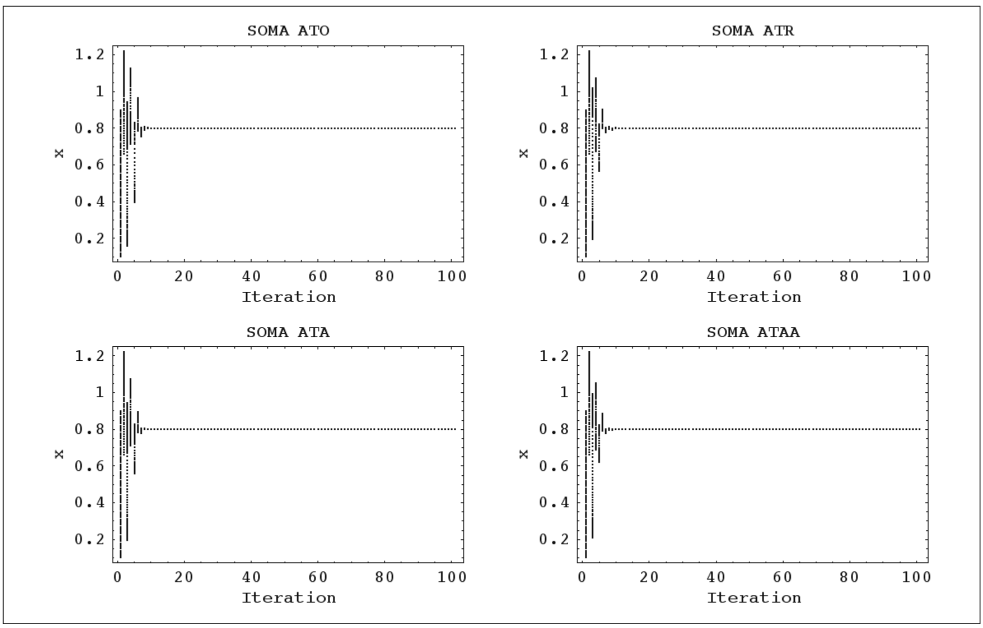


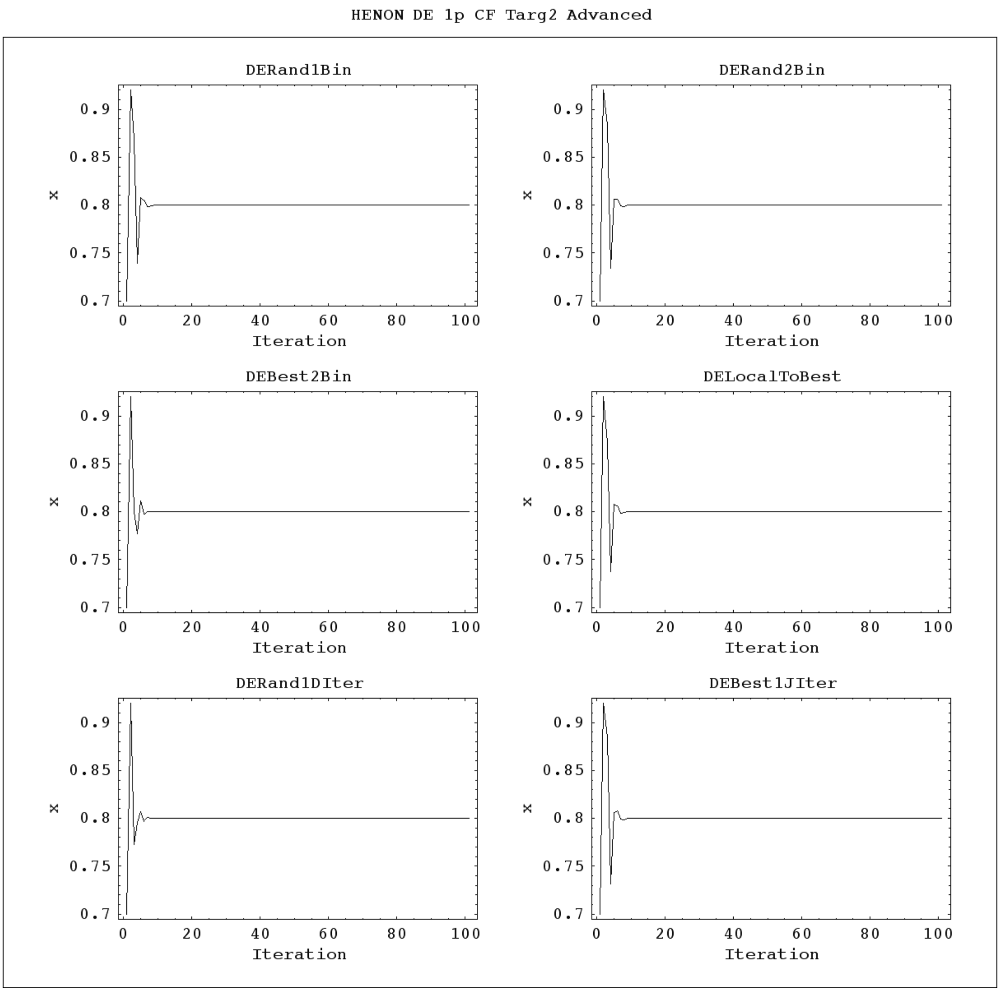
8.20 Summary of results, Case study 7, CF Targ2 Advanced, HENON

HENON SOMA 1p CF Targ2 Advanced

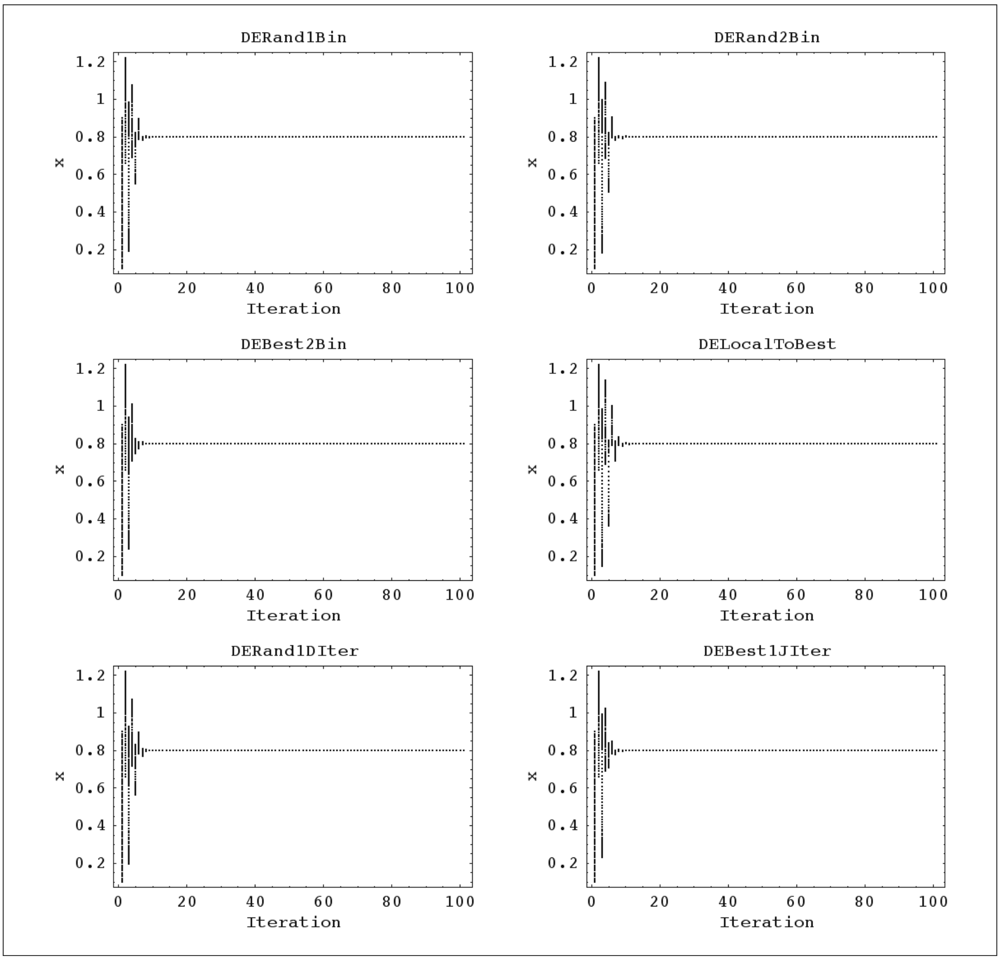


Henon SOMA 1p CF Targ2 Advanced

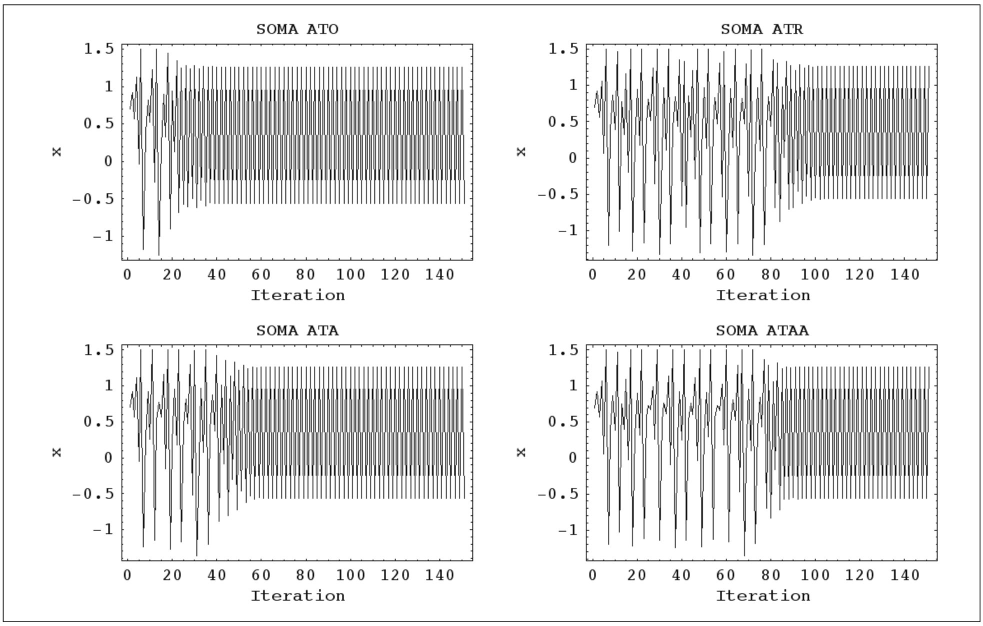



HENON DE 1p CF Targ2 Advanced
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration
0.7
0.75
0.8
0.85
0.9
0
20
40
60
80
100

Henon DE 1p CF Targ2 Advanced



HENON SOMA 2p CF Targ2 Advanced



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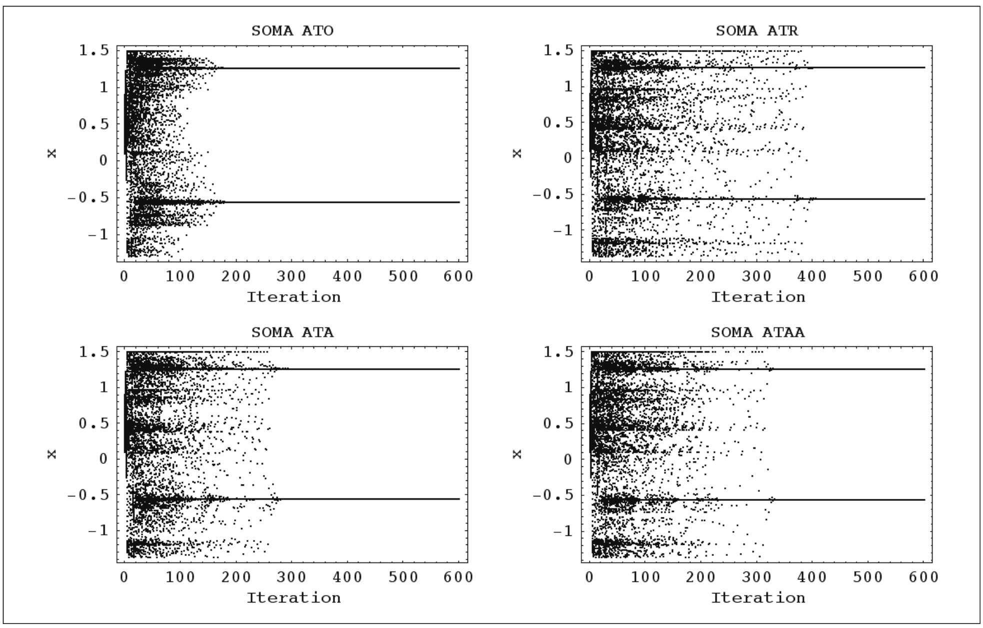
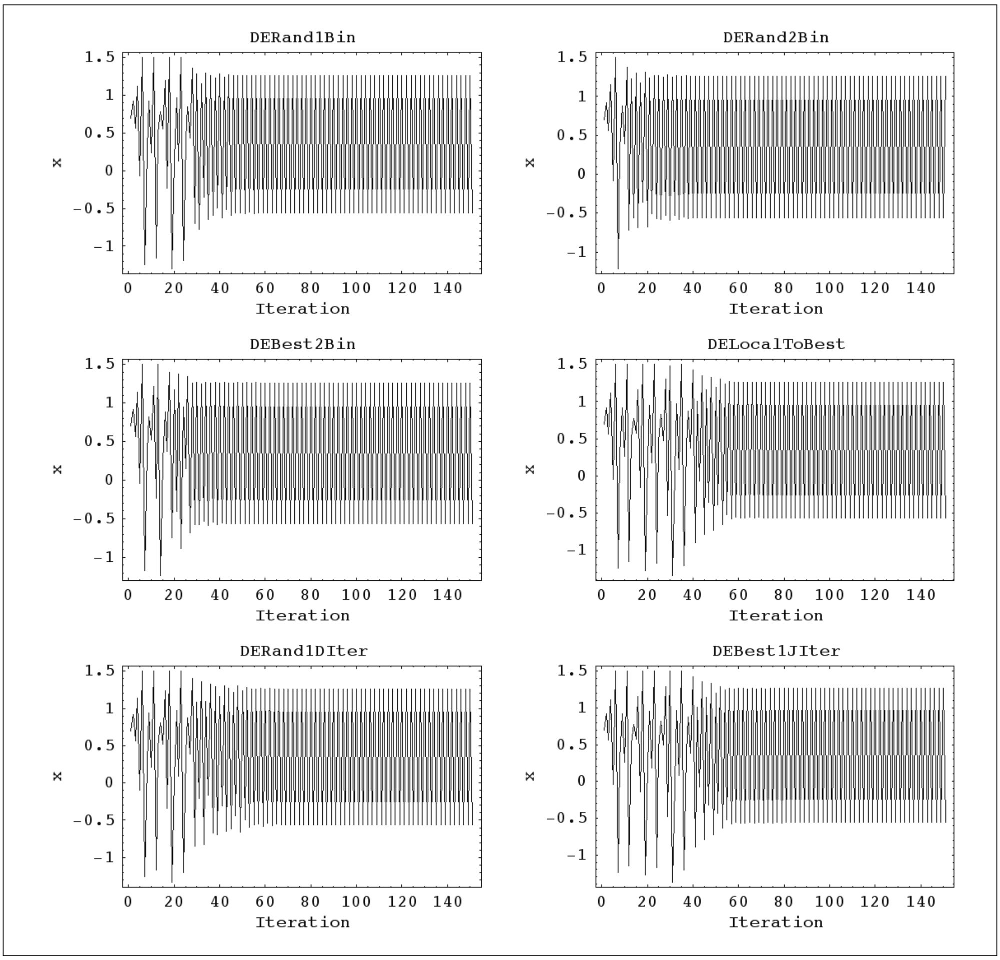
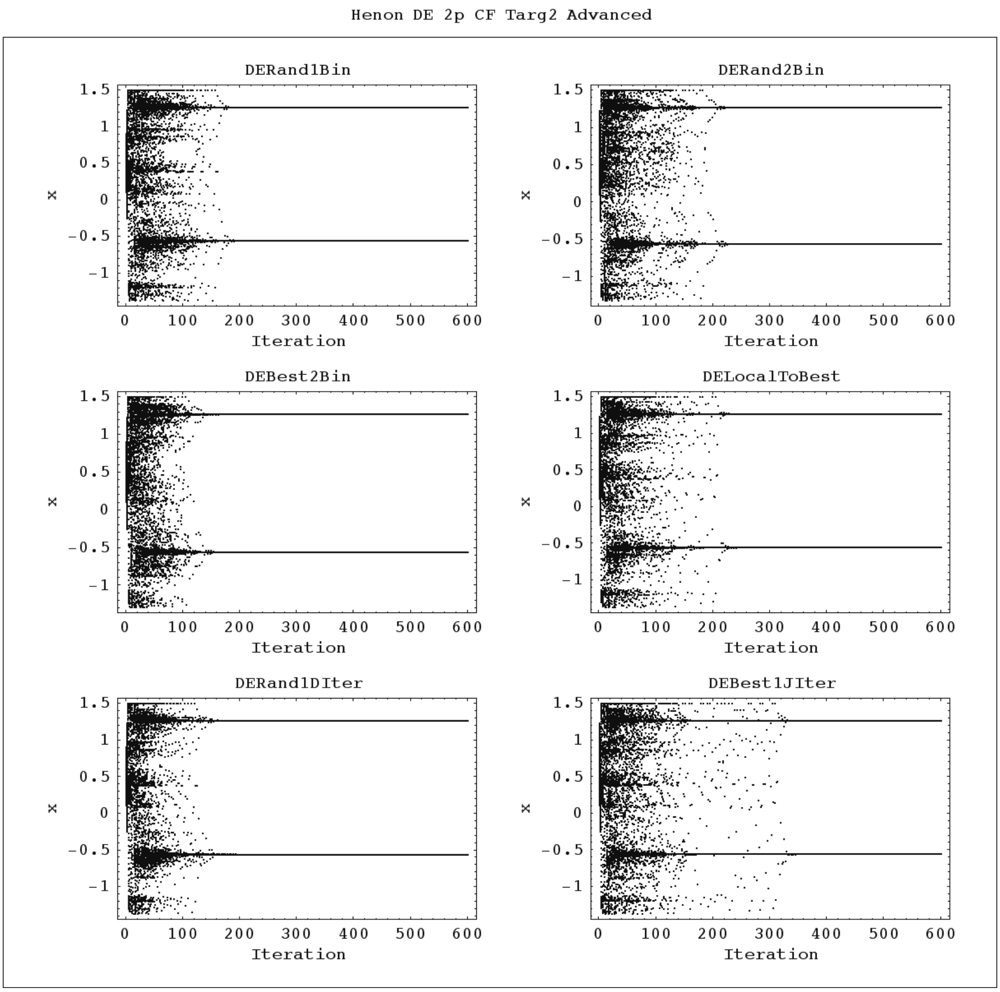
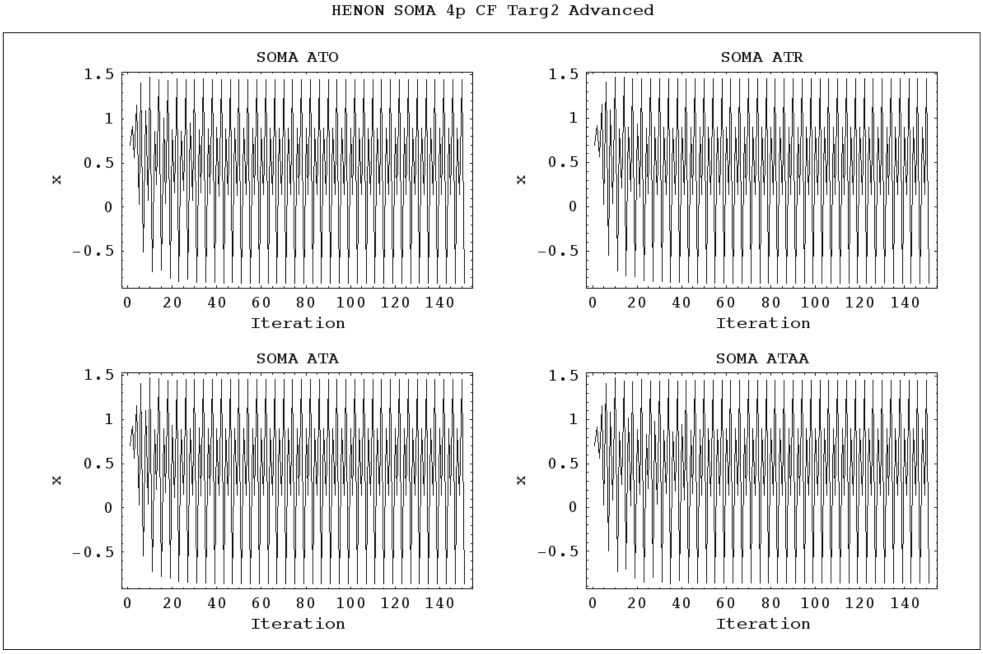


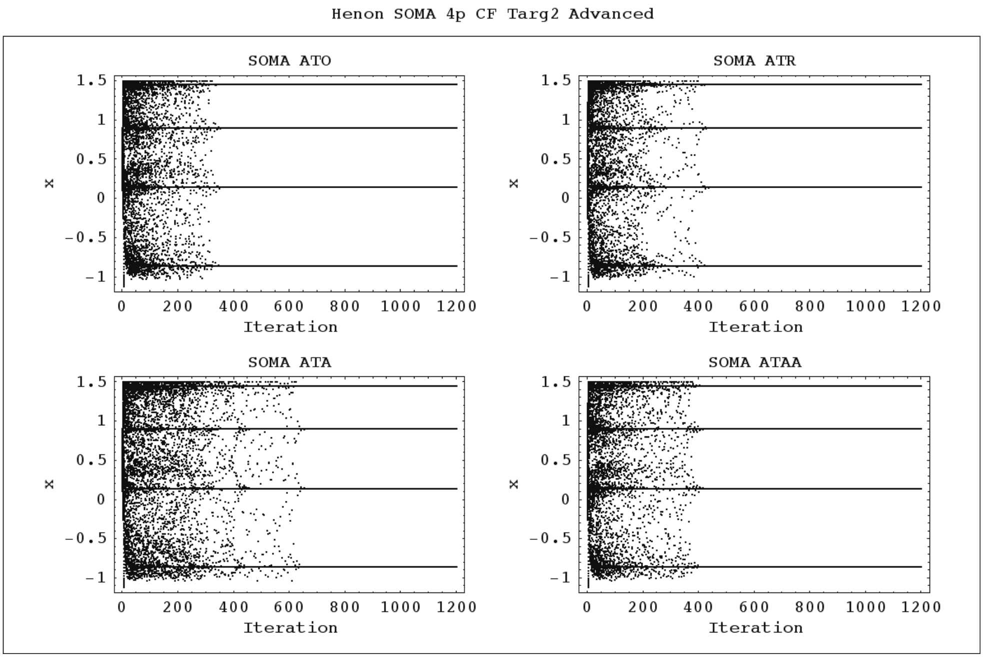
HENON DE 2p CF Targ2 Advanced



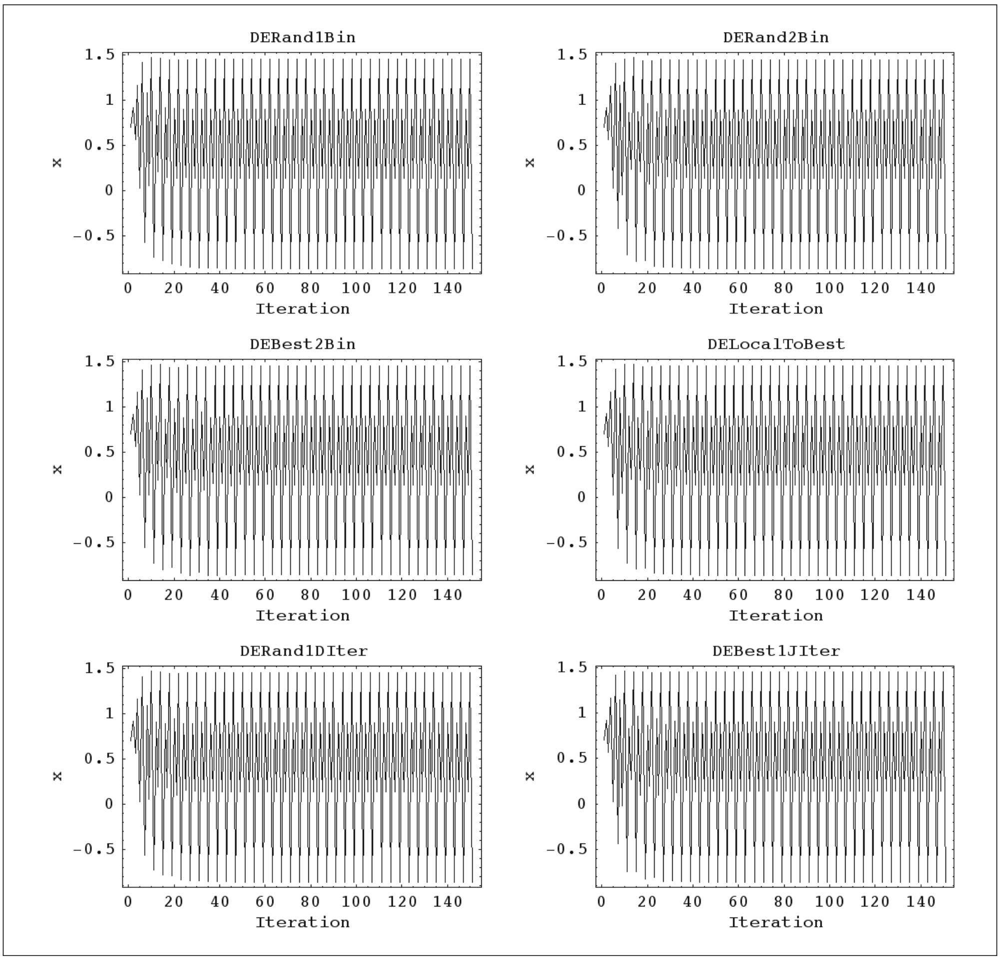


Henon DE 2p CF Targ2 Advanced
DERand1Bin
DERand2Bin
DEBest2Bin
DELocalToBest
DERand1DIter
DEBest1JIter
x
Iteration
1.5
1
0.5
0
-0.5
-1
0
100
200
300
400
500
600

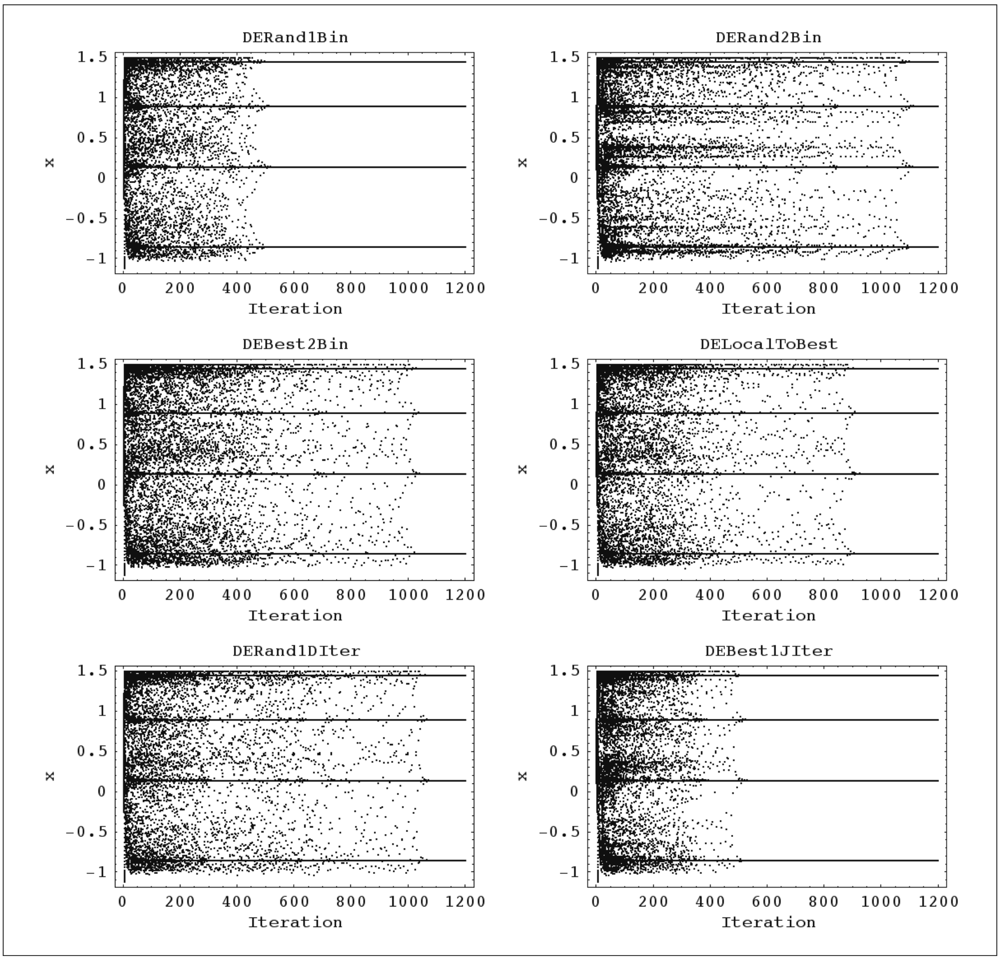

HENON SOMA 4p CF Targ2 Advanced
SOMA ATO
SOMA ATR
SOMA ATA
SOMA ATAA
x
Iteration
1.5
1
0.5
0
-0.5
0
20
40
60
80
100
120
140


Henon SOMA 4p CF Targ2 Advanced
SOMA ATO
SOMA ATR
SOMA ATA
SOMA ATAA
x
Iteration
1.5
1
0.5
0
-0.5
-1
0
200
400
600
800
1000
1200

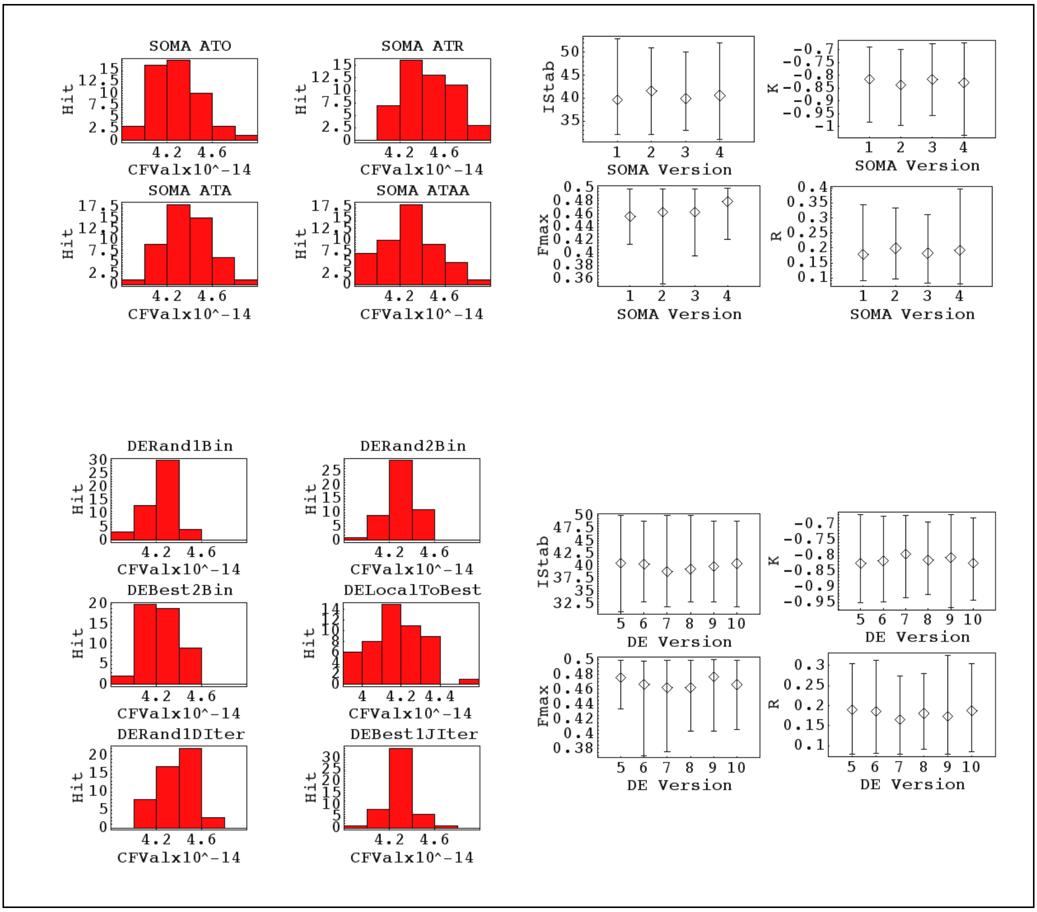
HENON DE 4p CF Targ2 Advanced



Henon DE 4p CF Targ2 Advanced



HENON SOMA & DE 1p CF Targ2 Advanced



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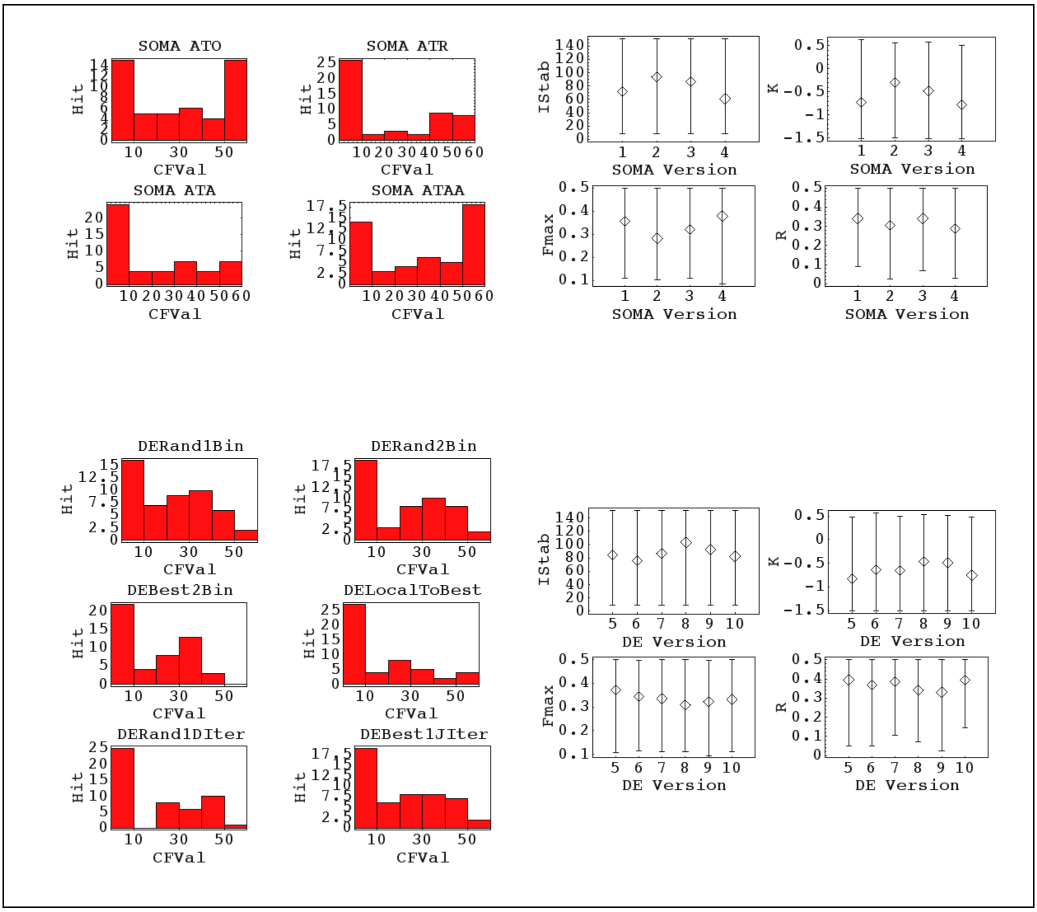
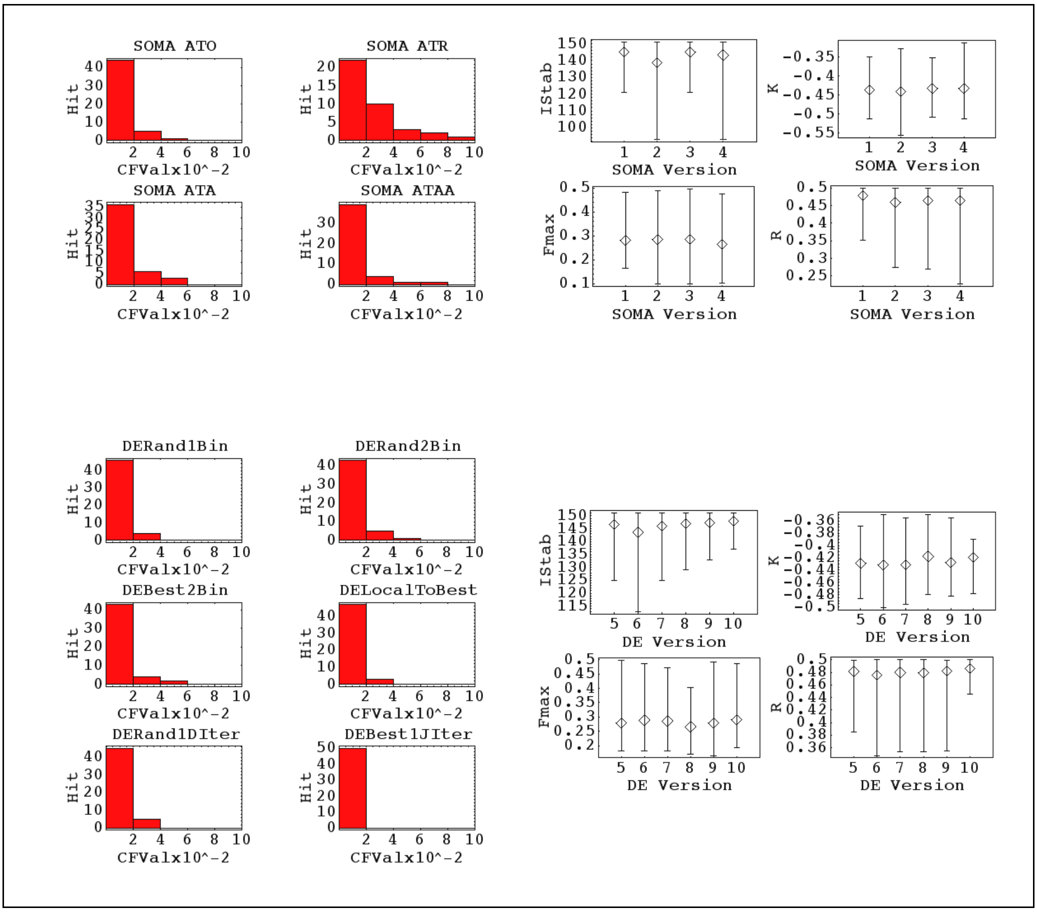


HENON SOMA & DE 4p CF Targ2 Advanced



LIST OF AUTHOR'S PUBLICATION ACTIVITIES

Journal articles

[1] Zelinka I., Senkerik R., Navratil E.,*Investigation on Evolutionary Optimitazion of Chaos Control*, CHAOS, SOLITONS & FRACTALS (2007), doi:10.1016/j.chaos.2007.07.045

Conference Papers

[1] Senkerik R., *Optimization and control of Chemical batch reactor by means of Evolutionary Algorithms,* Diploma thesis, FT, TBU Zlín, 2004

[2] Senkerik R., Zelinka I., *Optimization and Control of Batch Reactor by Evolutionary Algorithms*, ECMS 2005, In Proc. 19th European Conference on Modelling and Simulation 2005, Riga, Latvia, 1-4 June 2005, pages 59-65

[3] Senkerik R., Zelinka I., Navratil E., *Investigation on Evolutionary EDTAS Chaos Control*, ECMS 2006, In Proc. 20th European Conference on Modelling and Simulation 2006, Bonn, Germany, 28-31 May 2006, pages 507-512, ISBN 0-9553018-0-7

[4] Navratil E., Zelinka I., Senkerik R., *Preliminary Results of Deterministic Chaos Control Through Complexity Measures*, ECMS 2006, In Proc. 20th European Conference on Modelling and Simulation 2006, Bonn, Germany, 28-31 May 2006, pages 513-518,ISBN 0-9553018-0-7

[5] Senkerik R., Zelinka I., Navratil E., *Design of Cost function for Deterministic Chaos Control*, MENDEL 2006, In Proc. 12th International Conference on Soft Computing 2006, Brno, Czech Republic, 31 May – 2 June 2006, pages 73-78, ISBN 80-214-3195-4

[6] Navratil E., Zelinka I., Senkerik R., *Spatiotemporal Chaos Control Through Complexity Measures*, Mendel 2006, In Proc. 12th International Conference on Soft Computing, Brno, Czech Republic, 31 May - 2 June 2006, pages 68-72, ISBN 80-214-3195-4

[7] Senkerik R., Zelinka I., Navratil E., *Optimitazion of Feedback Control of Chaos by Evolutionary Algorithms*, CHAOS'06, In Proc. 1st IFAC Conference on Analysis and Control of Chaotic Systems, Reims, France, 28-30 June 2006, pages 97-102

[8] Zelinka I., Senkerik R., Navratil E., *Investigation on Real Time Deterministic Chaos Control by Means of Evolutionary Algorithms*, CHAOS'06, In Proc. 1st IFAC Conference on Analysis and Control of Chaotic Systems, Reims, France, 28-30 June 2006, pages 211-217

[9] Senkerik R., Zelinka I., *Optimitazion and Evolutionary Control of Chemical Reactor*, TMT 2006, In Proc. 10th International Research/Expert Conference, Barcelona – Lloret de Mar, Spain, 11-15 September 2006, pages 1171-1174, ISBN 995861730-7

[10] Senkerik R., Zelinka I., Navratil E., *Evolutionary Optimitazion of Chaos Control*, Znalosti 2007 – WETDAP 2007, In Proc. 1st Workshop onEvolutionary Techniques in Data-processing 2007, Ostrava, Czech Republic, February 2007, pages 60-71, ISBN 978-80-248-1332-5

[11] Navratil E., Zelinka I., Senkerik R., *Complexity Measures and Evolutionary Algorithms in Spatiotemporal Chaos Control*, Znalosti 2007 – WETDAP 2007, In Proc. 1st Workshop on Evolutionary Techniques in Data-processing 2007, Ostrava, Czech Republic, February 2007, pages 42-53, ISBN 978-80-248-1332-5

[12] Senkerik R., Zelinka I., Navratil E., *Design of Targeting Cost function for Evolutionary Optimization of Chaos Control*, ECMS 2007, In Proc. 21st European Conference on Modelling and Simulation 2007, Prague, Czech Republic, 4-6 June 2007, pages 345-350, ISBN 978-0-9553018-2-7

[13] Senkerik R., Zelinka I., Navratil E., *Cost function Design for Evolutionary Optimization of Chaos Control*, ECC'07, In Proc. 9th European Control Conference 2007, Kos, Greece, 2-5 July 2007, pages 1682-1687, ISBN 978-960-89028-5-5

[14] Senkerik R., Zelinka I., Navratil E.,*Optimitazion of Chaos Control by means of Evolutionary Algorithms*, DEXA 2007 – ETID 2007, In Proc. 1st International Workshop on Evolutionary Techniques in Data-processing 2007, Regensburg, Germany, 3-7 September 2007, pages 163-167, ISBN 0-7695-2932-1

[15] Senkerik R., Zelinka I., Davendra D., *Comparison of Evolutionary Algorithms in the Task of Chaos Control Optimization*, CEC'07, In Proc. IEEE Congres on Evolutionary Computation 2007, Singapore, Singapore, 25-28 September 2007, pages 3952-3958, ISBN 1-4244-1340-0

CURRICULUM VITAE

PERSONAL INFORMATION		
	Name	Roman Šenkeřík
	Date of birth	28 July 1981
	Present address	Tř.T.Bati 813 760 01 Zlín
	Marital status	Single
	Contact	phone: +420 777 621 792, email: senkerik@fai.utb.cz
EDUCATION		
	1995 – 1999	Technical High School in Zlin, specialization: Technical lyceum GCE in Czech language, Mathematics, Physics, Programming
	1999 – 2004	TBU in Zlín, Faculty of technology, master degree program Chemical and process engineering, specialization Automation and control technology in consumer goods industry
	June 2004	State exam and defence of diploma thesis: Optimization and control of Chemical batch reactor by means of Evolutionary Algorithms
	September 2004 – January 2007	Student of a doctoral program in Technical Cybernetics in full-time study
	Since February 2007	Further study in the doctoral program as a part-time student

SCHOLARSHIPS AND ATTENDENCE AT CONFERENCES		
	March – May 2005	Scholarship under the program ERASMUS at The Strathclyde University, Department of Elecrical and Electronic Engineering, Industry Control Center, Glasgow, Scotland.
	2006	Attendence at international conference: ECMS 2006 in Bonn, Germany Mendel'06 in Brno, Czech Republic IFAC Chaos´06 in Reims, France TMT 2006 in Lloret de Mar, Spain
	2007	Attendence at international conference: Znalosti 2007 in Ostrava, Czech Republic ECMS 2007 in Prague, Czech Republic IFAC-ECC 2007 in Kos, Greece DEXA - ETID 2007 in Regensburg, Germany IEEE-CEC 2007 in Singapore, Singapore

MEMBERSHIP	
	IPC member of ECMS 2006 in Bonn, Germany
	IPC member of ECMS 2007 in Prague, Czech Republic
	Section chair for IFAC - ECC 2007, Kos, Greece

LANGUAGE KNOWLEDGE		
	Czech language	native
	English language	active – doctoral exam – June 2006
	German language	basic

EMPLOYMENT		
	Since Feb. 1, 2007	Lecturer at TBU in Zlín, Faculty of applied informatics Seminars and laboratories – Applied informatics, Artificial intelligence (neural networks, evolutionary algorithms,), Basics of informatics (information theory, software – mathematica), Programming in VHDL language (FPGA chips), Database systems (SQL language, MS Access)
OTHER ACTIVITIES		
	2004 – 2007	Supervising of several bachelor and master theses.